Consequences of phase advance differences between IPs on beam-beam effects

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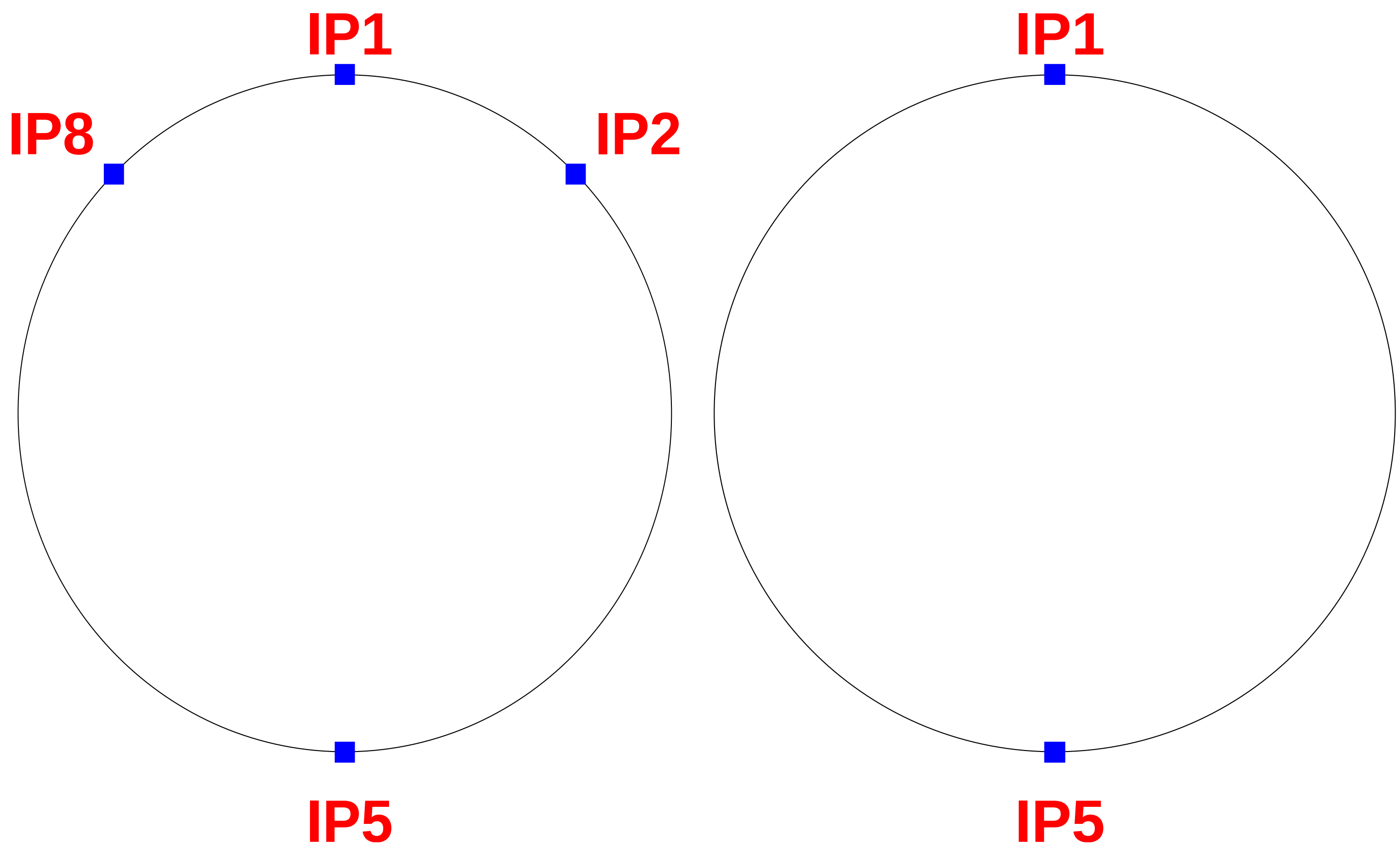
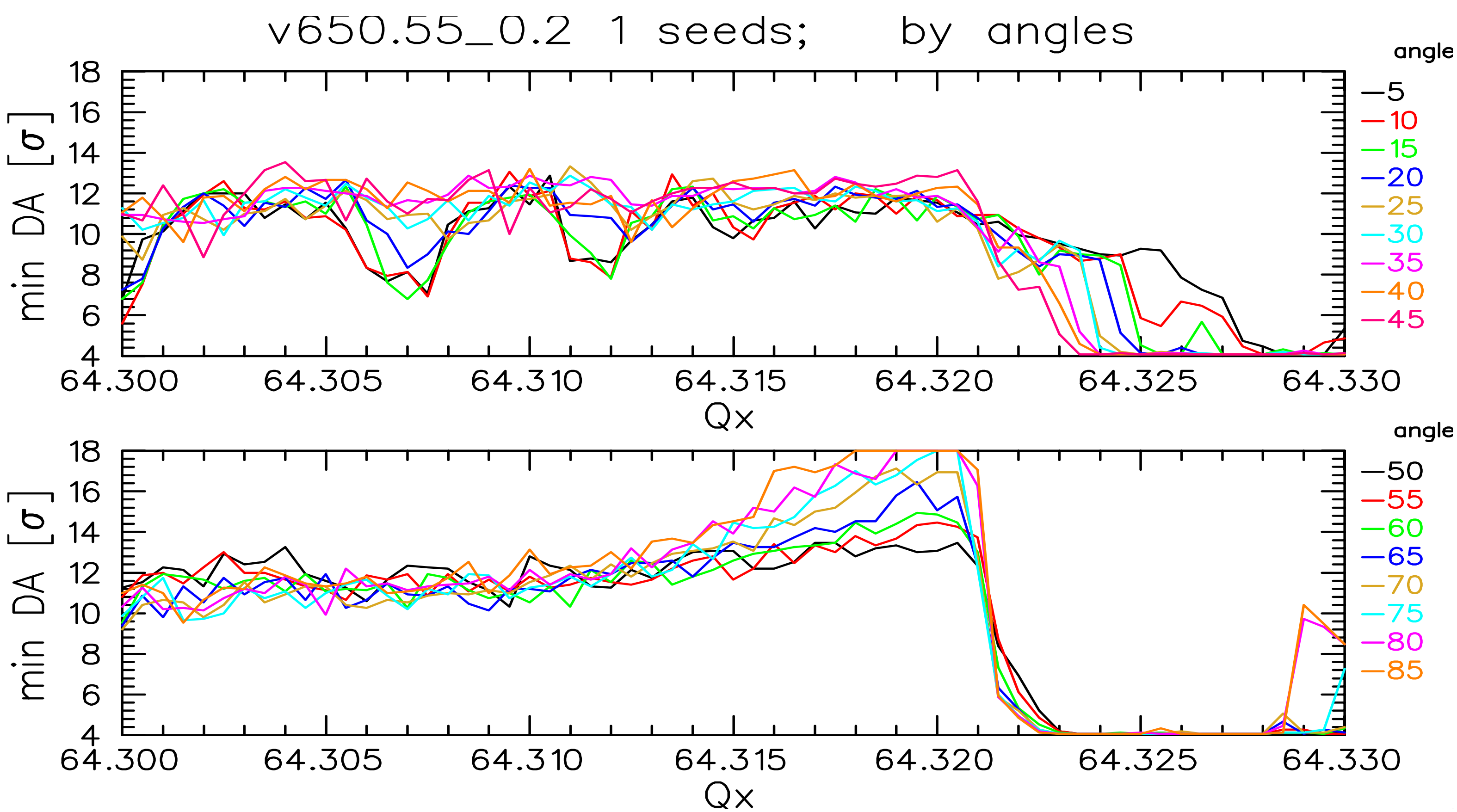
Effect of phase advance between beam-beam interactions

- **Possible good effect: suppression of some resonances**
- **Possible bad effect: many (resonances, orbit, ...)**
- **For demonstration simplified model:**
  - **one dimension**
  - **treat head-on interactions only in the first step**
  - **first order in beam-beam strength ξ**
  - **no other non-linearities**

Standard and simplified collision scheme



Tune scan results (from 2006)



Tune scan results (from 2006)

- **In horizontal tune scan: two bad regions visible around working point**
- **Dynamic aperture reduced by up to 5** σ
- **Questions:**
  - **Which resonances ?** → **4/13 and 5/16**
  - **Can they be compensated by adjusting the phase advance ?**

Start with single IP

- Interaction point at beginning (end) of the ring (very local interactions, δ-functions)

"Classic" (B.C.) approach:

- s-dependent Hamiltonian and perturbation theory:

$$\mathcal{H} \;=\; .... + \delta(s)\epsilon V$$

- Disadvantages:
  - for many IPs endless mathematics
  - can lead to stupid conclusions (e.g. 4th order resonance cannot be driven by sextupoles)

→ conceptually and computationally easier method

Effect on invariants - start with single IP

Look for invariants h, (see e.g. Chao[1)]), and evaluate for different number of interactions and phase advance.

