Status of the HPPS studies

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Outline

- Introduction
- Layout and parameters
- Optics studies
- Magnet design
- Collimation
- Impedance budget
- Summary and next steps

HPPS

- HPPS: High Power Proton Synchrotron
- 50/75 GeV 2MW p-beam for neutrino studies (LAGUNA-LBNO)

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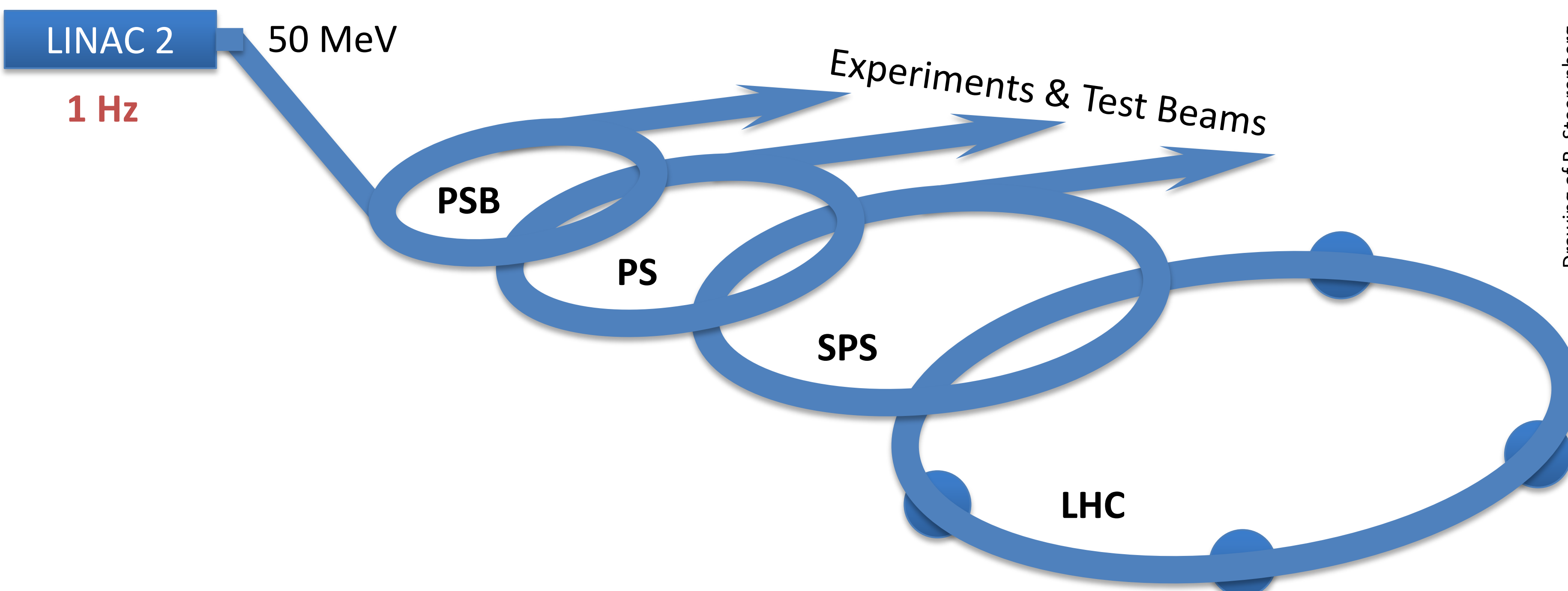
- Present:

