Emittance analysis for the LHC beam along a 160 MeV plateau in the PSB

M. Martini

Contents

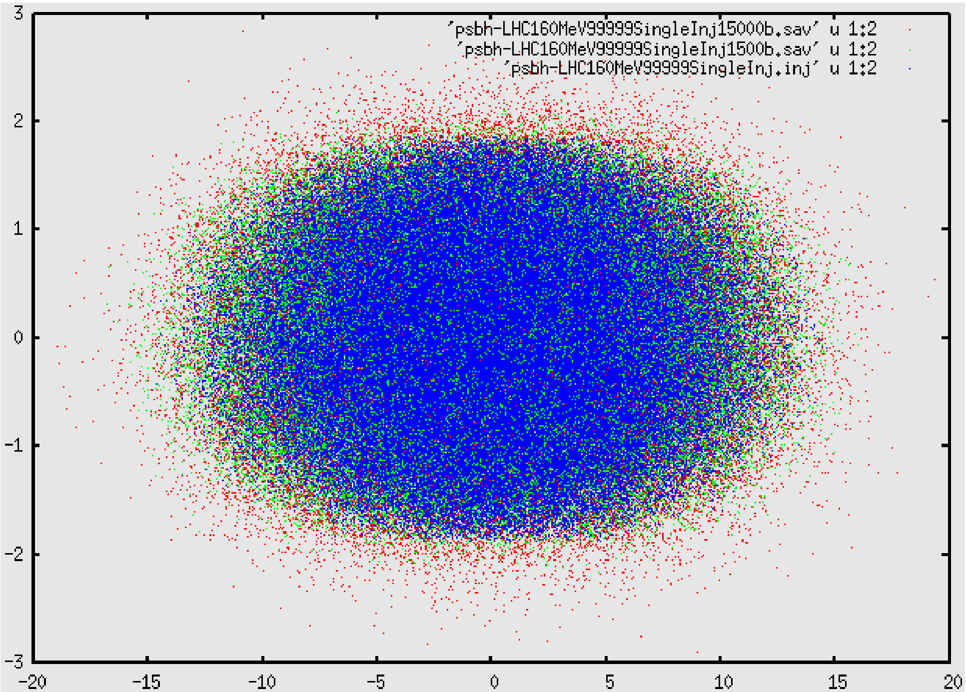
- Simulation scenario and overall results
- LHC nominal beam: analysis on the 1st turn
- LHC nominal beam: analysis on the 1500th turn
- LHC nominal beam: analysis on the 15000th turn
- Summary

Simulation scenario and overall results

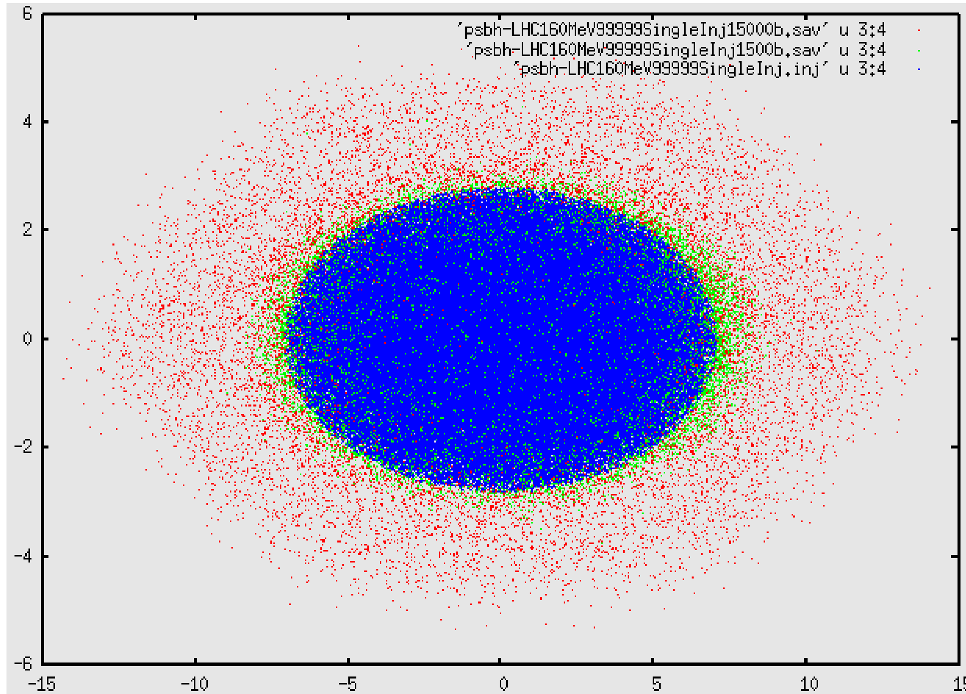
LHC nominal beam (high brightness beam - 3.25 $\times 10^{12}$ protons)

- **PSB single turn injection with Accsim on a 160 MeV plateau**
  - 99999 macro-particles injected and stored for 15000 turns (3.25 $\times 10^{12}$ real particles). The initial transverse normalized rms emittances are 2.5 μm.
  - The phase and energy half-widths of limiting injected bunch ellipse are 100.2 deg and 1.03 MeV. The bunch length is about 550 ns.
  - No H^- injection took place, the total proton beam intensity is injected on the 1^{st} turn onto an 8 kV bucket.
  - Proton beams injected in the middle of the PSB-ring section L1 where $\alpha_{H,V}$=0 to avoid transverse mismatch and subsequent emittance blow-up. Likewise, the short closed orbit bump (BS1-BS4) was disabled to avoid optics distortions.
  - Simulation made using the working point Q_H=4.28, Q_V=5.47.
  - Simulations done with the keyword TSCBUNCH=True in Accsim (which enables to scale the transverse space charge force in line with the local longitudinal charge density in the bunch). The transverse space charge fields were calculated using grid arrays with 0.5 mm spacing of grid points.
  - The following emittance analysis is based on Accsim output data (x,x',y,y',ϕ,ΔE) stored on the 1^{st}, 1500^{th} and 15000^{th} tracking turns.

Simulation scenario and overall results



X-X' scatter-plot [mm-mrad] at turns 1, 1500 and 15000

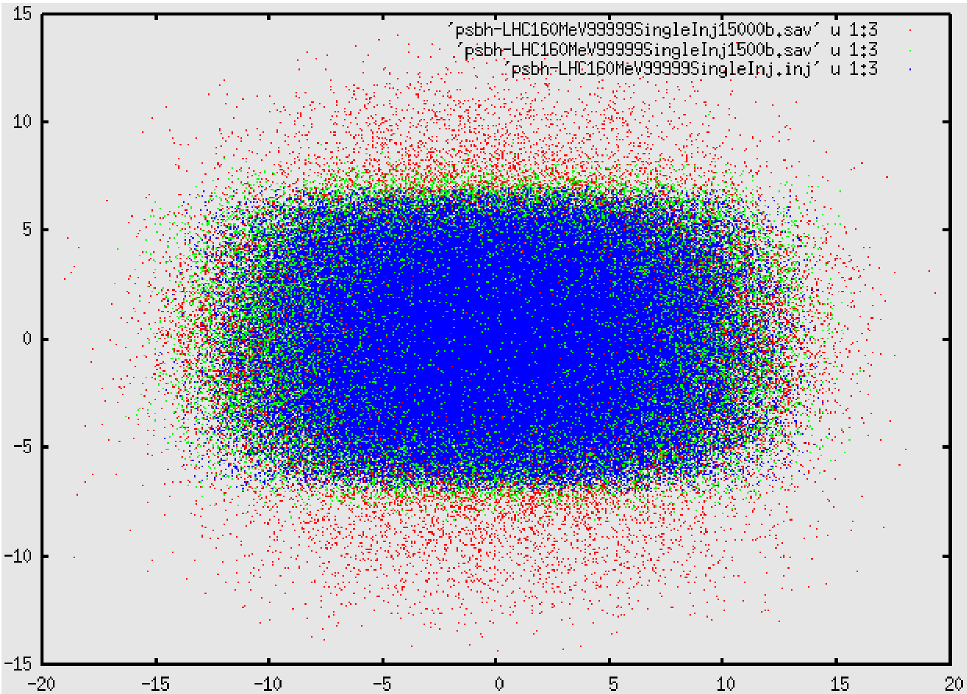


Y-Y' scatter-plot [mm-mrad] at turns 1, 1500 and 15000

Cf. LIS Section meeting held on 2nd April 2007

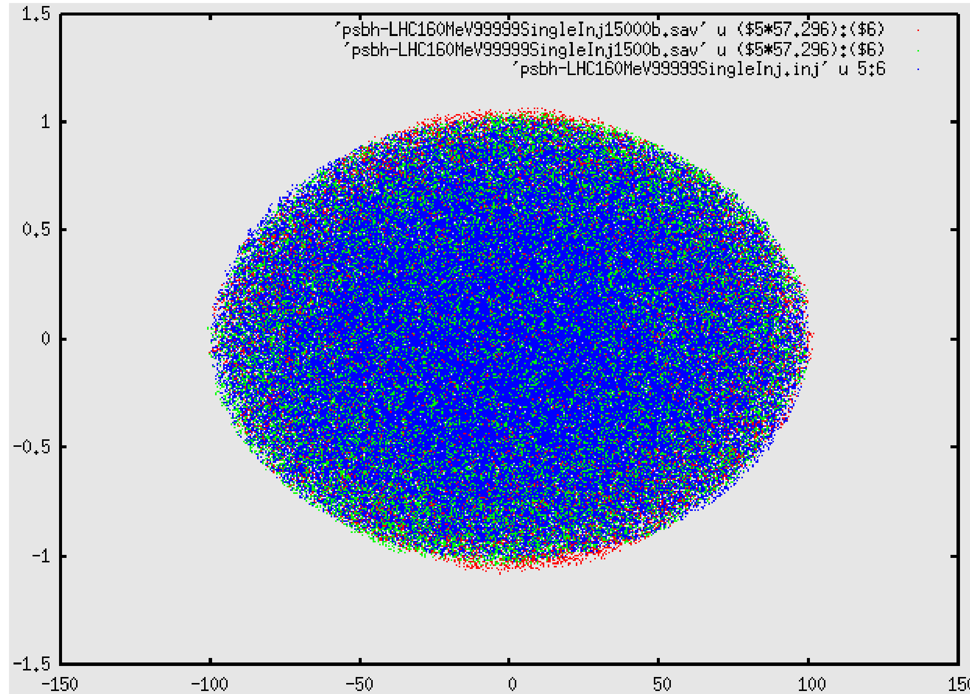
Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