Very well suited for local distortions (e.g. beam-beam kick)

Linear transfer $e^{:f_2:}$ and beam-beam interaction $e^{:F:}$, i.e.:

$$e^{:f_2:} \cdot e^{:F:} = e^{:h:}$$

with

$$f_2 = -\frac{\mu}{2}\left(\frac{x^2}{\beta} + \beta p_x^2\right)$$

and

$$F = \int_0^x dx' f(x')$$

1) A. Chao, Lecture Notes on Topics in Accelerator Physics, 2001

Effect on invariants

using for a Gaussian beam f(x):

$$f(x) = \frac{2Nr_0}{\gamma x}(1 - e^{\frac{-x^2}{2\sigma^2}})$$

as usual go to action angle variables Φ, A:

$$x = \sqrt{2A\beta}\sin\Phi, \qquad p = \sqrt{\frac{2A}{\beta}}\cos\Phi$$

and write F(x) as Fourier series:

$$F(x) = \sum_{n=-\infty}^{\infty} c_n(A)e^{in\Phi}$$

We need:

REMEMBER: with this transform:

$$f_2 = -\mu A$$

and useful properties of Lie operators (any textbook[2]):

$$: f_2 : g(A) = 0, \qquad : f_2 : e^{in\Phi} = in\mu e^{in\Phi}, \qquad g(: f_2 :)e^{in\Phi} = g(in\mu)e^{in\Phi}$$

and the formula (any textbook[2]):

$$e^{:f_2:}\ e^{:F:} \ = \ e^{:h:} \ = \ \exp\left[: f_2 + \left(\frac{: f_2 :}{1 - e^{-:f_2:}}\right) F + \mathcal{O}(F^2) :\right]$$

[2] E. Forest, "Beam Dynamics, A New Attitude and Framework", 1998

Single IP

gives immediately for h:

$$h = -\mu A + \sum_n c_n(A) \frac{in\mu}{1 - e^{-in\mu}} e^{in\Phi}$$

$$h = -\mu A + \sum_n c_n(A) \frac{n\mu}{2\sin(\frac{n\mu}{2})} e^{(in\Phi + i\frac{n\mu}{2})}$$

away from resonance normal form transformation gives:

$$h \ = \ - \mu A + c_0(A) \ = const.$$

$$\left[homework: \qquad \frac{dc_0(A)}{dA} \right]$$

Single IP - analysis of h

$$h = -\mu A + \sum_n c_n(A) \frac{n\mu}{2\sin(\frac{n\mu}{2})} e^{(in\Phi + i\frac{n\mu}{2})}$$

On resonance:

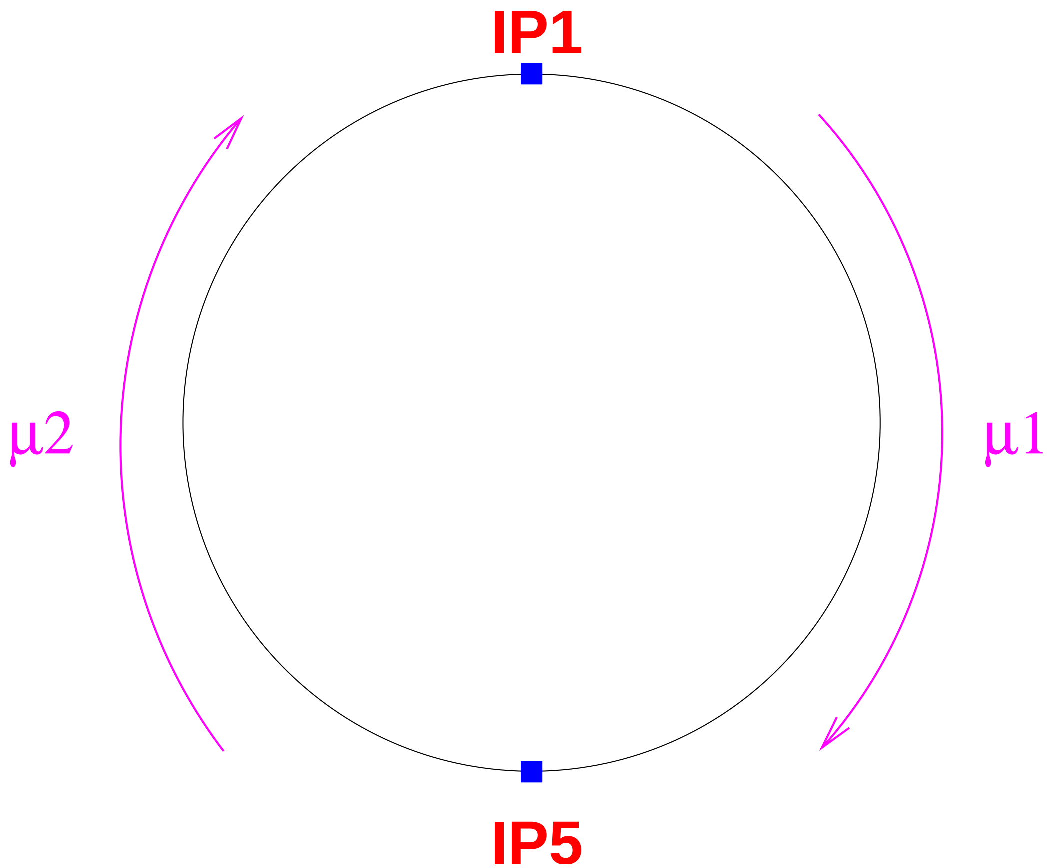
$$Q = \frac{p}{n} = \frac{\mu}{2\pi}$$

with $c_n \neq 0$**:**

$$\sin(\frac{n\pi p}{n}) = \sin(p\pi) \equiv 0 \qquad \forall \quad integer \quad p$$

and h **diverges**

Collision scheme - two IPs



$$\mu = \mu 1 + \mu 2$$

Two IPs

➡ **two transfers f_2^1, f_2^2 and two beam-beam kicks F^1, F^2, first IP at μ_1, second IP at μ:**

$$= \quad e^{:f_2^1:}\ e^{:F^1:}\ e^{:f_2^2:}\ e^{:F^2:} \ = \ e^{:h_2:}$$

$$= \quad e^{:f_2^1:}\ e^{:F^1:}\ e^{-:f_2^1:}\ e^{:f_2^1:}\ e^{:f_2^2:}\ e^{:F^2:} \ = \ e^{:h_2:}$$

$$= \quad e^{:f_2^1:}\ e^{:F^1:}\ e^{-:f_2^1:}\ e^{:f_2:}\ e^{:F^2:}\ e^{-:f_2:}\ e^{:f_2:} = \ e^{:h_2:}$$

$$= \quad e^{:e^{-:f_2^1:}F^1:}\ e^{:e^{-:f_2:}F^2:}\ e^{:f_2:} = \ e^{:h_2:}$$

$$f_2 \ = \ -\mu A, \quad f_2^1 \ = \ -\mu_1 A, \quad and \quad f_2^2 \ = \ -\mu_2 A$$

Two IPs

here a miracle occurs (remember $g(:f_2:)e^{in\Phi} = g(in\mu)e^{in\Phi}$**):**

$$e^{:f_2^1:}e^{in\Phi} \;=\; e^{in\mu_1}e^{in\Phi} \;=\; e^{in(\mu_1+\Phi)}$$

i.e. the Lie transforms of the perturbations are phase shifted[2]**. Therefore:**

$$e^{:e^{-:f_2^1:}F^1:}\;e^{:e^{-:f_2:}F^2:}\;e^{:f_2:} = \;e^{:h_2:}$$

becomes simpler with substitutions of $\Phi_1 \;=\; \Phi+\mu_1$ **and** $\Phi \;=\; \Phi+\mu$ **in** F^1 **and** F**:**

$$e^{:F^1(\Phi_1):}e^{:F(\Phi):}e^{:f_2:} \quad\Rightarrow\quad e^{:F^1(\Phi_1)+F(\Phi):}e^{:f_2:}$$

2) E. Forest, "Beam Dynamics, A New Attitude and Framework", 1998

Two IPs

gives for h_2:

$$h_2 = -\mu A + \sum_{n=-\infty}^{\infty} \frac{n\mu c_n(A)}{2\sin(n\frac{\mu}{2})} e^{-in(\Phi+\mu/2+\mu_1)} + e^{-in(\Phi+\mu/2)}$$

$$h_2 = -\mu A + 2c_0(A) + \underbrace{\sum_{n=1}^{\infty} \frac{2n\mu c_n(A)}{2\sin(n\frac{\mu}{2})} \cos(n(\Phi + \frac{\mu}{2} + \frac{\mu_1}{2}))\cos(n\frac{\mu_1}{2})}_{\text{interesting part}}$$

Nota bene, because of:

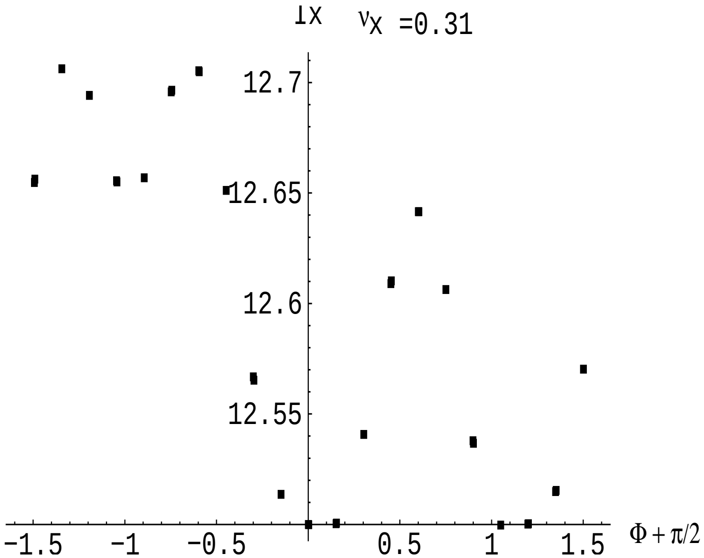
$$e^{:F(\Phi):} e^{:f_2:} \;\rightarrow\; e^{:F^1(\Phi_1)+F(\Phi):} e^{:f_2:}$$

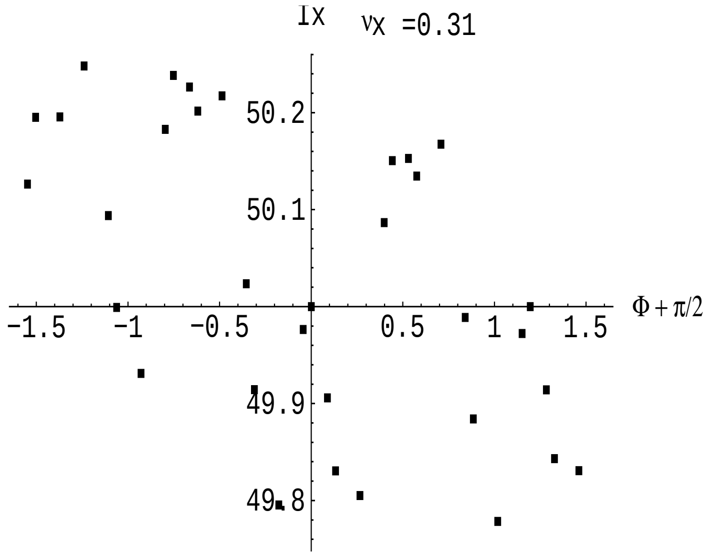
can be generalized to more interaction points ...