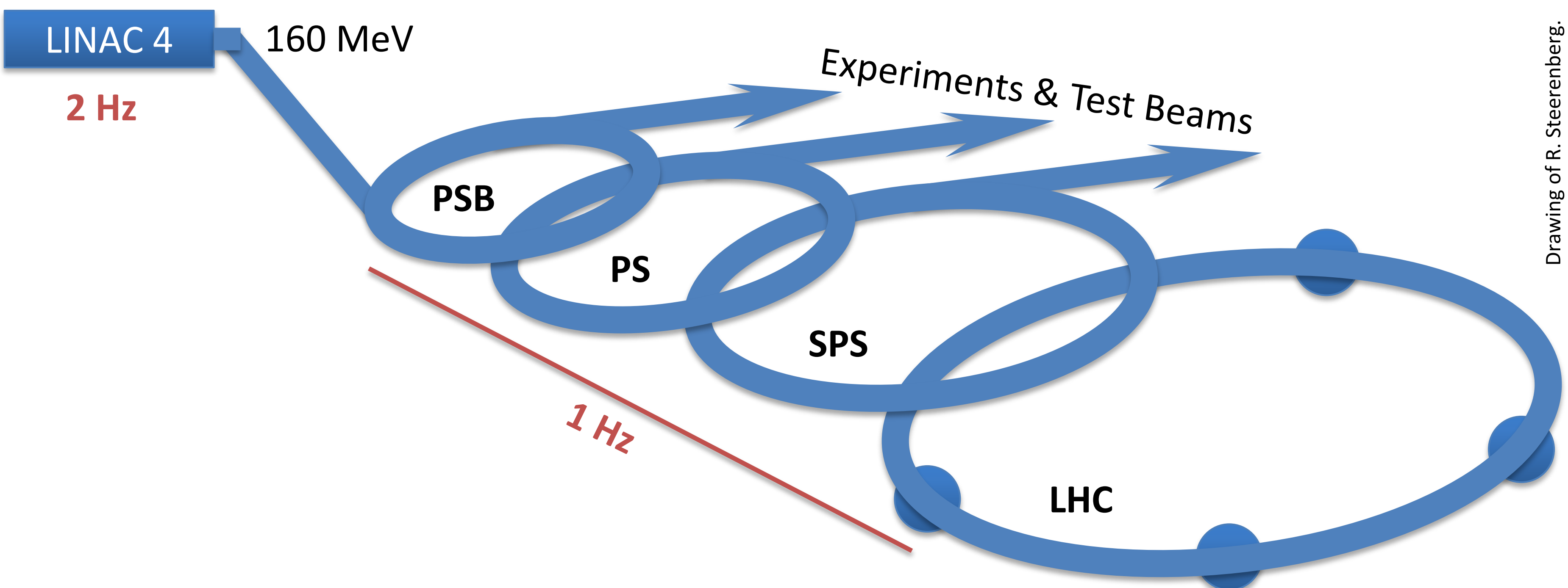


Drawing of R. Steerenberg.