Simulation scenario and overall results



X-Y scatter-plot[1] [mm-mm] at turns 1, 1500 and 15000

[1] The physical cross-section of the injected beam is rectangular as no correlation is assumed between horizontal and vertical planes

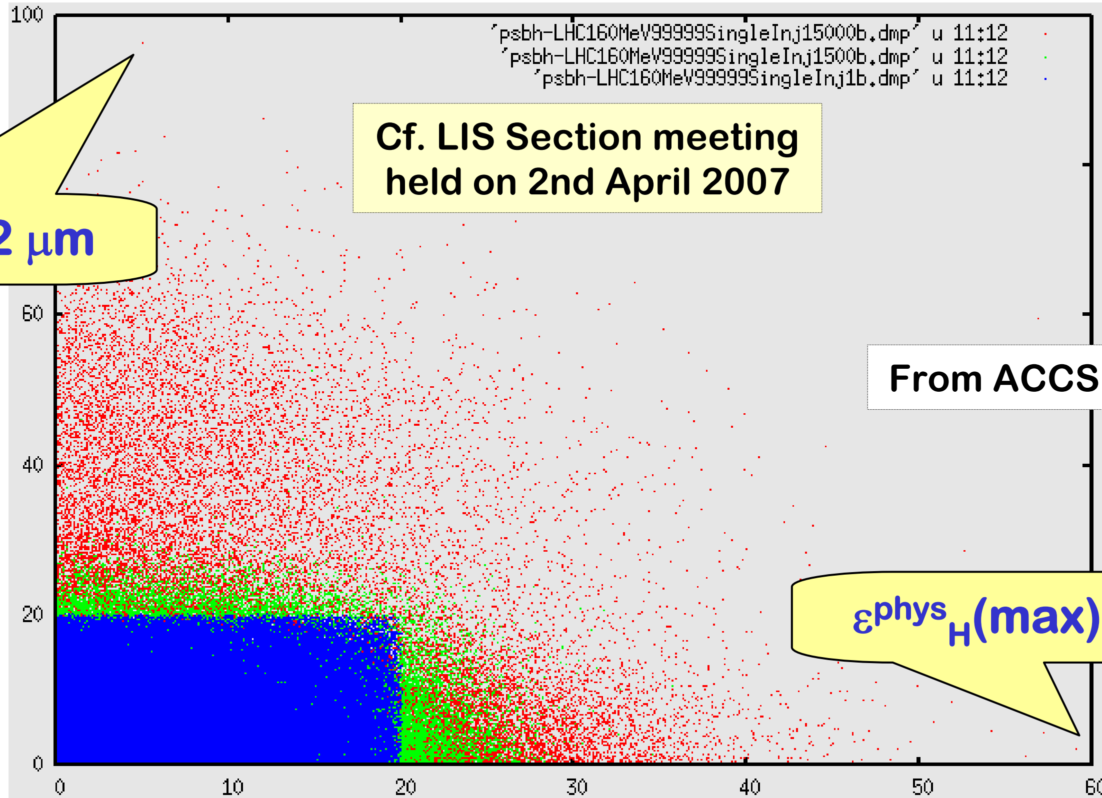


ϕ-ΔE scatter-plot [deg-MeV] at turns 1, 1500 and 15000

Cf. LIS Section meeting held on 2nd April 2007

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

Simulation scenario and overall results



Physical emittance[(1)(2)] [μm] scatter-plot at turns 1, 1500 and 15000

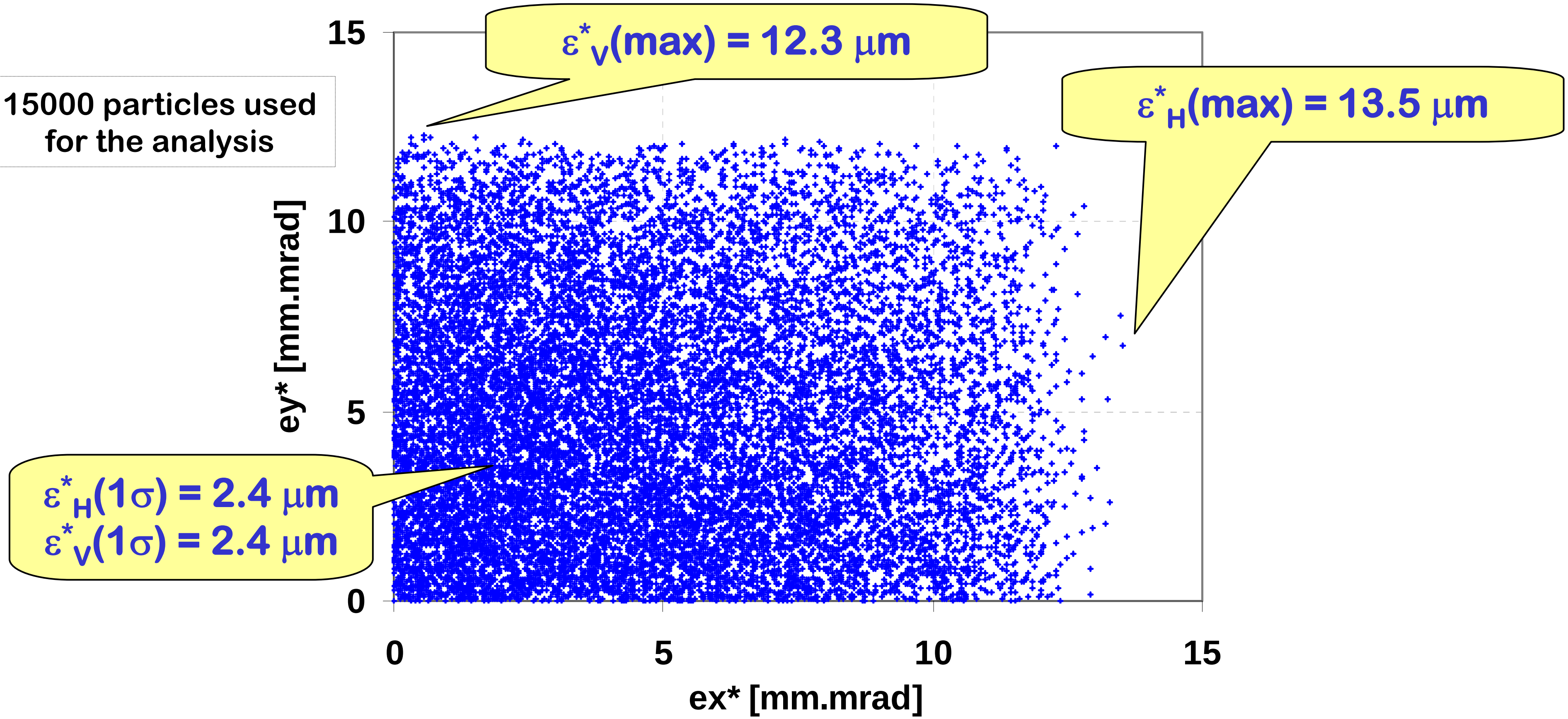
(1) Calculated Courant-Snyder invariants for individual particles

(2) The limiting 20 μm physical emittances correspond to 2.43 μm normalized rms emittances