Invariant versus tracking

- **Is it useful what we obtained ?**
  - **Debug and compare ("benchmark")**
- **Compare to very simple tracking program:**
  - **linear transfer between interactions**
  - **beam-beam kick for round beam**
  - **compute action** $I = \frac{\beta^*}{2\sigma^2}(\frac{x^2}{\beta^*} + p_x^2\beta^*)$
  - **and phase** $\Phi = \arctan(\frac{p_x}{x})$
  - **compare** I **with** h

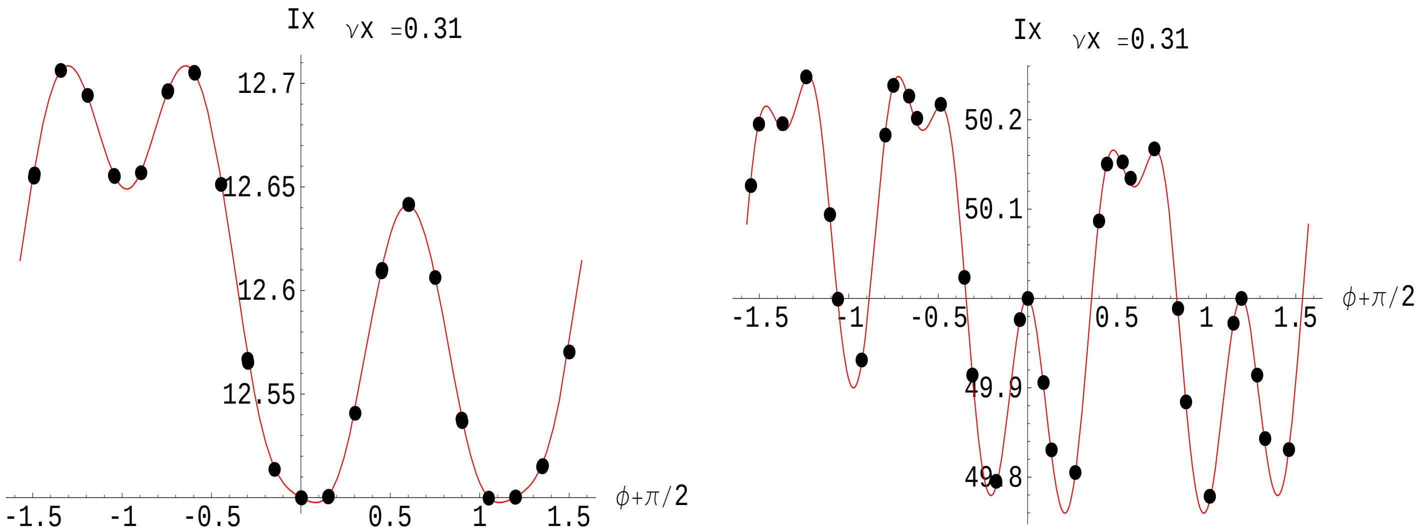
Invariant from tracking: one IP





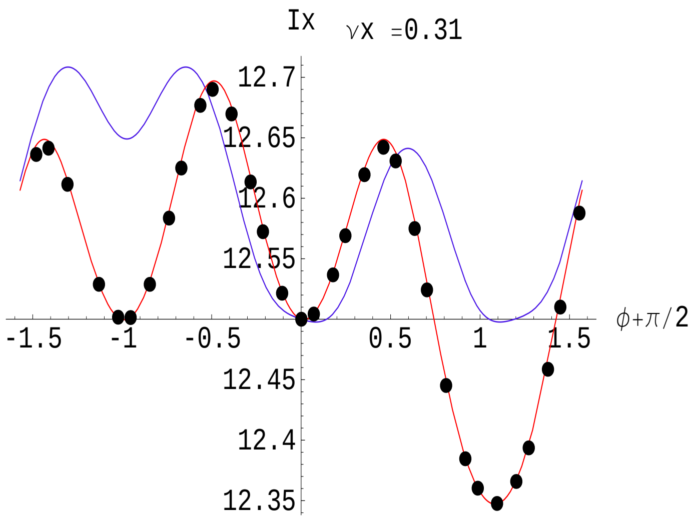
➡ **Shown for $5\sigma_x$ and $10\sigma_x$**

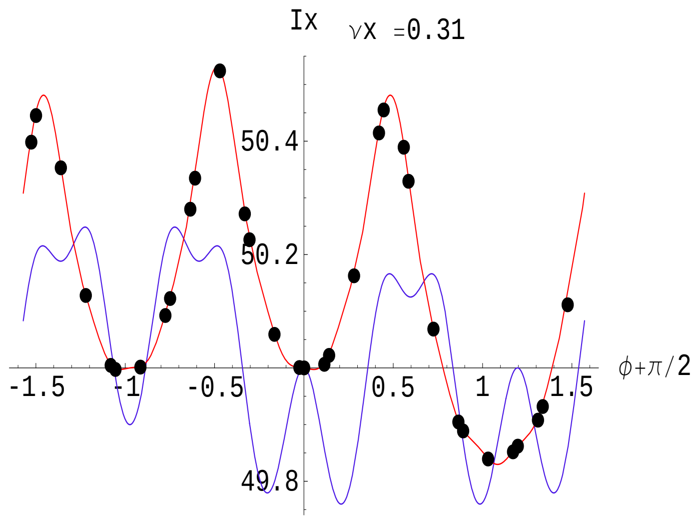
Invariant versus tracking: one IP



➡ **Shown for $5\sigma_x$ and $10\sigma_x$**

Invariant versus tracking: two IPs





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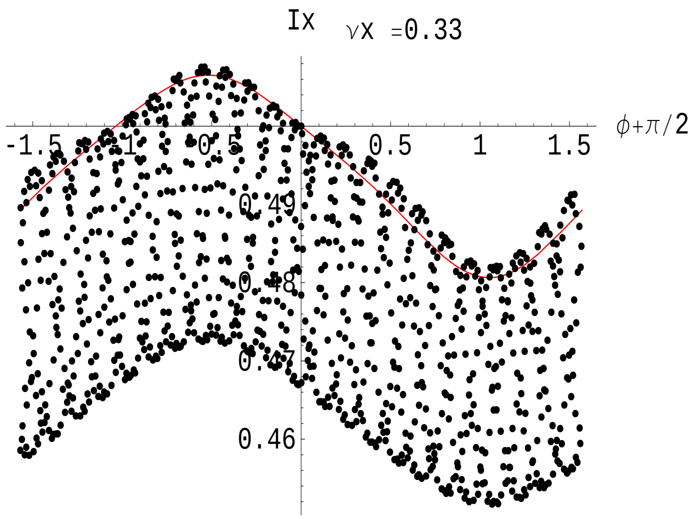
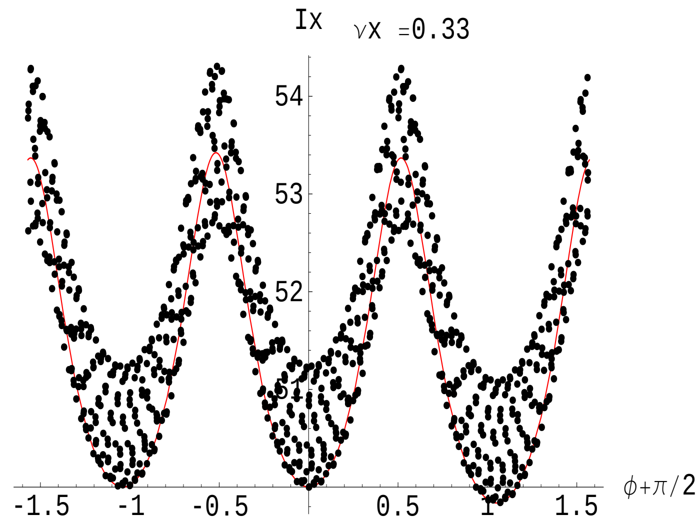
Invariant versus tracking: two IPs