HPPS

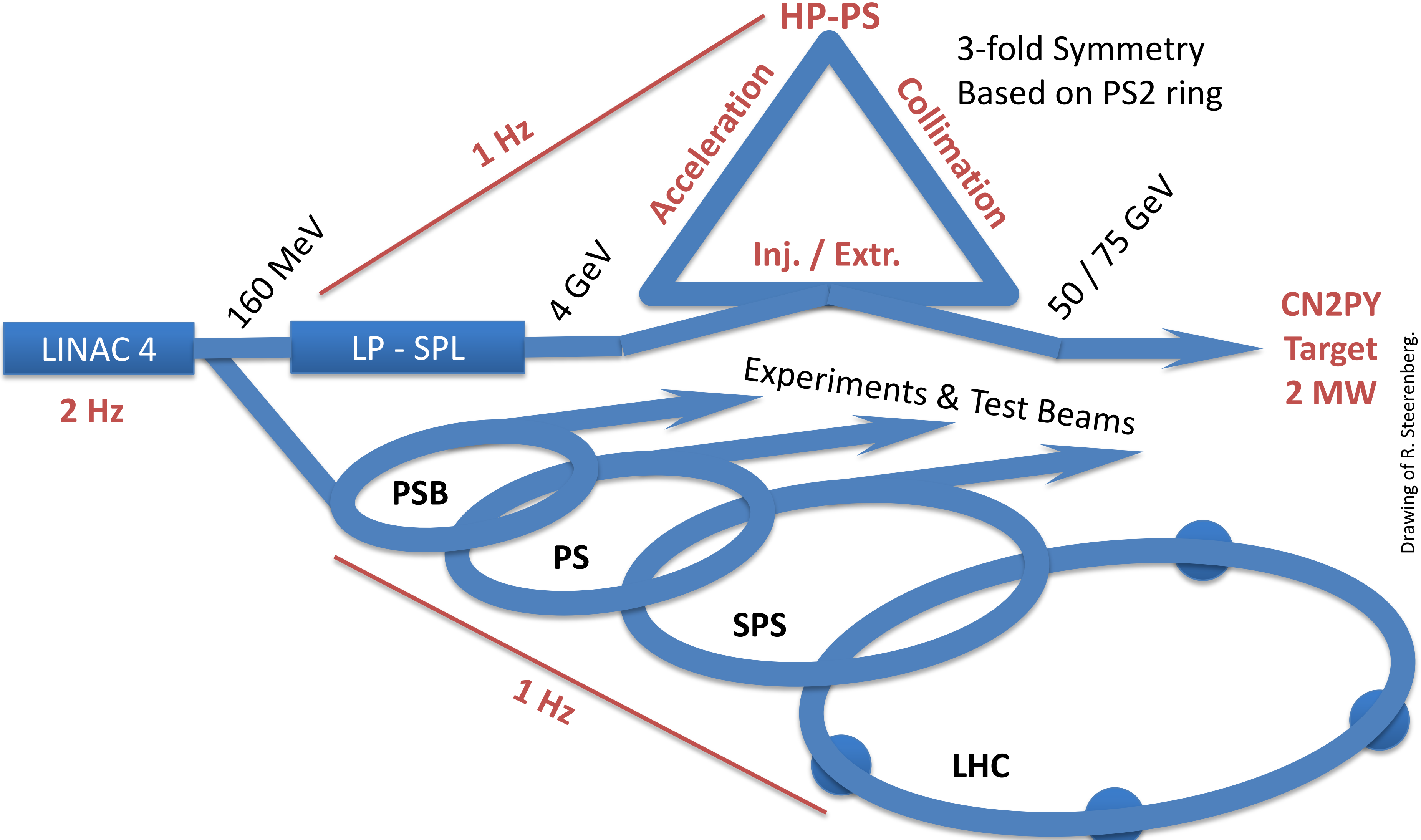
- HPPS: High Power Proton Synchrotron
- 50/75 GeV 2MW p-beam for neutrino studies (LAGUNA-LBNO)

- Linac 4 upgrade:



LINAC 4 is **an H- source, allowing phase space painting**.

HPPS


HP-PS
3-fold Symmetry
Based on PS2 ring
1 Hz
Acceleration
Collimation
Inj. / Extr.
160 MeV
4 GeV
50 / 75 GeV
LINAC 4
LP - SPL
2 Hz
CN2PY
Target
2 MW
Experiments & Test Beams
PSB
PS
SPS
1 Hz
LHC
Drawing of R. Steerenberg.

Beam Power

$$P = q f_r E_k N_p \rightarrow 2MW$$

- **High repetition rate (f_r)**
  - Maximum rep rate defined by source/linac (2Hz for LINAC4)
  - Drives magnet ramp rate (magnet design and elect. consumption)
- **High energy (E_k)**
  - For fixed magnetic field, it is proportional to circumference (cost)
  - For fixed circumference, proportional to magnetic field at extraction, linked also to ramp rate (magnet technology)
- **High pulse population (N_p)**
  - Through space-charge limit (impose tune-shift < 0.2) minimum emittance values and thereby constrains geometrical acceptance, for given ring optics (magnet technology, losses control, collimation system)
  - Can be reduced for higher rep. rate and/or energy