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

LHC nominal beam: analysis on the 1st turn

15000 particles used for the analysis

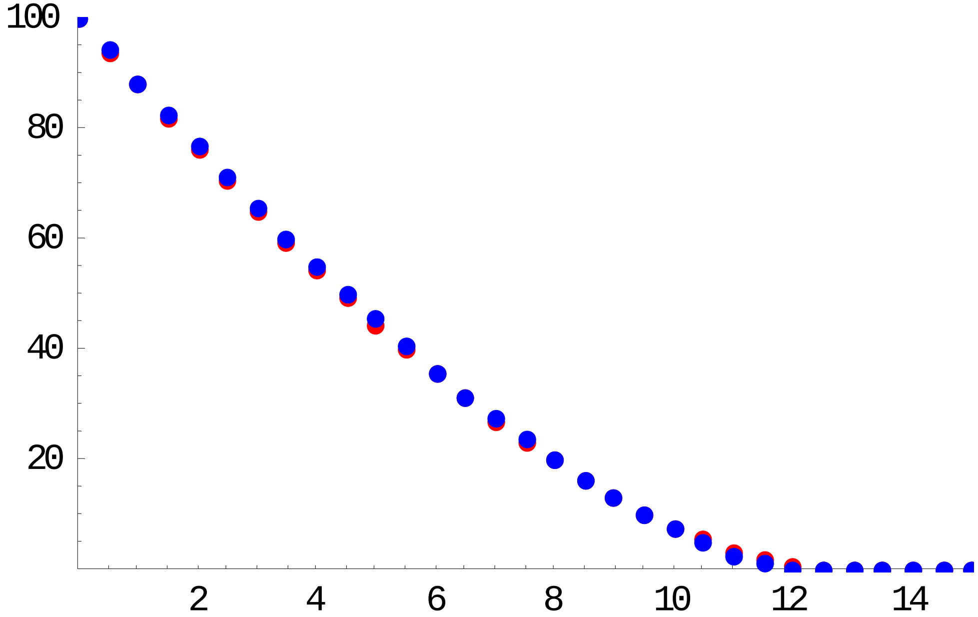


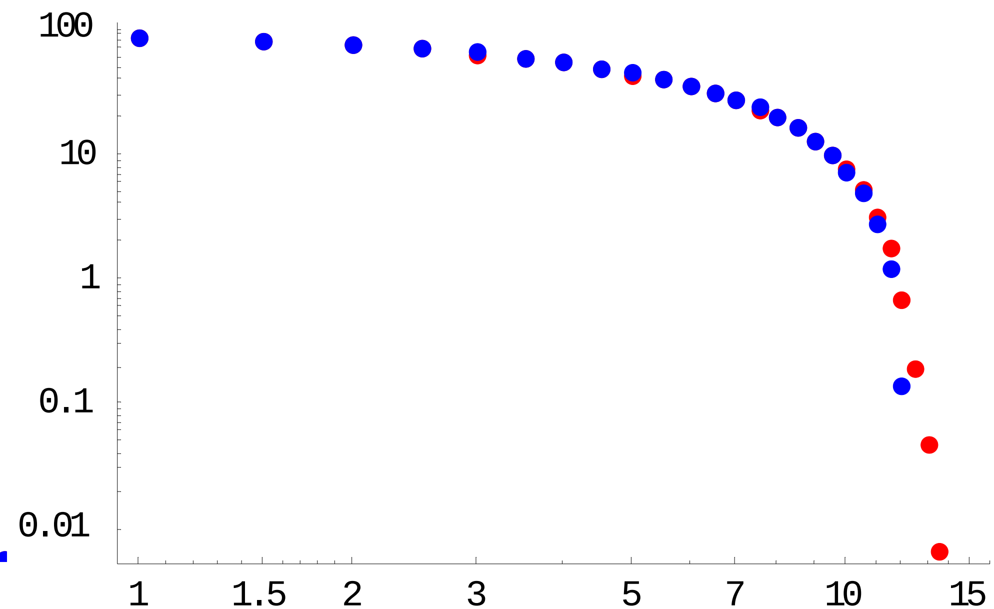
Normalised emittance(1) [µm] scatter-plot on the 1st turn

(1) Calculated Courant-Snyder invariant for individual particles

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

LHC nominal beam: analysis on the 1st turn





Horizontal & vertical normalised emittance 1-CPDF [1] plot [%] at the 1st turn

[1] Cumulative probability density function

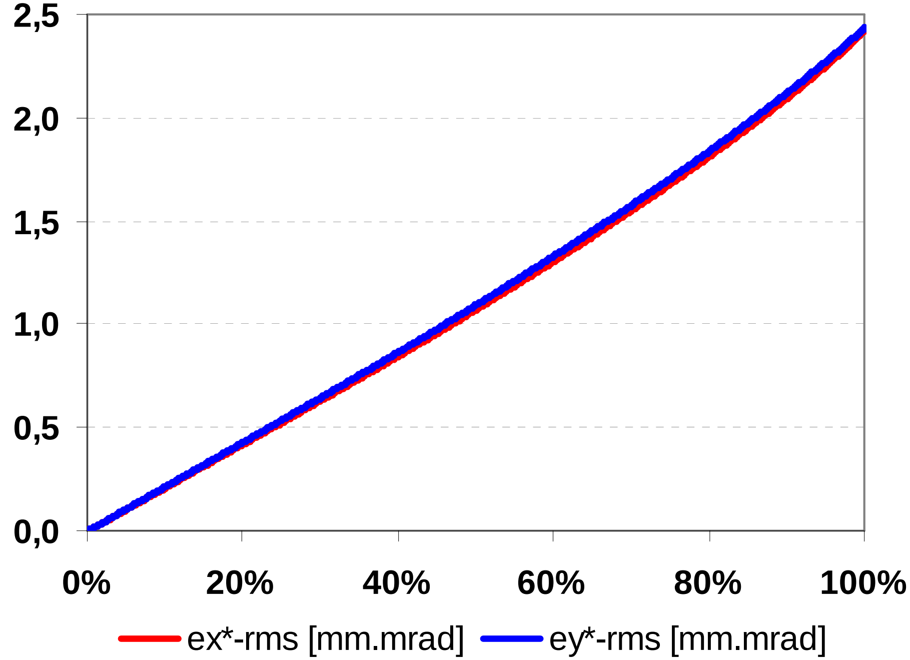
Horizontal & vertical normalised emittance log-log 1-CPDF [1] plot [%] at the 1st turn

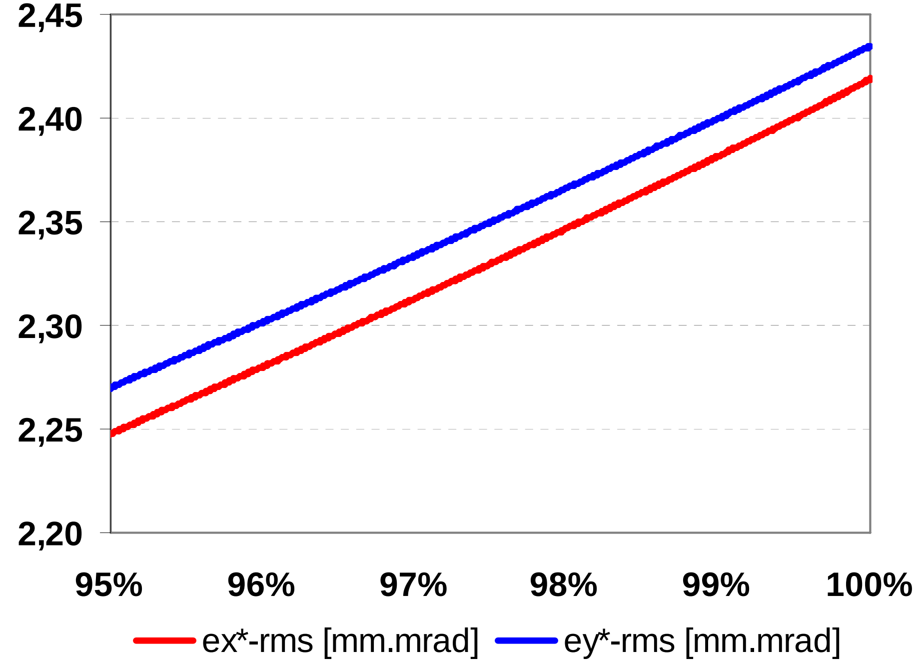
[1] Outliers at end tails removed

15000 particles used for the analysis

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

LHC nominal beam: analysis on the 1st turn





Horizontal & vertical normalised rms emittances[(1)] at the 1st turn vs. p%-acceptance (defined as the fraction of the particle beam with emittance[(2)] less than a given value)

(1) Calculated from the beam "sigma" matrix

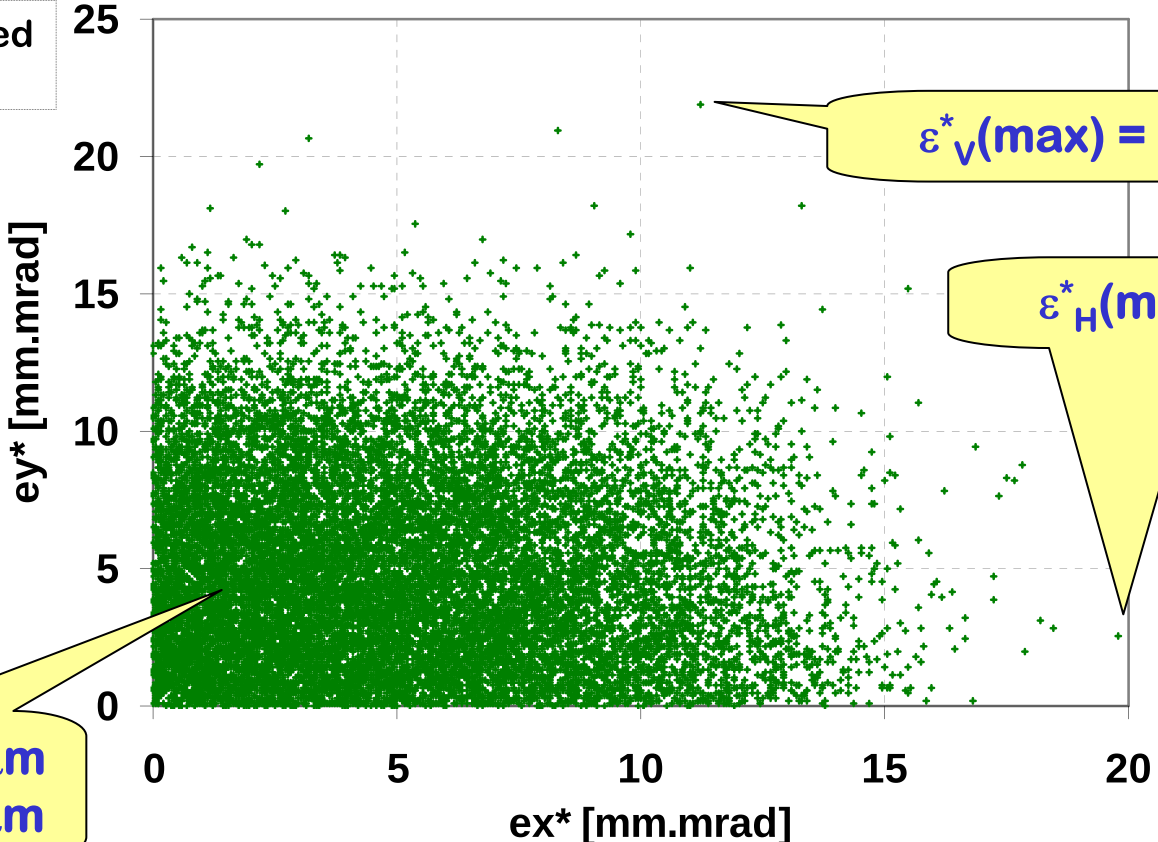
(2) Calculated for individual particles from the Courant-Snyder invariant with Twiss parameters at injection