➡ **one resonance: $Q_x = 0.33$ .....**

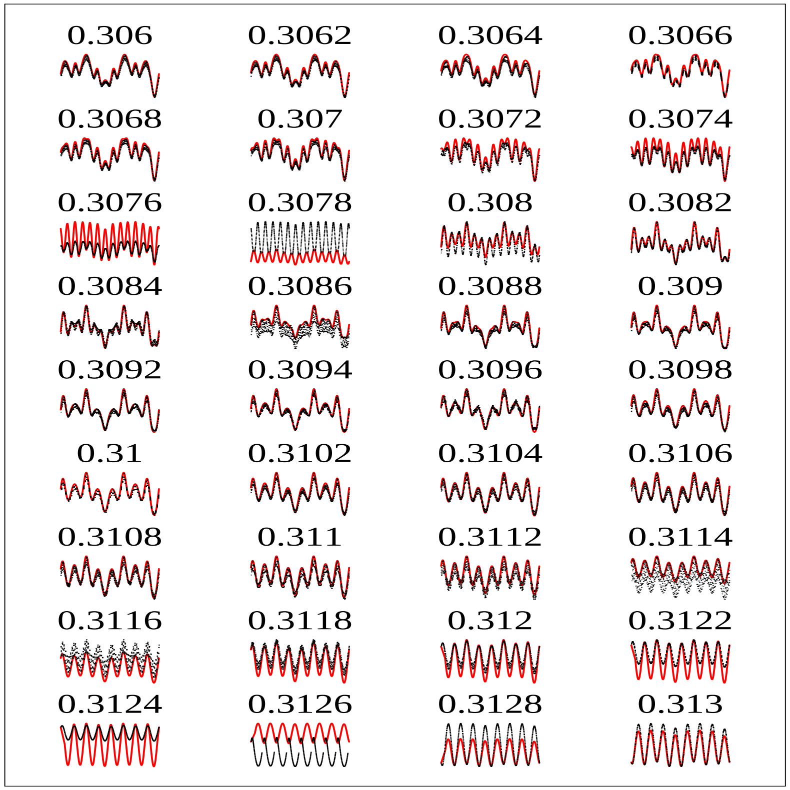
Two IPs - analysis of h_2

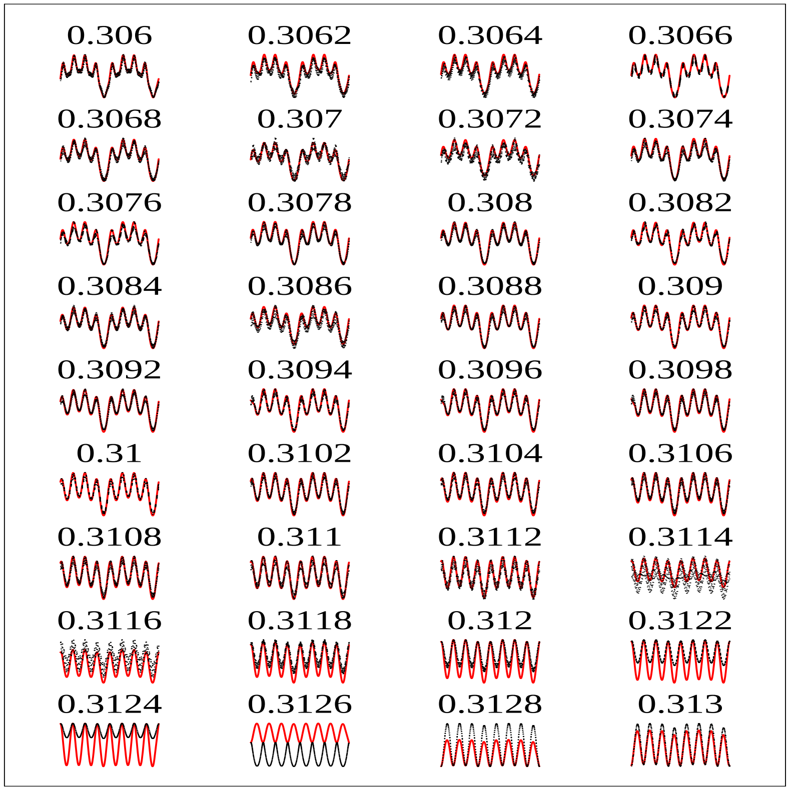
$$h_2 \approx \sum_n 2n\mu c_n(A) \;\cdot\; \frac{\cos(n\frac{\mu_1}{2})}{2\sin(n\frac{\mu}{2})} \;\cdot\; \cos(n(\Phi + \frac{\mu}{2} + \frac{\mu_1}{2}))$$

- **For phase advance $\mu_1 = \frac{\pi}{2} \cdot k$ and n divisible by 4: resonant**
- **For $n = 2\cdot$ (odd integer) finite**

→ **Q = 4/13 can be cancelled, Q = 5/16 can not**

Invariant versus tracking: two IPs





- **For $\mu_1 = 0.2604$ and $\mu_1 = 0.2500$**

Two IPs - analysis of h_2

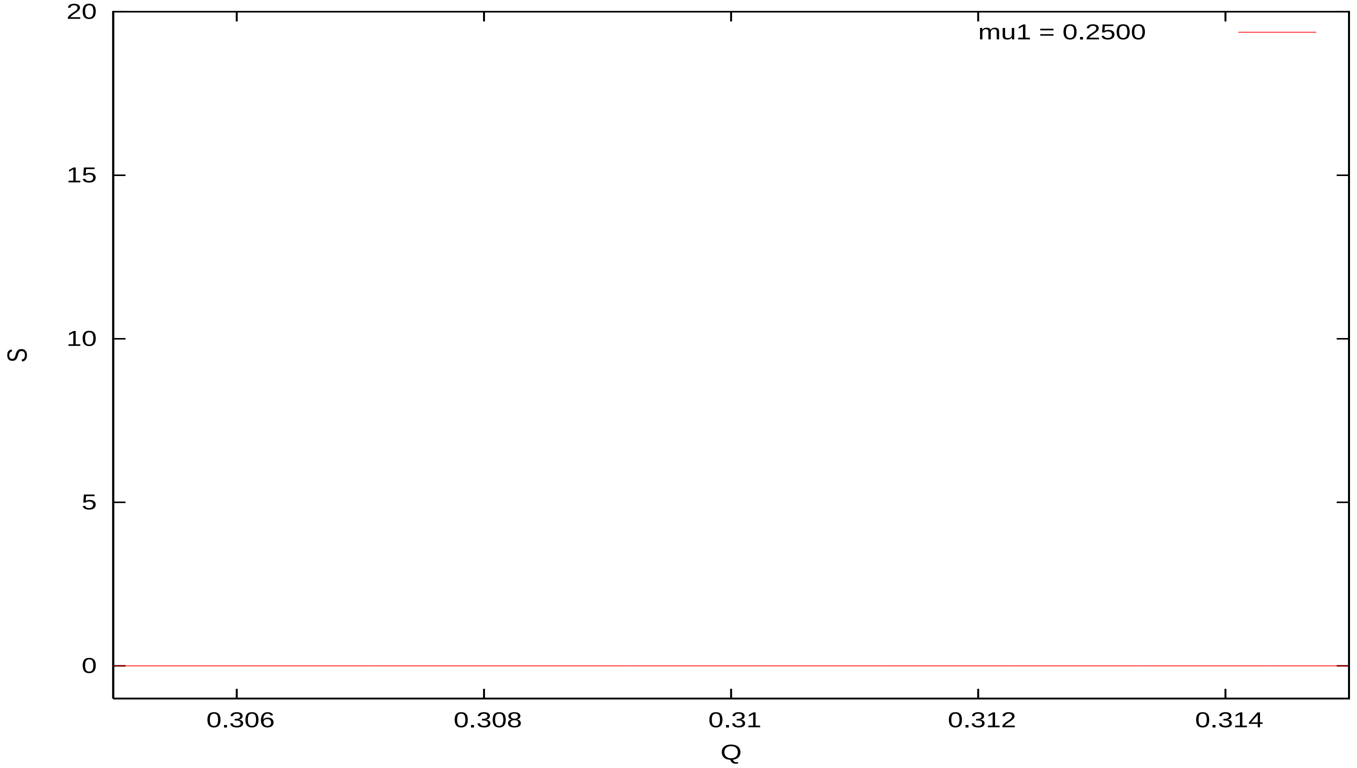
$$h_2 \approx \sum_n 2n\mu c_n(A) \;\cdot\; \frac{\cos(n\frac{\mu_1}{2})}{2\sin(n\frac{\mu}{2})} \;\cdot\; \cos(n(\Phi + \frac{\mu}{2} + \frac{\mu_1}{2}))$$

- **For phase advance $\mu_1 = \frac{\pi}{2} \cdot k$ and n divisible by 4: resonant**
- **For $n = 2\cdot$ (odd integer) finite**

→ **Q = 4/13 can be cancelled, Q = 5/16 can not**

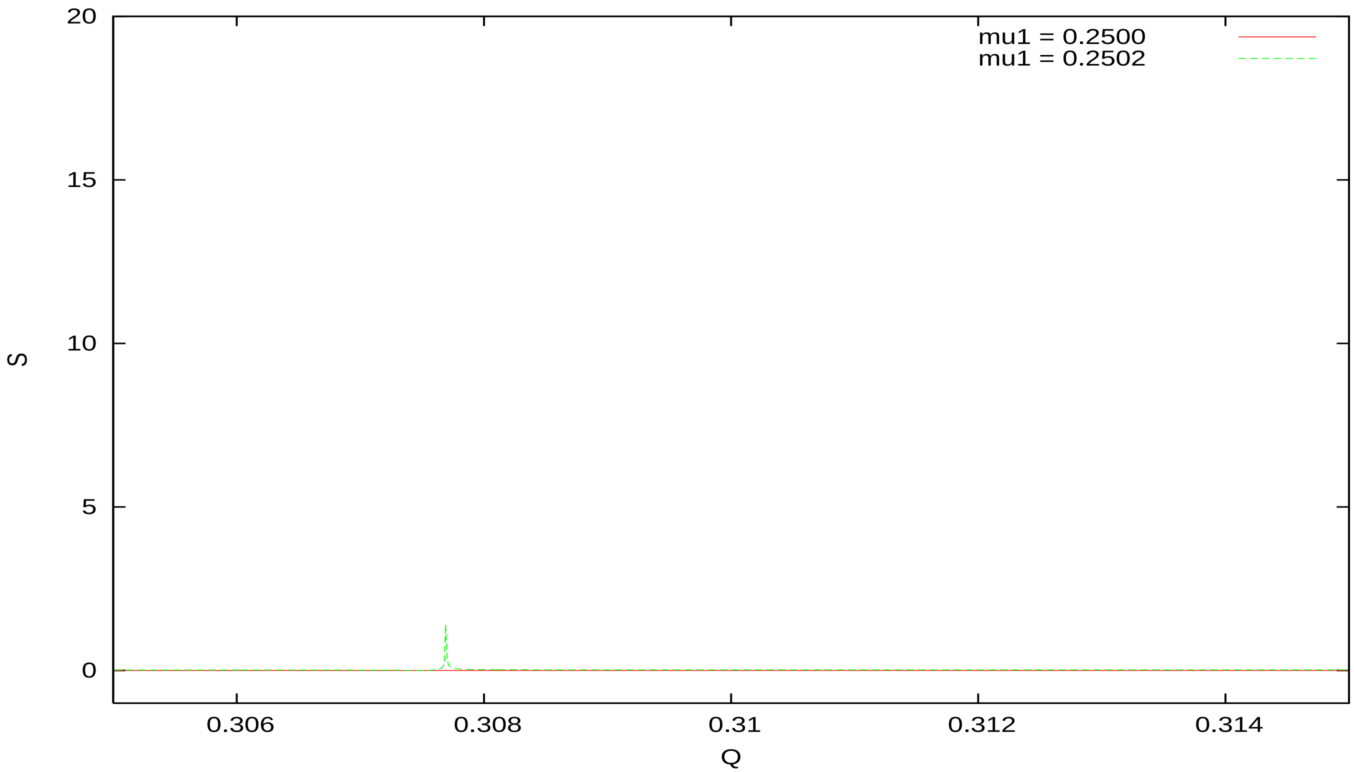
- **Which precision is needed for $\mu_1 = \frac{\pi}{2}$?**
- **Unfortunately LHC has many interaction points**

