

Currently known issues of MAD-X tracking (with $\delta \neq 0$)



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Tracking with MAD-X - Why talk about it?

X: ==> TRACK,DELTAP=double,ONEPASS,DAMP,QUANTUM,DUMP,ONETABLE,FILE;
performs element per element tracking of particle trajectories either for single passage (option `onepass` in the command), or for many turns (default option).
Only thin elements are allowed. Any lattice can be converted into a "thin element" lattice by invoking the `makethin` command.

- AK new responsible for the tracking module, since A.Verdier retires.
→ revision of code might be helpful
- MAD(X) is a historically grown program.
→ revision of code might be advisable
- Doubts about correct implementation of tracking off-momentum particles ($\delta \neq 0$) exist since years.
→ Check of code needed, definite answers desired
- Limitations of tracking with MAD-X?

Revision of theory and code is in progress. This is a summary of known facts so far.

Tracking with MAD-X - The basis



$$H(x, y, \sigma, p_x, p_y, p_\sigma; s) = p_\sigma - (1 + \delta) \cdot [1 + K_x x + K_y y] \cdot \left\{ 1 - \frac{\left(p_x - \frac{e}{p_0 \cdot c} A_x\right)^2 + \left(p_y - \frac{e}{p_0 \cdot c} A_y\right)^2}{(1 + \delta)^2} \right\}^{1/2} - [1 + K_x x + K_y y] \cdot \frac{e}{p_0 \cdot c} A_s \quad [*]$$

Early versions of MAD: Solutions of equations of motion derived from above Hamiltonian with *expanded* square-root term!

→ **Approximation**

Today: Formulae from *unexpanded* Hamiltonian.

→ **Exact solution of equations of motion**, unfortunately not everywhere yet

Thin Lenses: In thin lens tracking the only "long" element is the drift. Any other element (multipole) has zero length, hence a thick element is always replaced by (drift + thin element).

$$H_{ThickLens} = H_{Drift} + \hat{H}_{ThinLens} \cdot \Delta s \cdot \delta(s - s_0)$$

→ **Symplecticity conserved**

[*] **G.Ripken**, *A symplectic six-dimensional thin-lens formalism for tracking*, DESY 95-063, (1995)



Tracking with MAD-X - Coordinate systems

Standard coordinates (Trace space coordinates)

→ Plausible and easy to visualize, used very often. However: Not canonical pairs and not symplectic.

$$(x, x'), \quad (y, y'), \quad (\Delta s, \frac{\Delta p}{p_0})$$

Canonical coordinates The tracking module uses these variables exclusively!

→ Solution of equations of motion easier, symplectic.

$$(x, p_x), \quad (y, p_y), \quad (\sigma = s - v_0 t, p_\sigma) \quad [\text{Ripken}]$$

$$(x, p_x), \quad (y, p_y), \quad (T = -c\Delta t, p_T = \frac{\Delta E}{p_0 c}) \quad [\text{MAD}]$$

$$\text{where } p_\sigma = \frac{p_T}{\beta} \quad \text{and} \quad \sigma = \beta \cdot T$$

And to complicate things further, when $\delta \neq 0$ is used, the momenta are rescaled:

$$(x, \frac{p_x}{1 + \delta}), \quad (y, \frac{p_y}{1 + \delta}), \quad (-c\Delta t, \frac{\Delta E}{p_0(1 + \delta) c}) \quad [\text{MAD}]$$

(Action-Angle coordinates)

Tracking with MAD-X - Tracking scenarios (concerning coordinate systems)

4D $x, p_x, y, p_y \neq 0$ and $T, p_T = 0, \delta = 0$

In this case people usually assume $x' \approx p_x, y' \approx p_y$, therefore tracking should be ok even for standard coordinates. The approximation is however strictly speaking only acceptable for high-energy machines (see later).

5D $x, p_x, y, p_y \neq 0$ and $T, p_T = 0$, **but** $\delta \neq 0$

This is used to compute trajectories around an off-momentum closed orbit. Actually this set of start values creates the most confusion and **is not consistently treated in the tracking module currently (historical approximations are used)**.

6D $x, p_x, y, p_y \neq 0$ and $T, p_T \neq 0$ **and** $\delta = 0$

This is in principle the better choice (if any momentum offset is absorbed into the p_T coordinate), however as the current MAD-X implementation still uses some approximations (in the thin dipole and the cavity), results are not exact. Without specifying δ , the user does not get the trajectories around the off-momentum closed orbit, but the reference orbit.

(7D) $x, p_x, y, p_y, T, p_T, \delta \neq 0$

Ultimate goal? Not working at all yet!

Tracking with MADX - Transformation rules

Standard \rightarrow Canonical

$$p_x = \frac{x' (1 + \delta)}{\sqrt{1 + x'^2 + y'^2}}$$

$$p_y = \frac{y' (1 + \delta)}{\sqrt{1 + x'^2 + y'^2}}$$

$$p_T = \sqrt{(1 + \delta)^2 + \frac{1}{\beta^2}} - 1 - \frac{1}{\beta}$$

There is currently a subtlety about this transformation. When using $\delta \neq 0$, MAD adjust the relativistic beam parameters to those of the (new) off-momentum (reference) particle according to:

$$\eta_s = \beta_0 \cdot \gamma_0 \cdot (1 + \delta)$$

$$\gamma_s = \sqrt{1 + \eta_s^2}$$

$$\beta_s = \frac{\eta_s}{\gamma_s} = \frac{\beta_0 \cdot \gamma_0 \cdot (1 + \delta)}{\sqrt{1 + (\beta_0 \cdot \gamma_0 \cdot (1 + \delta))^2}}$$

Tracking with MADX - Transformation rules

Canonical \rightarrow Standard

$$x' = \frac{p_x}{\sqrt{(1 + \delta)^2 - p_x^2 + p_y^2}}$$

$$y' = \frac{p_y}{\sqrt{(1 + \delta)^2 - p_x^2 + p_y^2}}$$

$$\delta = \sqrt{1 + \frac{2p_T}{\beta_0} + p_T^2} - 1$$

From these formulae one can see, that the approximations

$$x' \approx p_x \quad \text{and} \quad y' \approx p_y$$

are only justified for high-energy machines and provided that δ is very small. Currently MAD-X uses at some points the approximation:

$$x' \approx \frac{p_x}{1 + \delta} \quad \text{and} \quad y' \approx \frac{p_y}{1 + \delta}$$

Tracking with MADX - Summary

- Revision of tracking module code (and also twiss) still in progress.
- Inconsistencies have been found and will be fixed. More tests have to be done however first.
- Be aware that the **tracking module requires canonical variables!**
- Tracking with $\delta \neq 0$ indeed is not entirely correct implemented in the tracking module. Actually the thin dipole and the cavity have to be revised more properly. Using the transformation rules given rectifies some problems.