$$\varepsilon_{\mathrm{u}}(\mathrm{rms}) = \sqrt{\langle u_{\mathrm{i}}^2\rangle\langle u_{\mathrm{i}}'^2\rangle - \langle u_{\mathrm{i}}\, u_{\mathrm{i}}'\rangle^2} \quad \text{for all i such that } \varepsilon_{\mathrm{u,i}} \le \varepsilon_{\mathrm{u}}(\mathrm{p\%}) \qquad \varepsilon_{\mathrm{u,i}} = \gamma_{\mathrm{u}}\, u_{\mathrm{i}}^2 + 2\alpha_{\mathrm{u}}\, u_{\mathrm{i}}\, u_{\mathrm{i}}' + \beta_{\mathrm{u}}\, u_{\mathrm{i}}'^2$$

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

LHC nominal beam: analysis on the 1500th turn

15000 particles used for the analysis



$\varepsilon^*_V(max) = 21.9\ \mu m$

$\varepsilon^*_H(max) = 19.8\ \mu m$

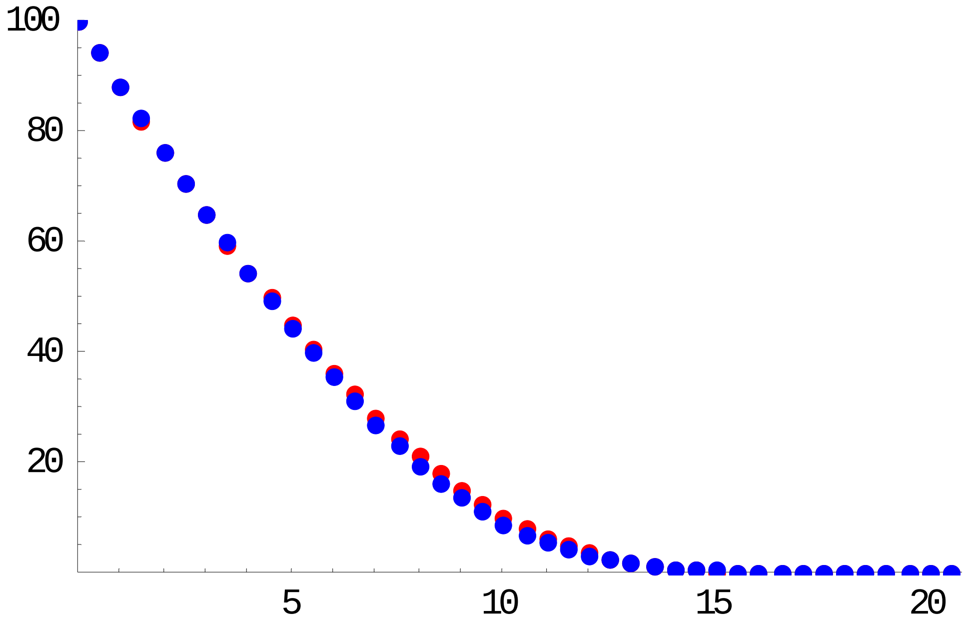
$\varepsilon^*_H(1\sigma) = 2.5\ \mu m$
$\varepsilon^*_V(1\sigma) = 2.5\ \mu m$

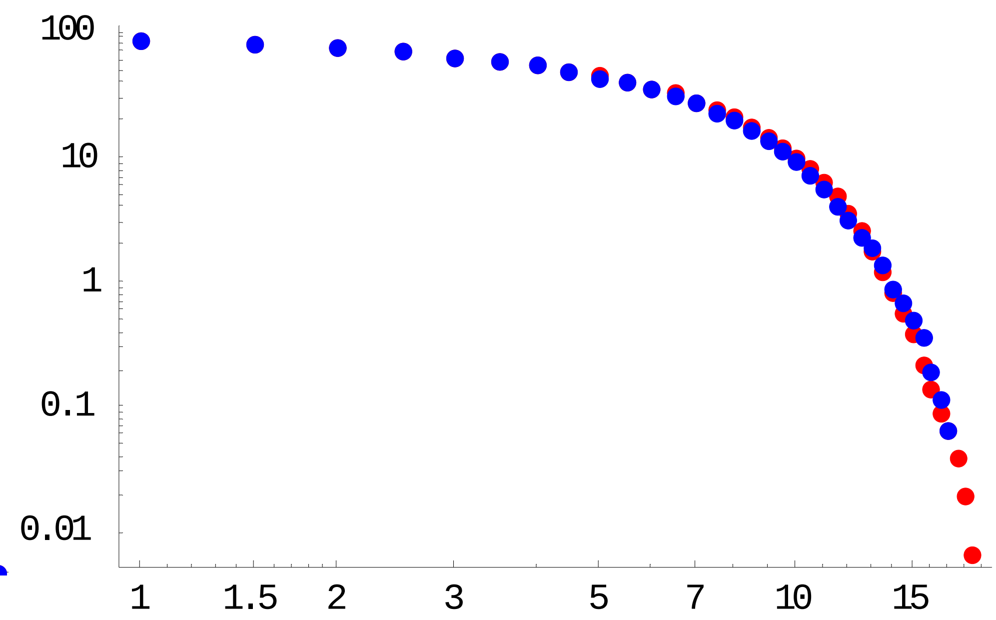
Normalised emittance(1) [μm] scatter-plot on the 1500th turn

(1) Calculated Courant-Snyder invariant for individual particles

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

LHC nominal beam: analysis on the 1500th turn





Horizontal & vertical normalised emittance 1-CPDF (1) plot [%] at the 1500th turn

(1) Cumulative probability density function

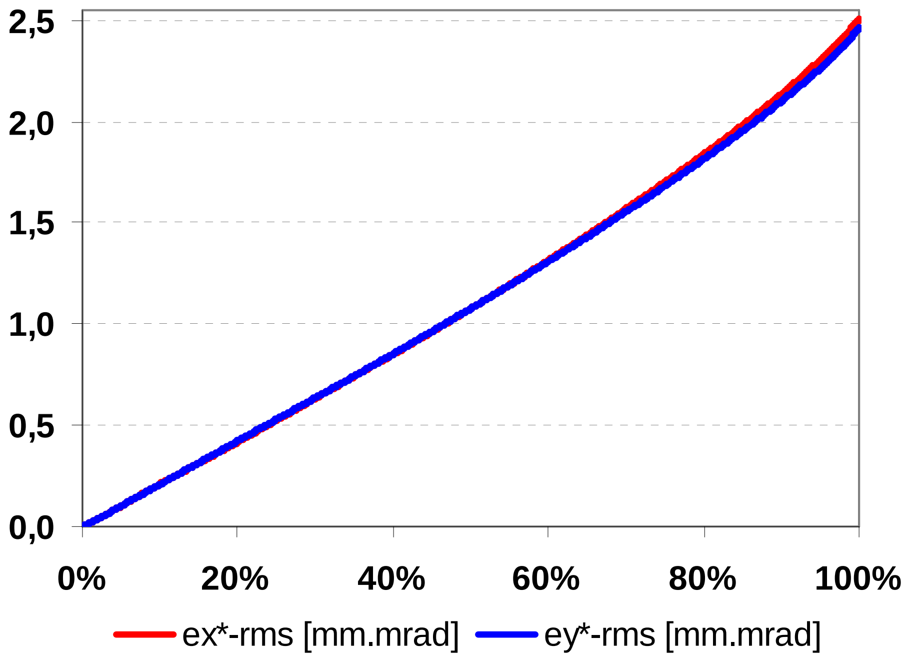
Horizontal & vertical normalised emittance log-log 1-CPDF (1) plot [%] at the 1500th turn