Sensitivity: $h_2(\mu)$, 13th order



- **Perfect cancelation for** $\mu_1 = \frac{\pi}{2}$

Sensitivity: $h_2(\mu)$, 13th order



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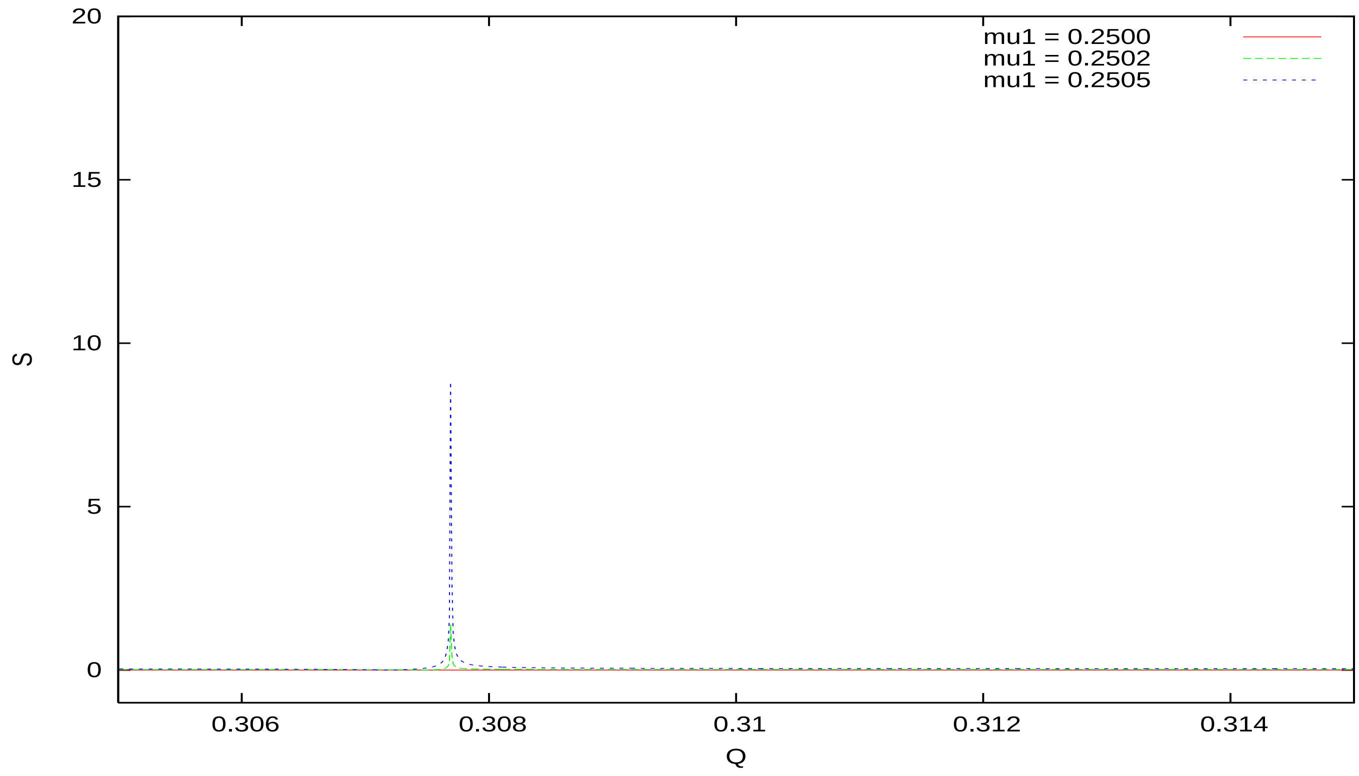

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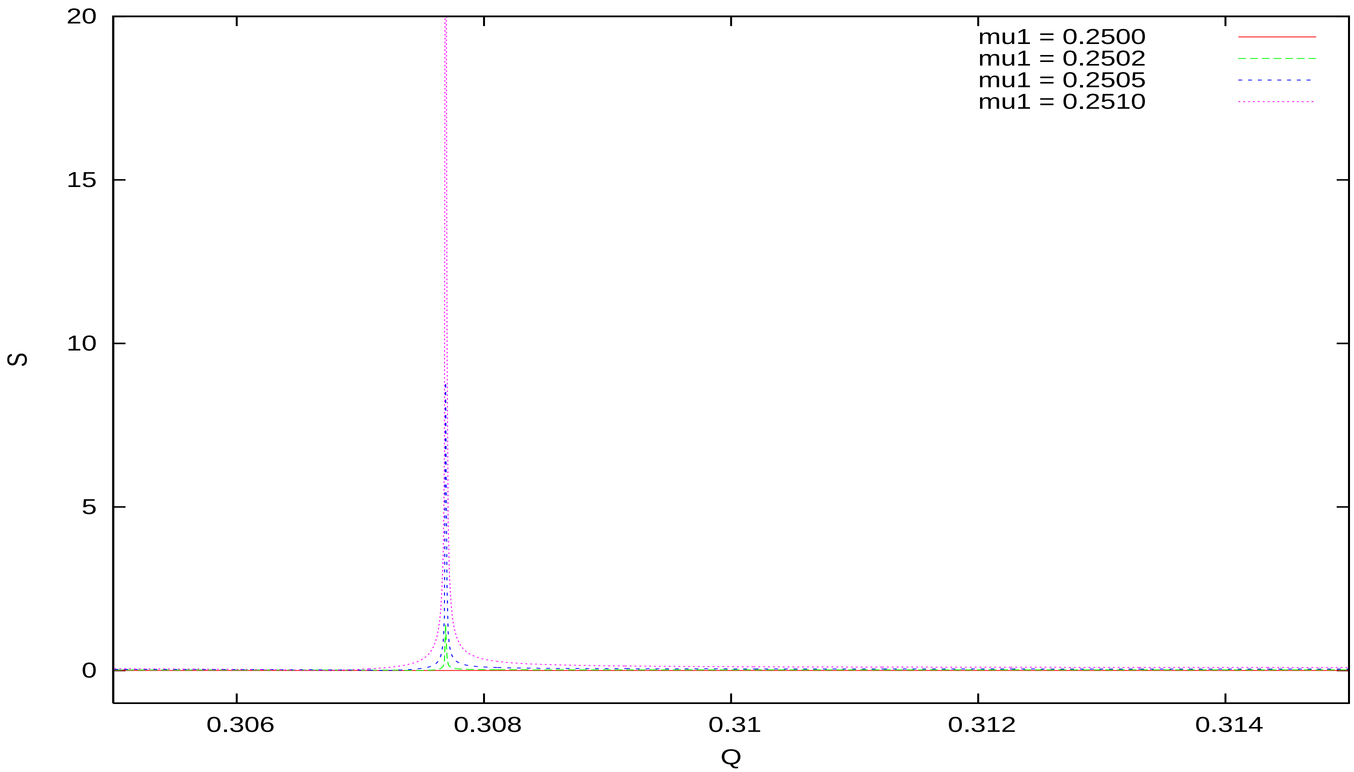


Another approach

- **Confidence in the method, but difficult to interprete for MANY interactions points (e.g. at N interaction points at azimuth Θ_i, not equally spaced)**
- **Invariant can be found in a form like[1]:**

$$h \;\approx\; N \cdot \xi \cdot U(A) + S \cdot V_n(A) \cdot \cos(n\Phi)$$

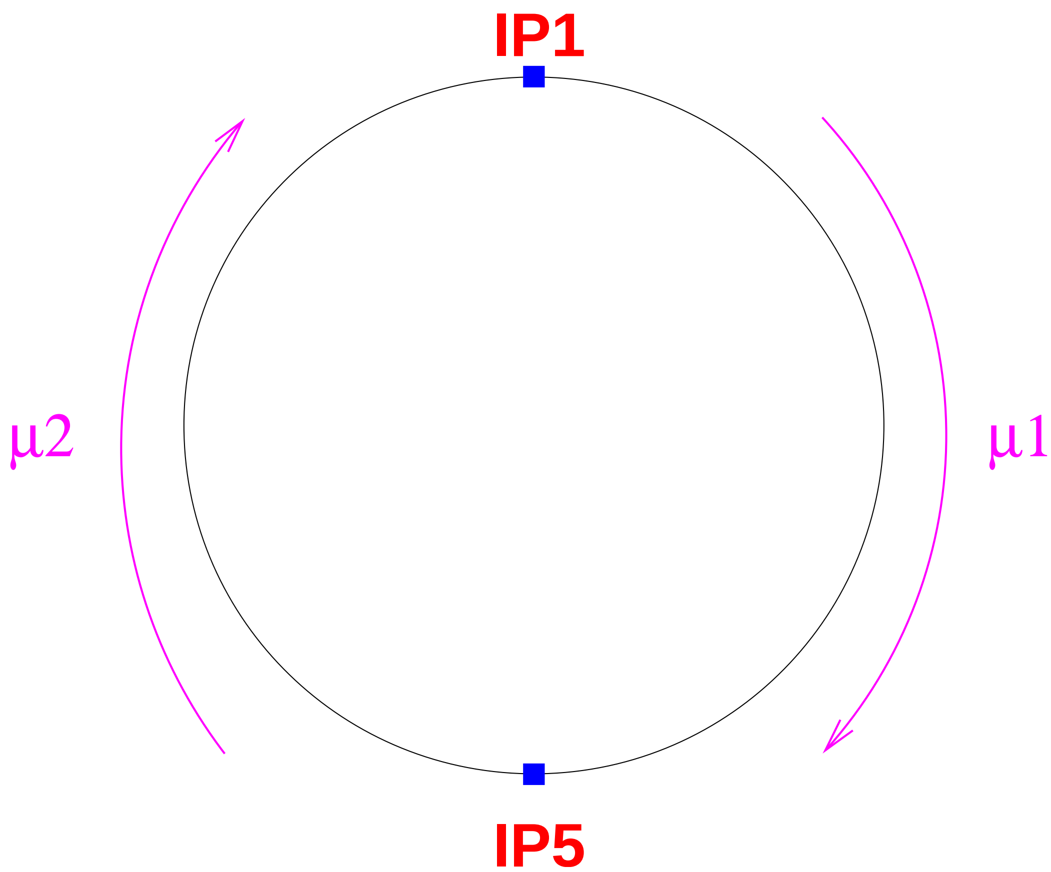
S **describes the periodicity and can be re-written as:**

$$S = \sqrt{(\sum_{i=1}^{N} \cos(p \cdot \Theta_i))^2 + (\sum_{i=1}^{N} \sin(p \cdot \Theta_i))^2}$$

(Θ_i is the azimuthal position of the ith beam-beam interaction, p is azimuthal harmonic)

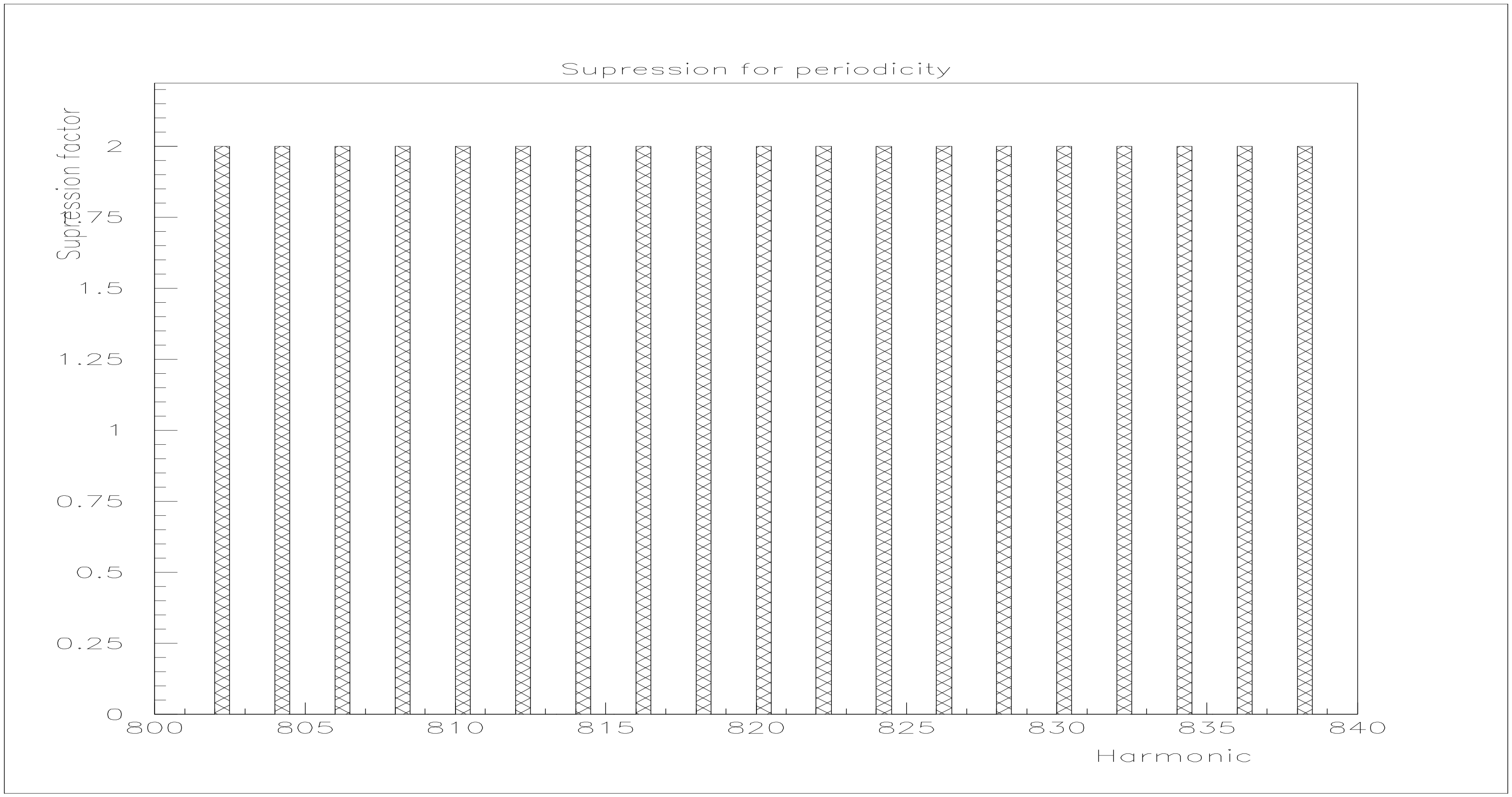
[1] W. Herr, LHC Project Report 49 (1996)