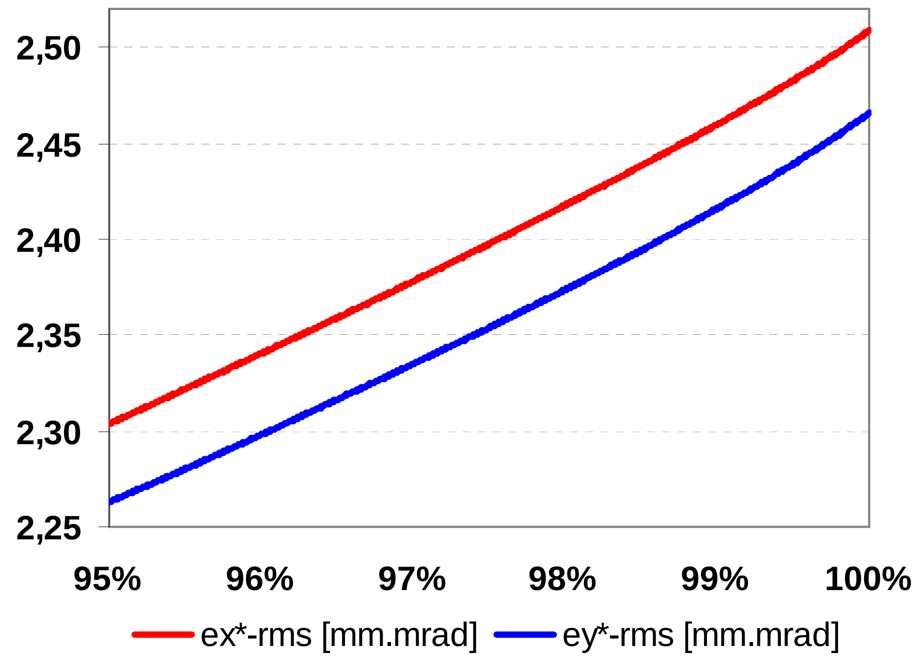
(1) Outliers at end tails removed

15000 particles used for the analysis

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

LHC nominal beam: analysis on the 1500th turn





Horizontal & vertical normalised rms emittances(1) at the 1500th turn vs. p%-acceptance (defined as the fraction of the particle beam with emittance(2) less than a given value)

(1) Calculated from the beam "sigma" matrix

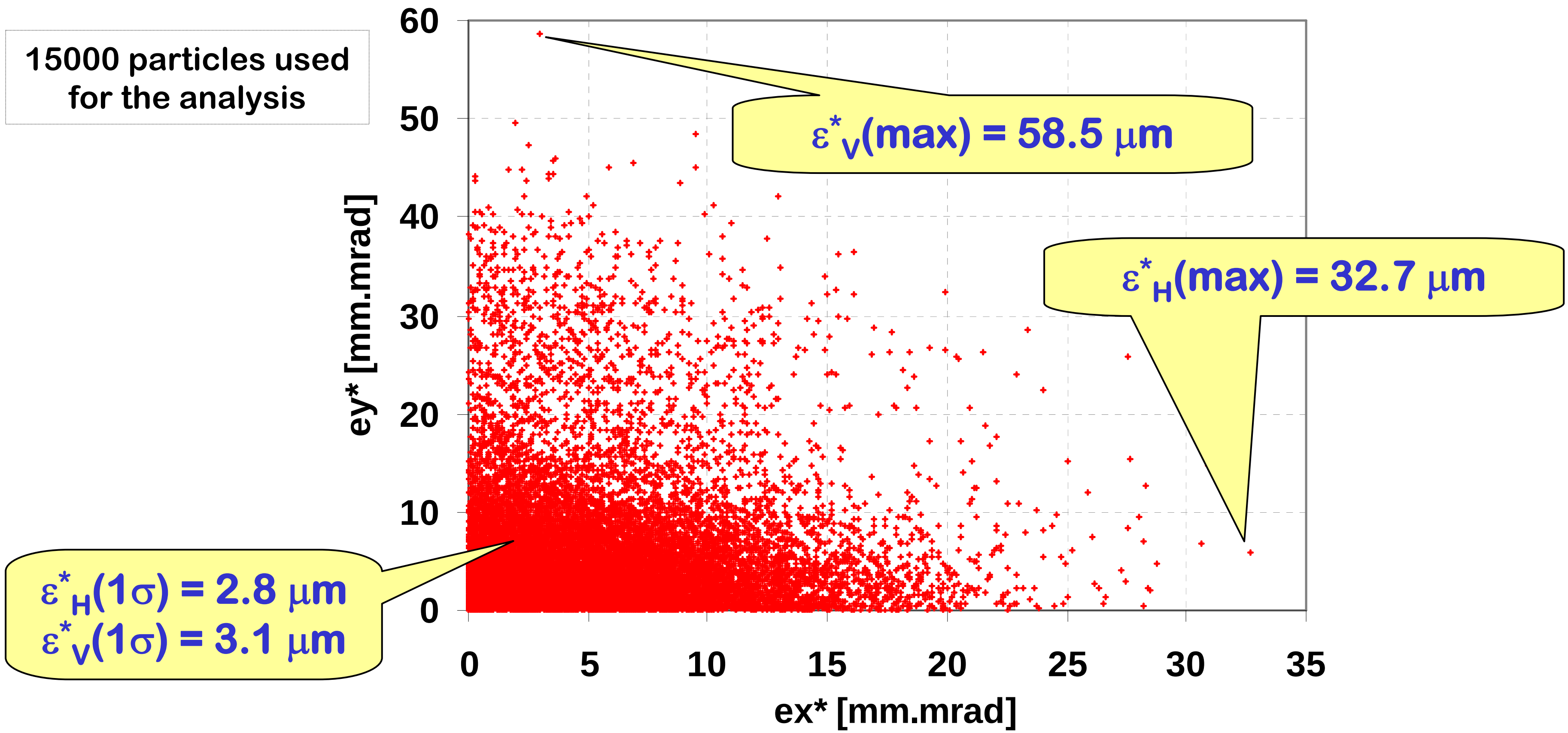
(2) Calculated for individual particles from the Courant-Snyder invariant with Twiss parameters at injection

$$\varepsilon_u(\text{rms}) = \sqrt{\langle u_i^2\rangle\langle u_i'^2\rangle - \langle u_i\, u_i'\rangle^2} \quad \text{for all i such that } \varepsilon_{u,i} \le \varepsilon_u(\text{p\%}) \quad \varepsilon_{u,i} = \gamma_u u_i^2 + 2\alpha_u u_i u_i' + \beta_u u_i'^2$$

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

LHC nominal beam: analysis on the 15000th turn

15000 particles used for the analysis

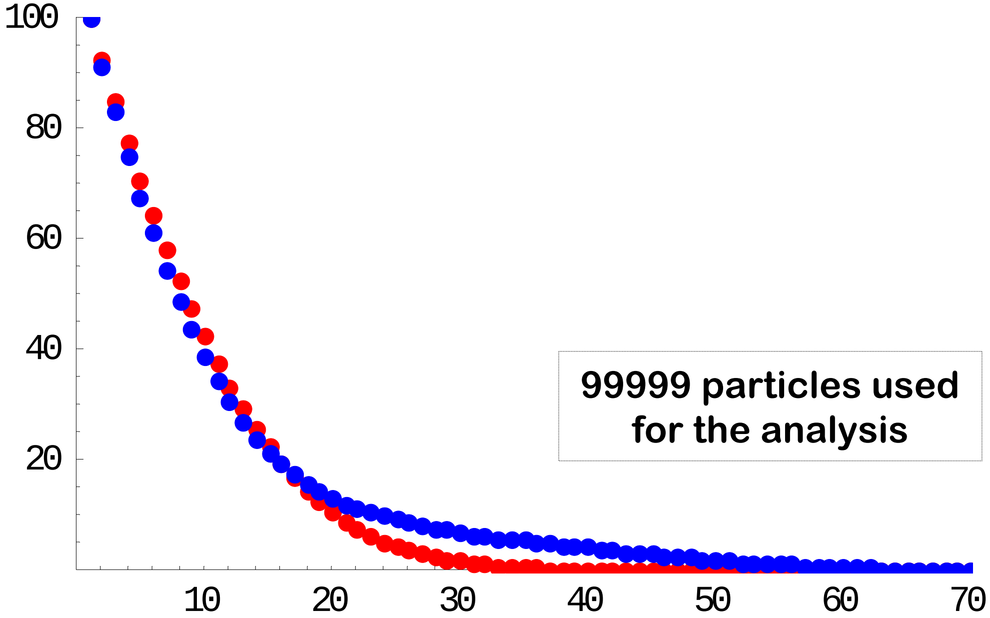


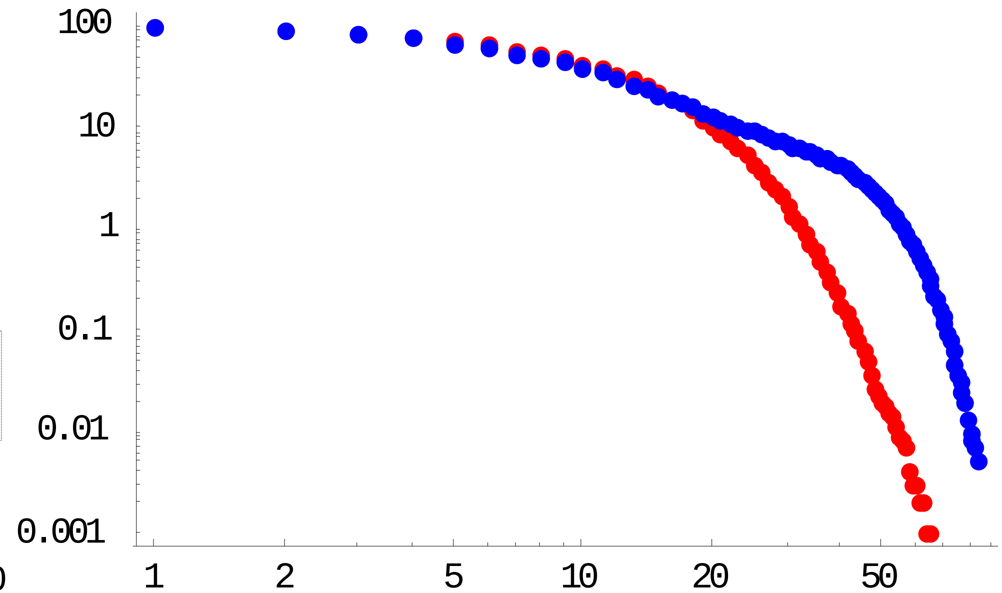
Normalised emittance(1) [μm] scatter-plot on the 15000th turn

(1) Calculated Courant-Snyder invariant for individual particles

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

LHC nominal beam: analysis on the 15000th turn





Horizontal & vertical normalised emittance 1-CPDF [(1)] plot [%] at the 15000th turn

[(1)] Cumulative probability density function

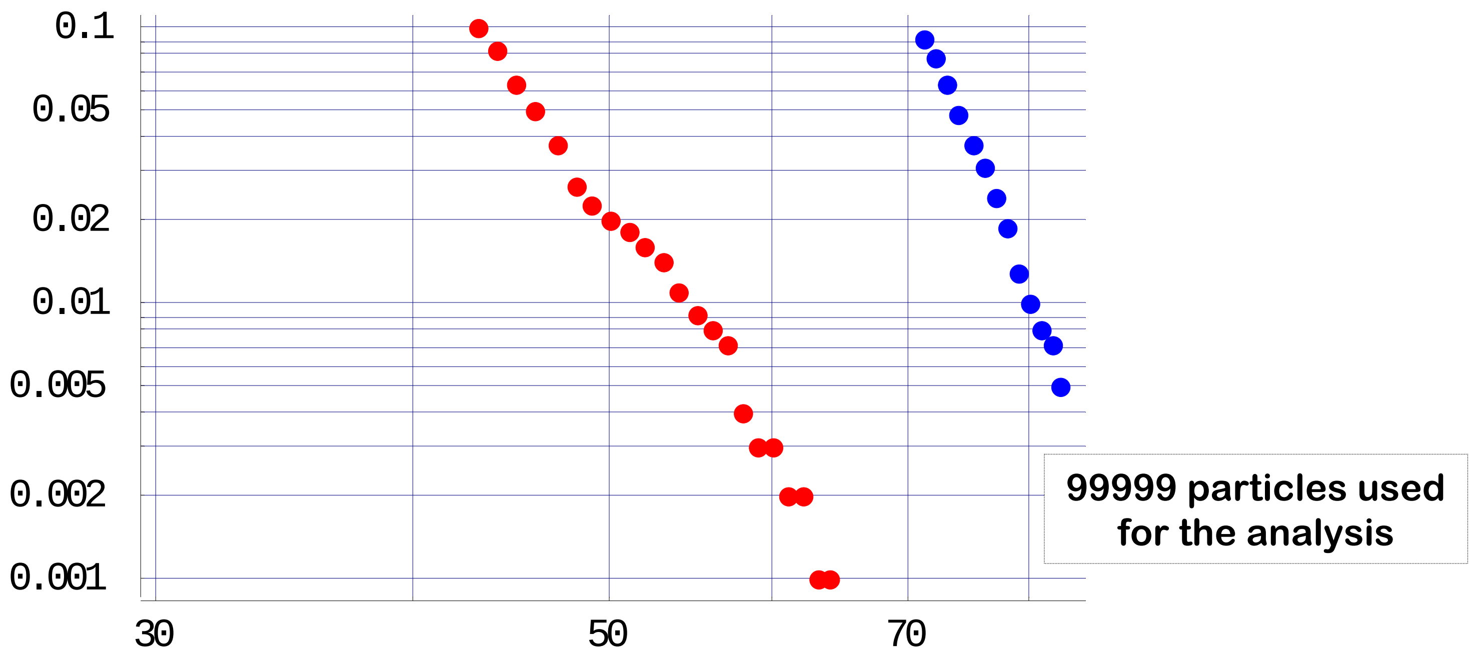
Horizontal & vertical normalised emittance log-log 1-CPDF [(1, 2)] plot [%]at the 15000th turn

[(1)] Leptokurtic distribution with power-law tails (α>2, non Levy-stable distribution)

[(2)] Outliers at end tails removed

Q_H=4.28 Q_V=5.47 N=3.25$\times 10^{12}$ protons (99999 macro-particles tracked)

LHC nominal beam: analysis on the 15000th turn



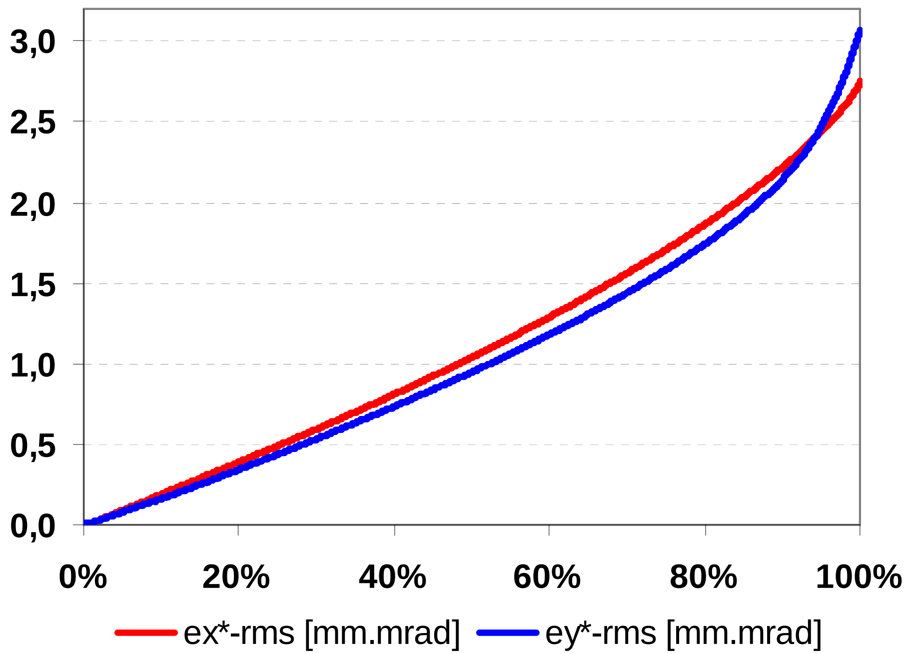
Horizontal & vertical normalised emittance log-log tail 1-CPDF plot [%] at the 15000th turn

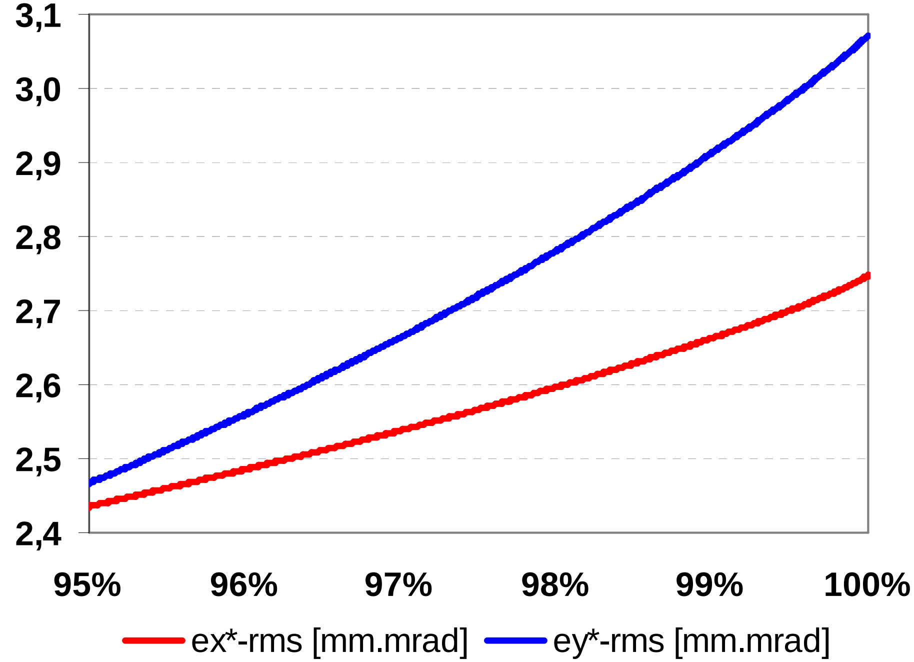
Convergence to a power-law tail (Pareto) at the 15000th turn
Estimation of the tail index (by moving power-law exponent fit) : $\alpha_H \approx 10.9$ $\alpha_V \approx 19.6$
Less than 0.1% of the particles serve to derive the tail index

$$\mathrm{Prob}(\varepsilon_{\mathrm{H,V}} > \varepsilon_{0,\mathrm{H,V}}) \to \mathrm{constant} \times \varepsilon_{\mathrm{H,V}}^{-\alpha_{\mathrm{H,V}}}$$

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

LHC nominal beam: analysis on the 15000th turn





Horizontal & vertical normalised rms emittances(1) at the 15000th turn vs. p%-acceptance (defined as the fraction of the particle beam with emittance(2) less than a given value)