Full two-fold symmetry

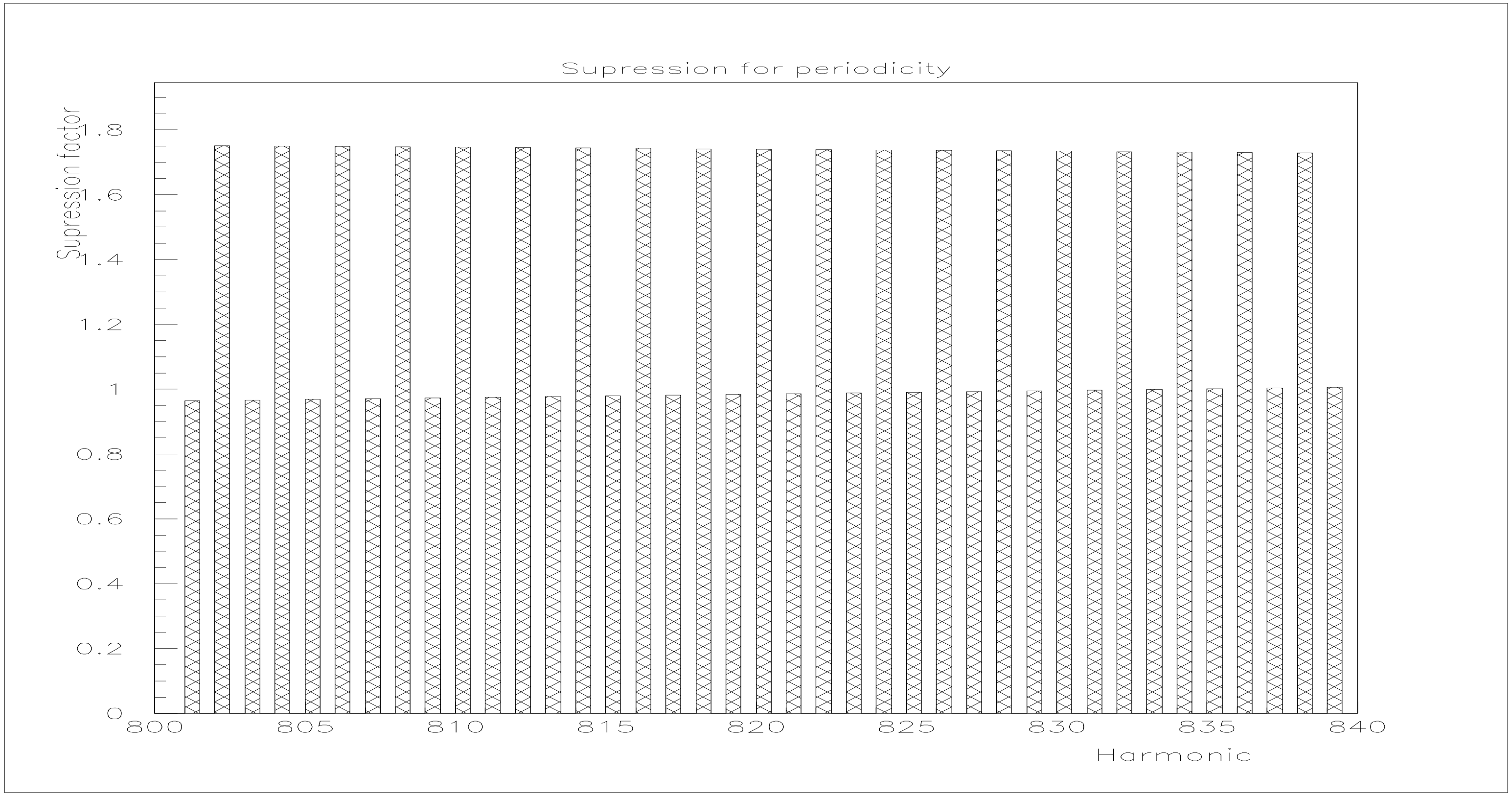


$\mu 1 = \mu 2$

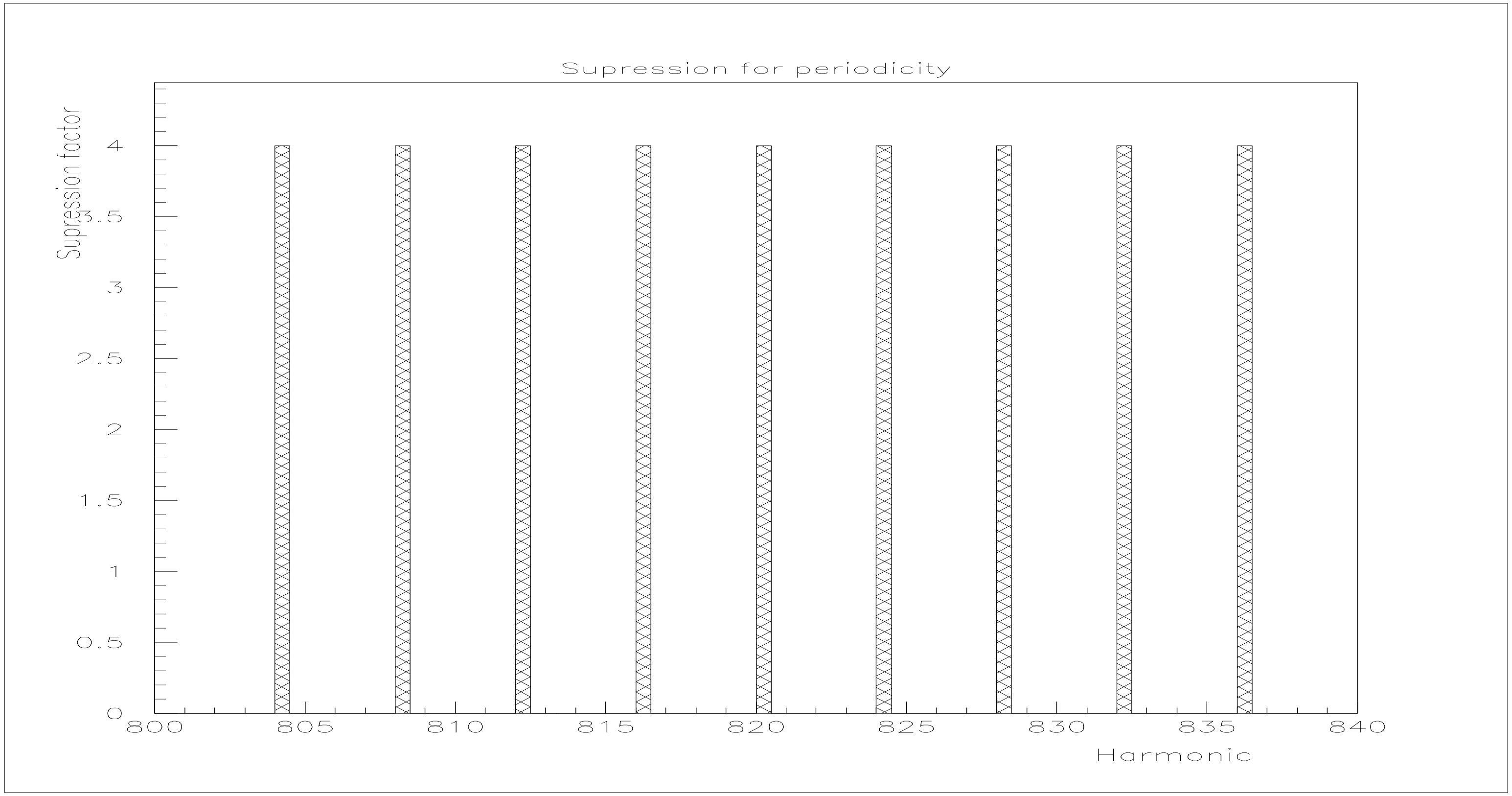
Full two-fold symmetry



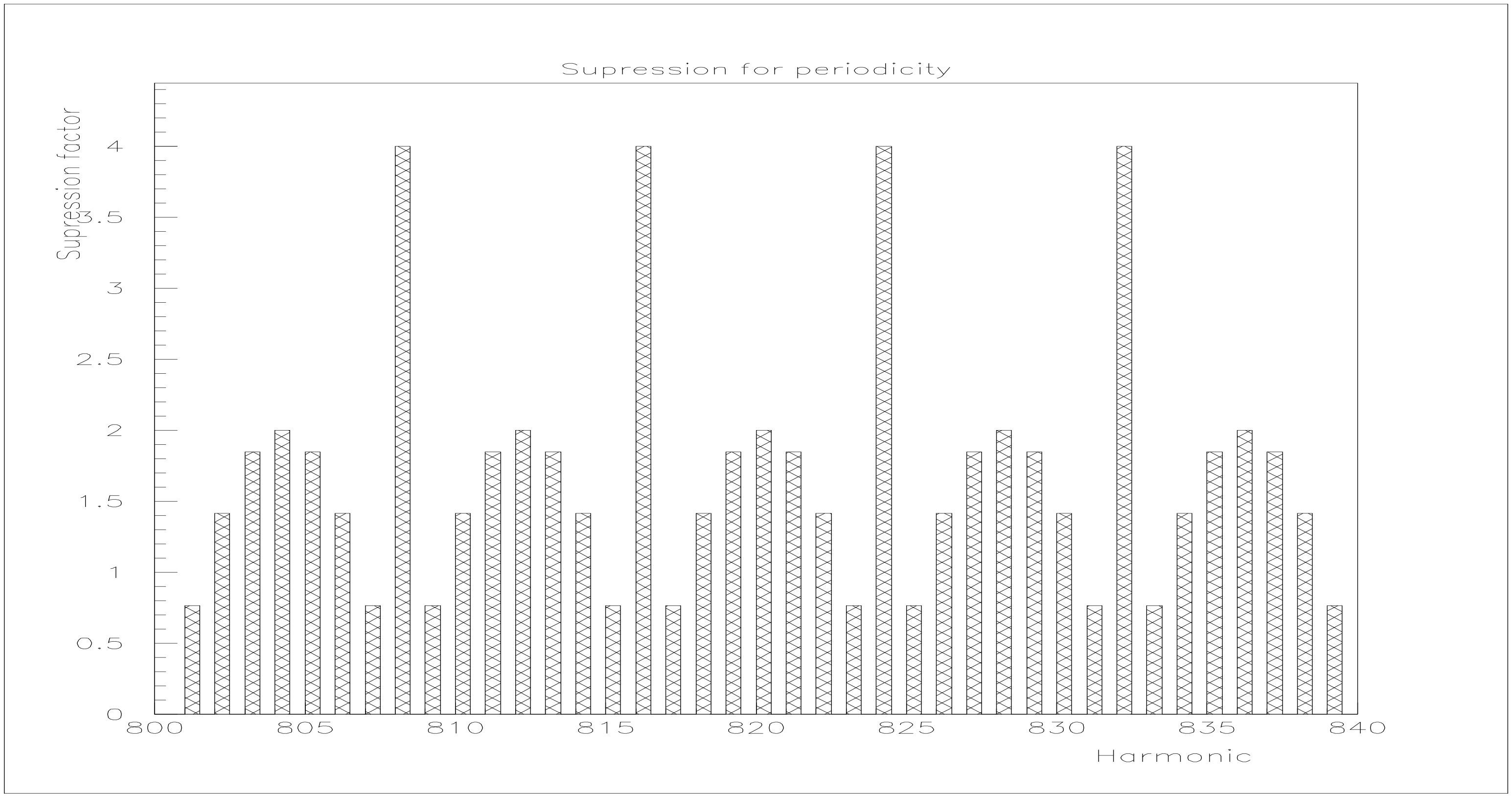
Two-fold symmetry with phase error



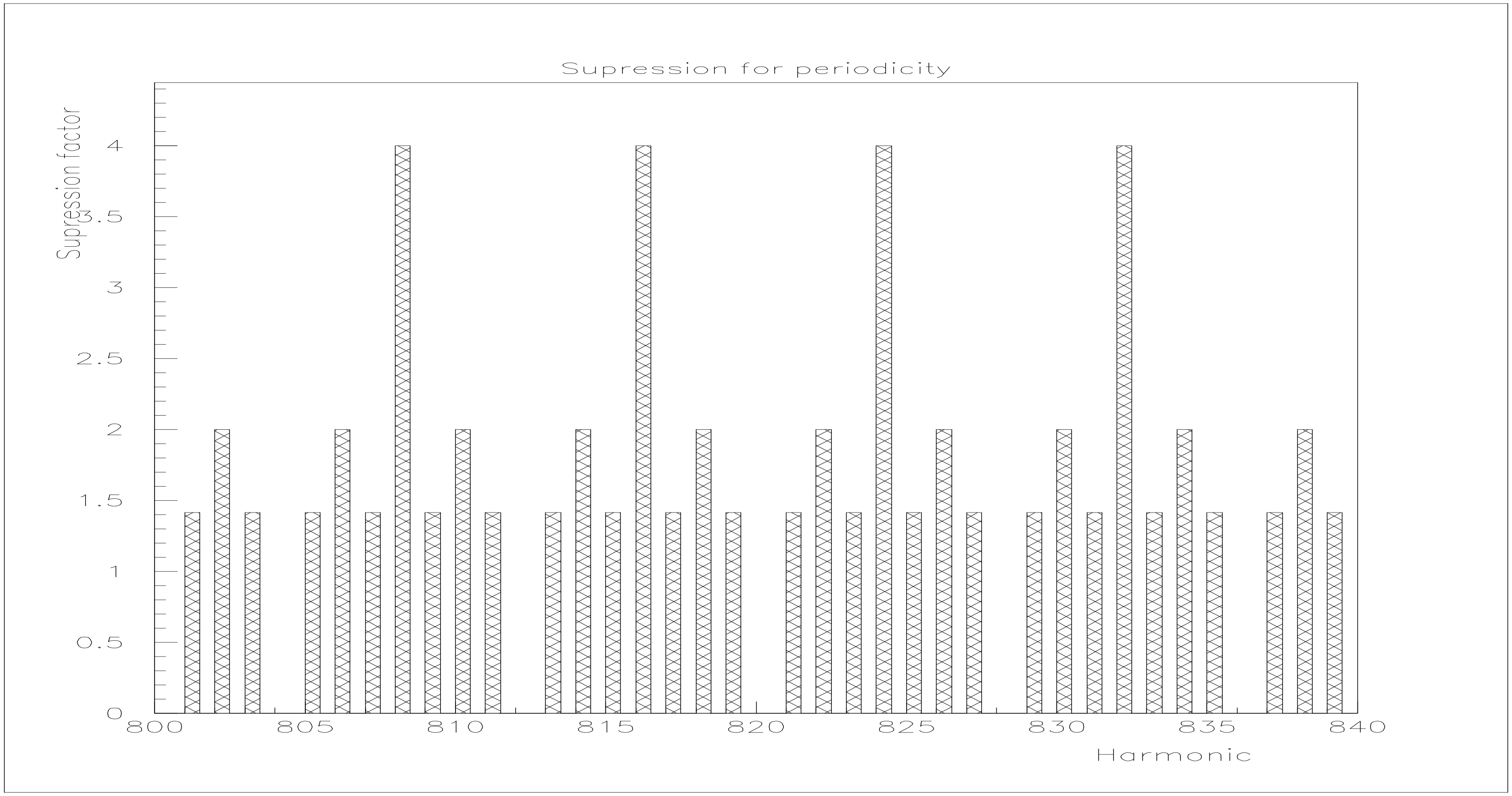
Full four-fold symmetry



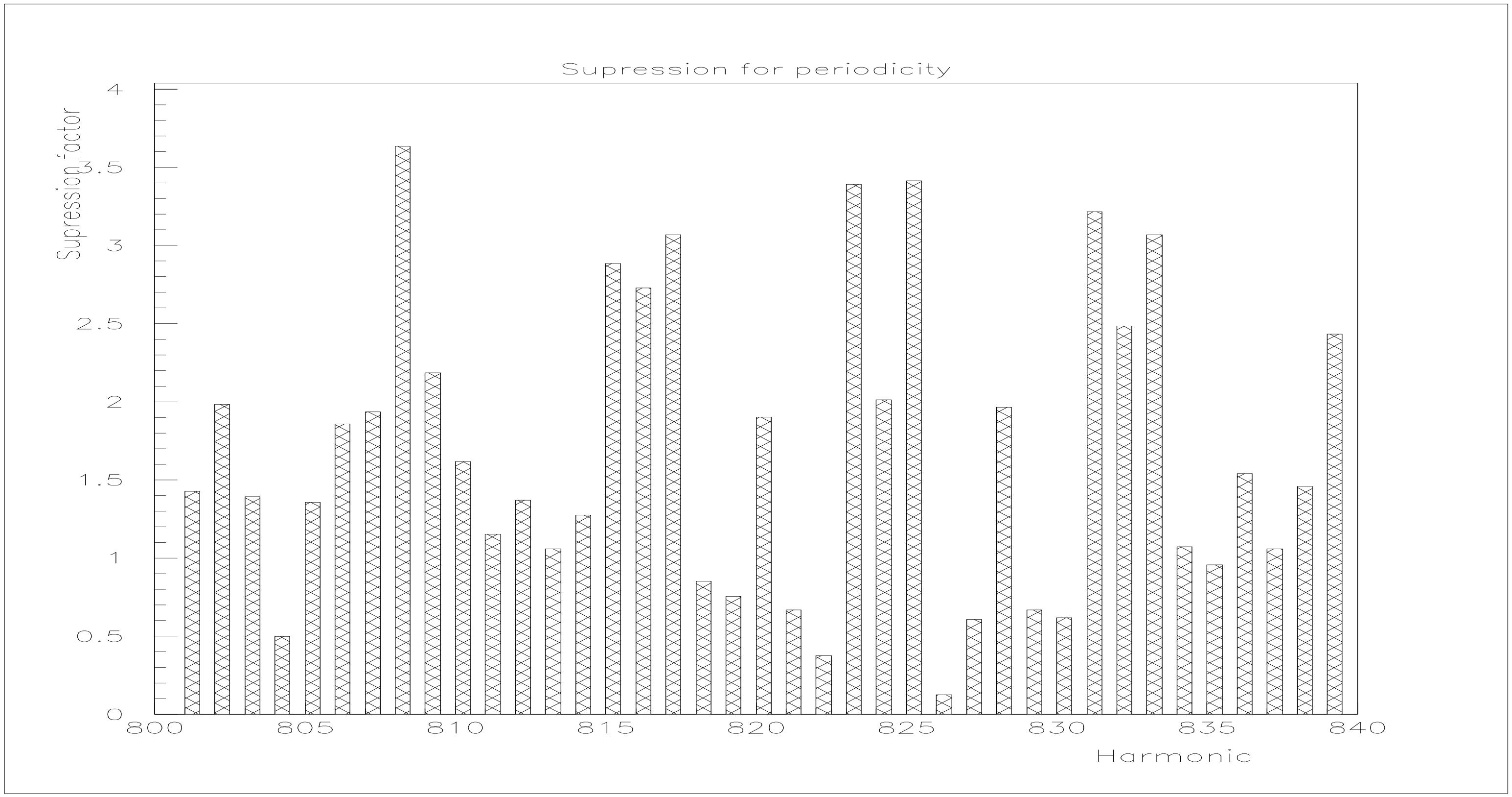
Four-fold symmetry with phase error



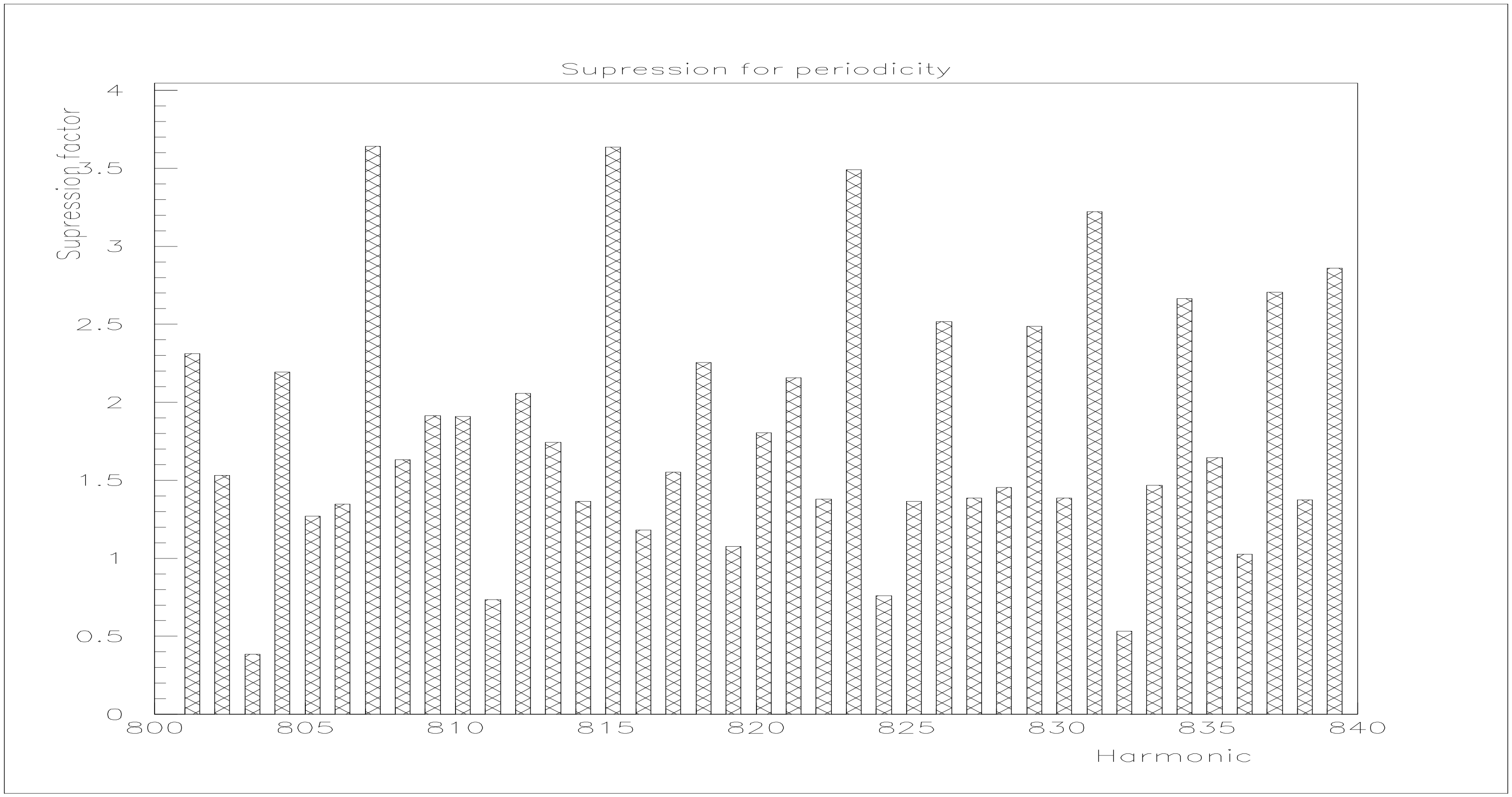
LHC collision scheme with eight-fold symmetry



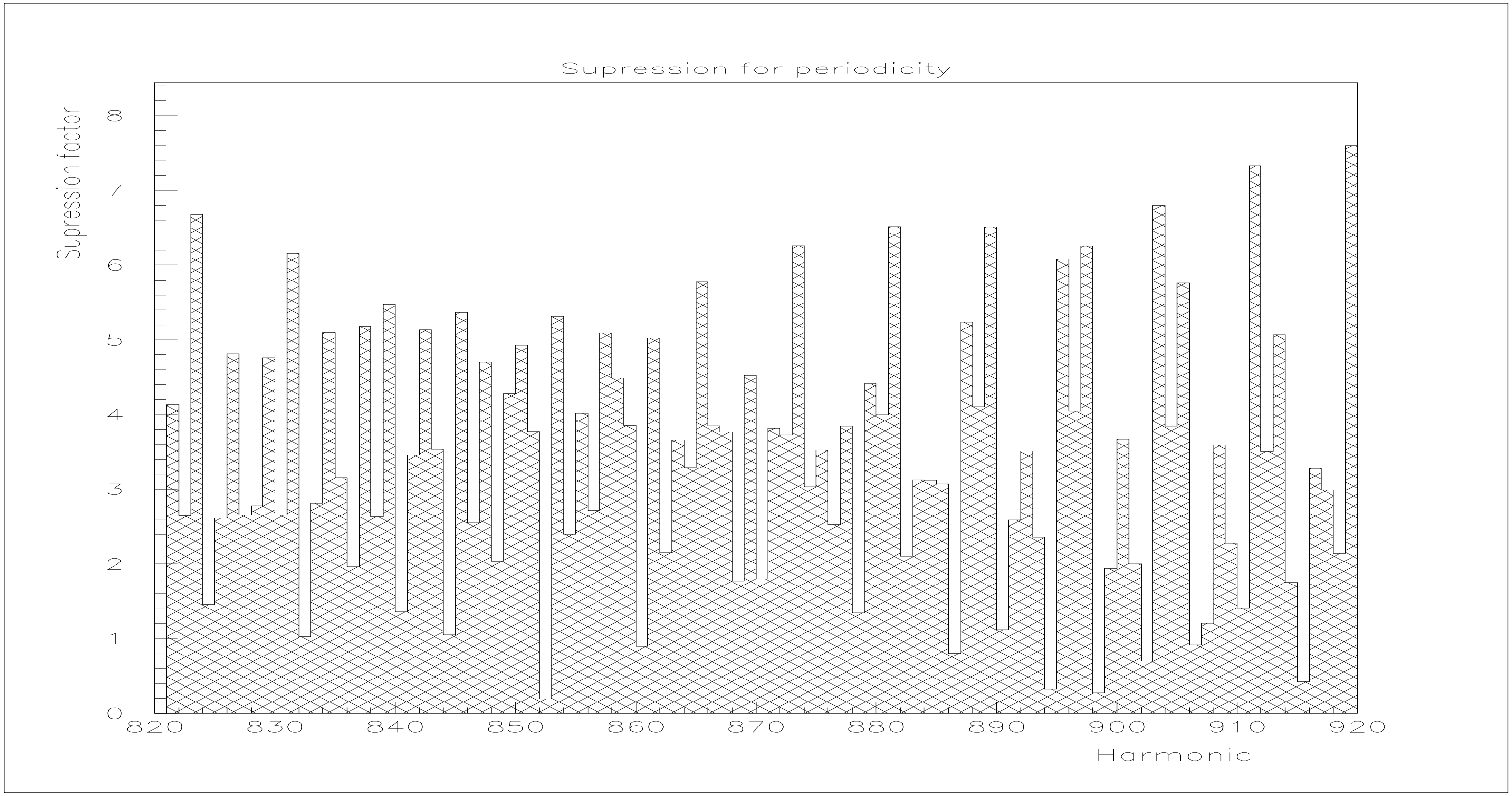
LHC collision scheme with eight-fold symmetry and phase erro



LHC collision scheme with nominal optics



LHC optics with long range



Conclusions

- **Potentially some resonances (not all) can be suppressed by good choice of phase advance**
- **Suppression needs tight control of the phase advance**
- **Including a second dimension was studied and makes it worse**
- **LHC is a dirty machine:**
  - **Additional IPs (2 and 8)**
  - **Long range effects**
  - **PACMAN effects**
- **Cannot rely on suppression**