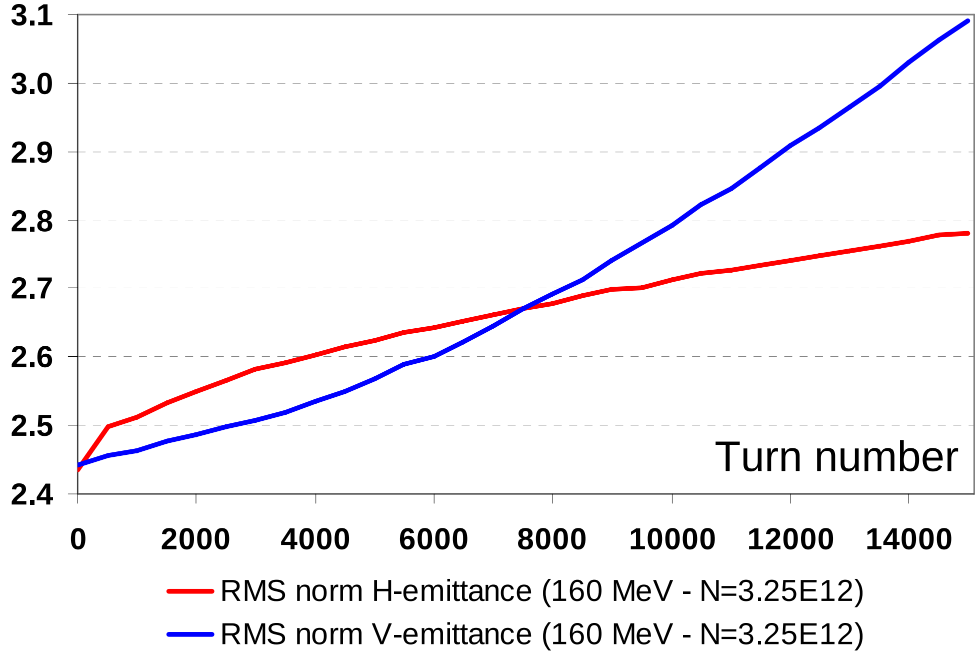
(1) Calculated from the beam "sigma" matrix

(2) Calculated for individual particles from the Courant-Snyder invariant with Twiss parameters at injection

$$\varepsilon_u(\mathrm{rms}) = \sqrt{\langle u_i^2\rangle\langle u_i'^2\rangle - \langle u_i\, u_i'\rangle^2} \quad \text{for all i such that } \varepsilon_{u,i} \le \varepsilon_u(\mathrm{p\%}) \qquad \varepsilon_{u,i} = \gamma_u\, u_i^2 + 2\alpha_u\, u_i\, u_i' + \beta_u\, u_i'^2$$

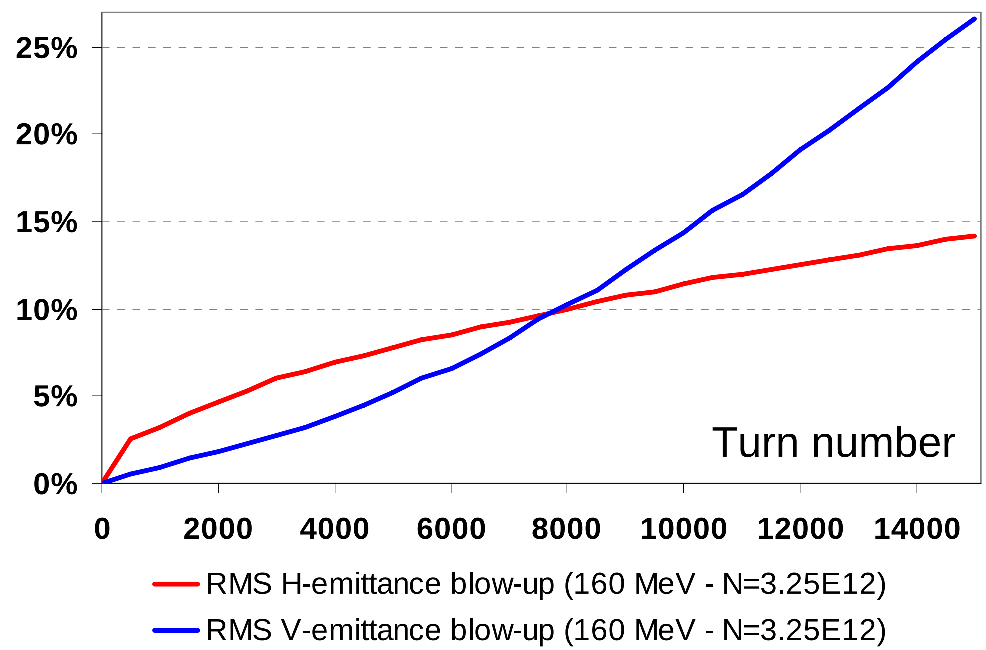
Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

Summary



Evolution of rms normalized emittances [μm]

From ACCSIM output

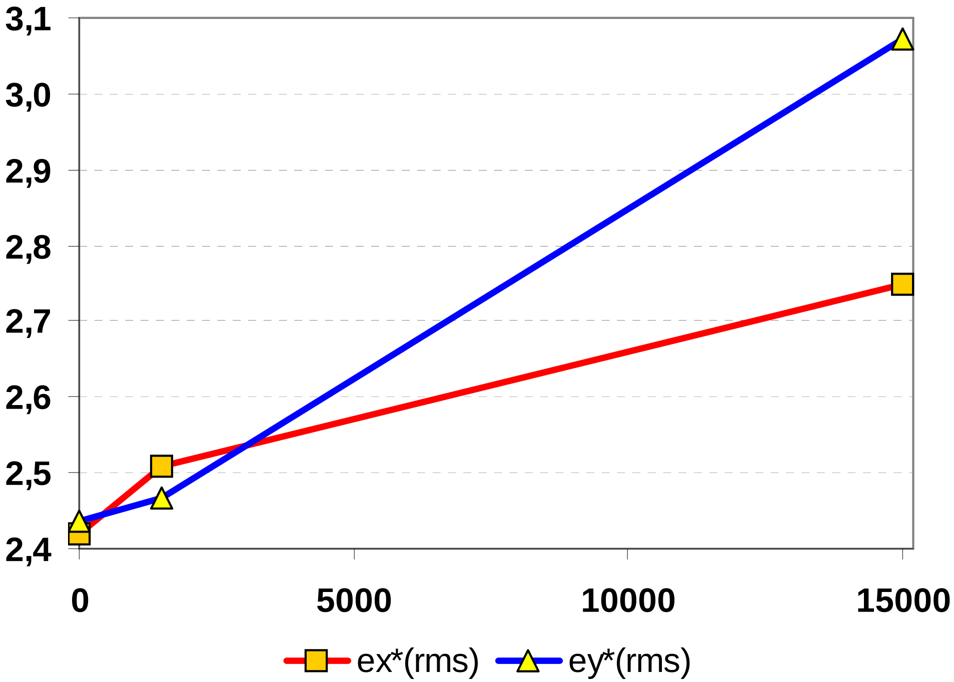


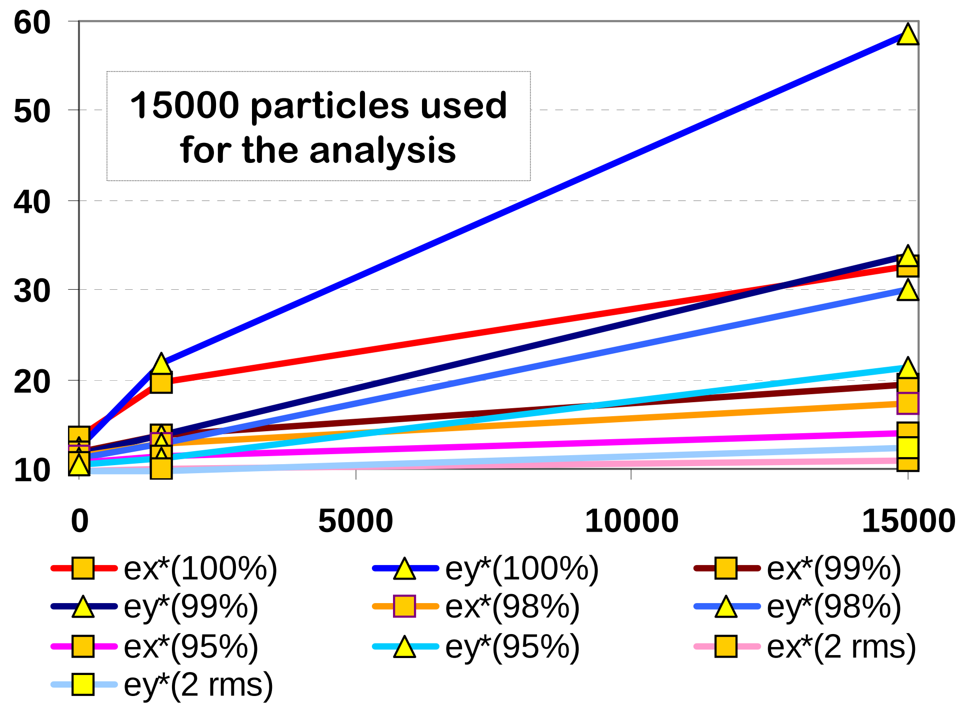
Evolution of rms normalized emittance blow-ups [%]

Cf. LIS Section meeting held on 2nd April 2007

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

Summary





Horizontal & vertical normalised rms emittances[1] vs. turns

[1] Calculated from the beam "sigma" matrix

$$\varepsilon_u(\mathrm{rms}) = \sqrt{\langle u_i^2\rangle\langle u_i'^2\rangle - \langle u_i\, u_i'\rangle^2} \quad \text{for all i}$$

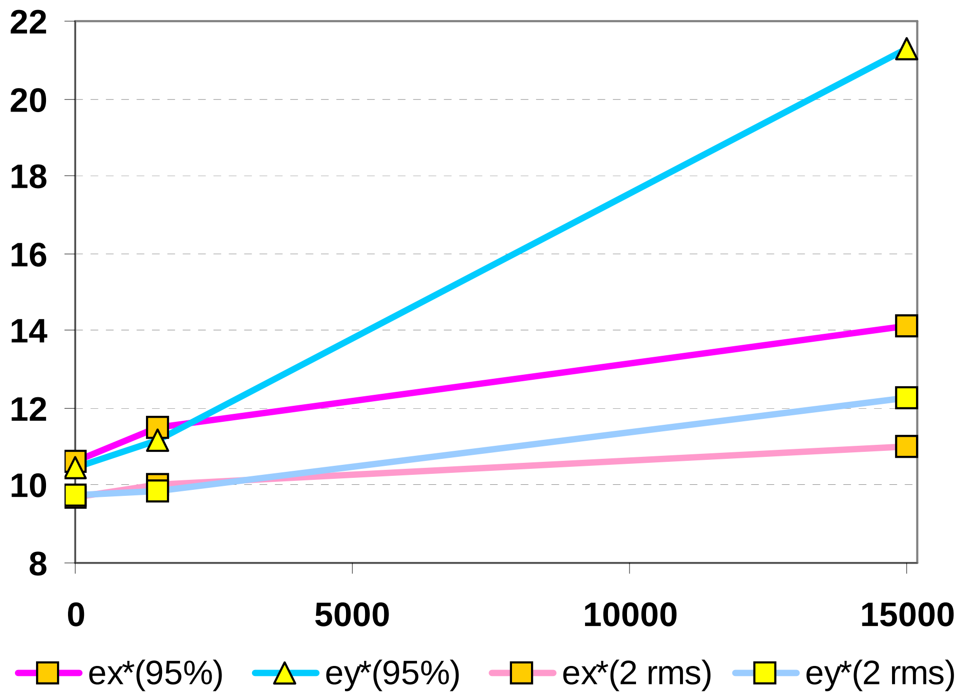
Horizontal & vertical normalised emittances at 100%, 99%, 98%, 95%[1] and 4 rms vs. turns

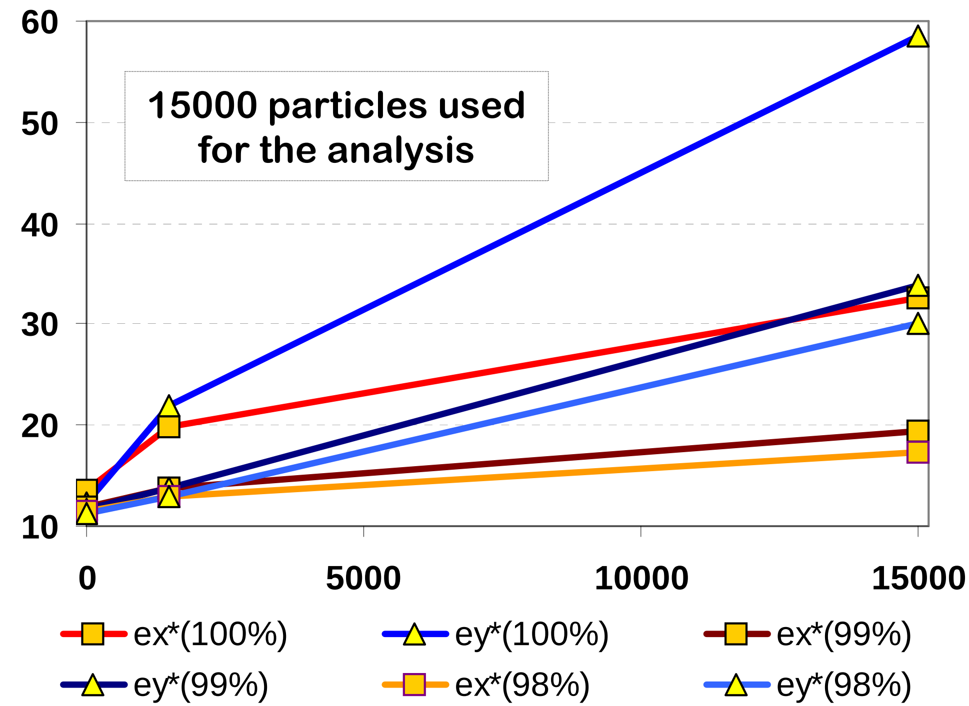
[1] Calculated Courant-Snyder invariant

$$\varepsilon_u(\max) = \max_i(\gamma_u\, u_i^2 + 2\alpha_u\, u_i\, u_i' + \beta_u\, u_i'^2)$$

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)

Summary





Horizontal & vertical normalised emittances at 95% (1) and 4 rms vs. turns

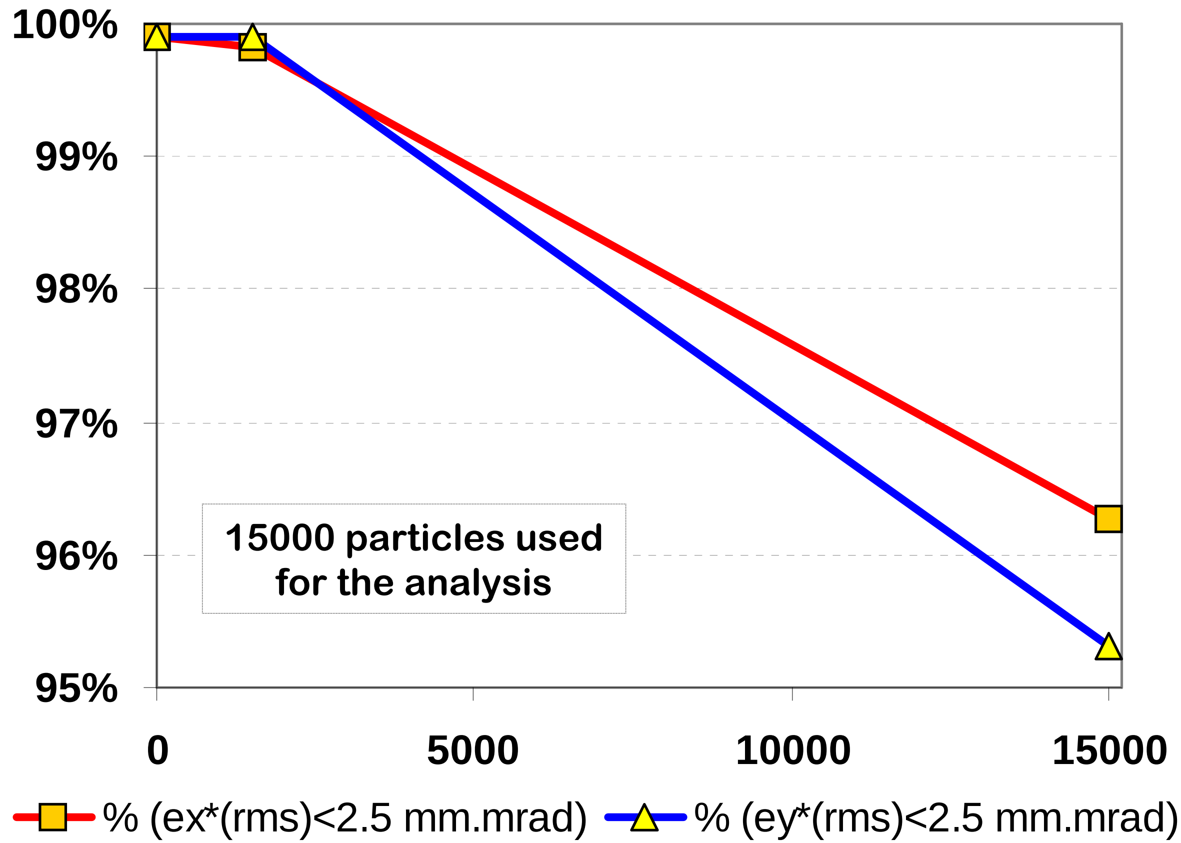
Horizontal & vertical normalised emittances at 100%, 99% and 98% vs. turns

$$\varepsilon_u(\text{rms}) = \sqrt{\langle u_i^2 \rangle \langle u_i'^2 \rangle - \langle u_i\, u_i' \rangle^2} \quad \text{for all i}$$

$$\varepsilon_u(\max) = \max_i(\gamma_u\, u_i^2 + 2\alpha_u\, u_i\, u_i' + \beta_u\, u_i'^2)$$

Q_H=4.28 Q_V=5.47 N=3.25×10[12] protons (99999 macro-particles tracked)

Summary



Fraction of the particle beam [%] with horizontal & vertical rms emittances[(1)] less than or equal to the 2.5 μm LHC nominal normalized rms emittances at PSB output vs. turns

[(1)] ***Calculated from the beam "sigma" matrix***

Q_H=4.28 Q_V=5.47 N=3.25×10^{12} protons (99999 macro-particles tracked)