Layout

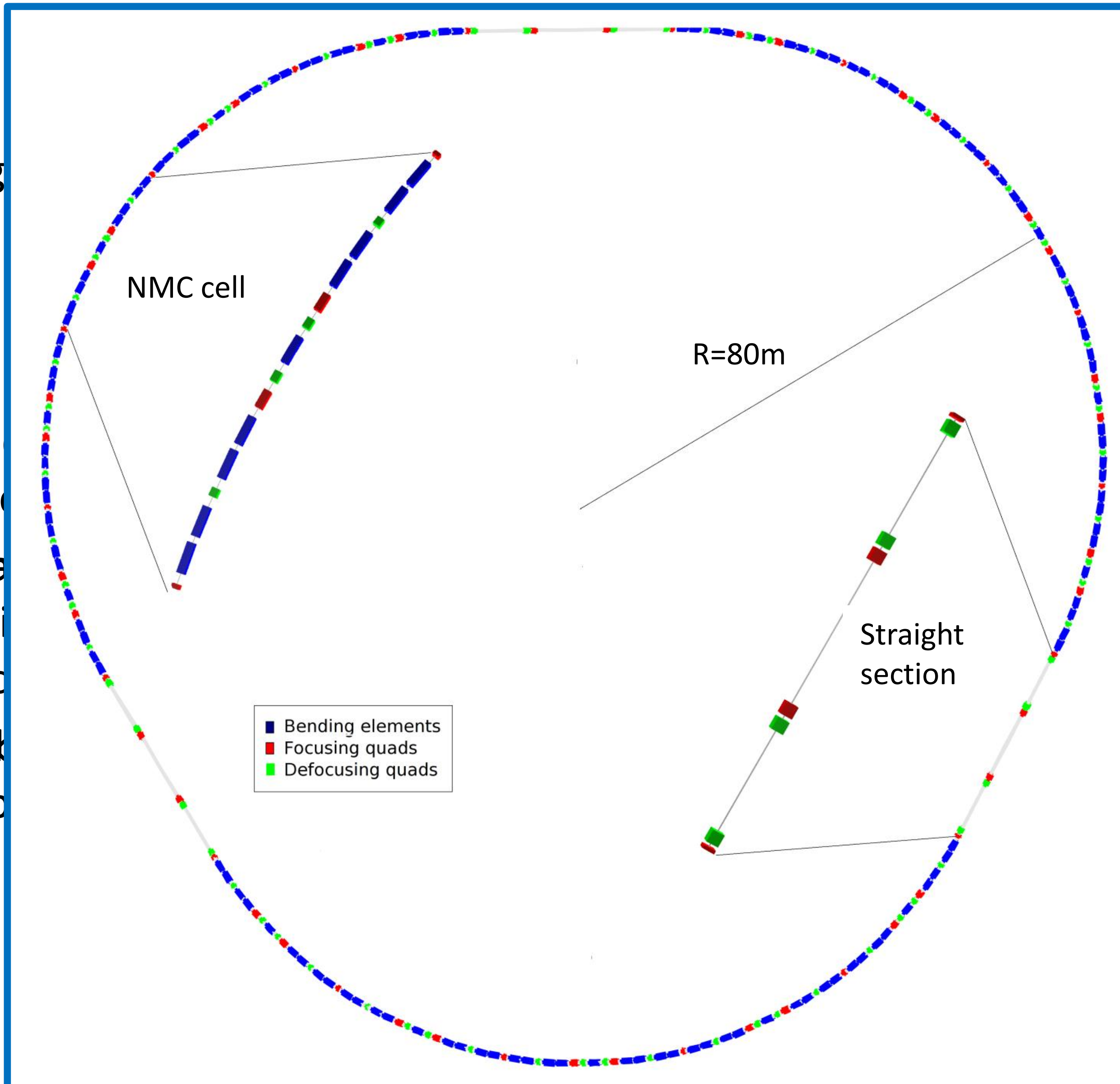
- Design based and adapted from PS2
- 2 options (50 and 75 GeV), based on same optics layout

- **3-fold symmetric** ring to accommodate in separate LSS the injection/ extraction, collimation and RF
- **Negative Momentum Compaction (NMC) arc cell** necessary to avoid transition (remain always below) and reduce losses. Use **resonant** arcs to increase filling factor (no Dispersion Suppressors)
- **Doublet Long Straight Section (LSS)** leave more space for BT equipment, collimation and RF

Layout and parameters

- Desig
- 2 opt

- **3-fol** ... /
  extrac
- **Nega** ... avoid
  transi ... **nt** arcs
  to inc
- **Doub**
  equip



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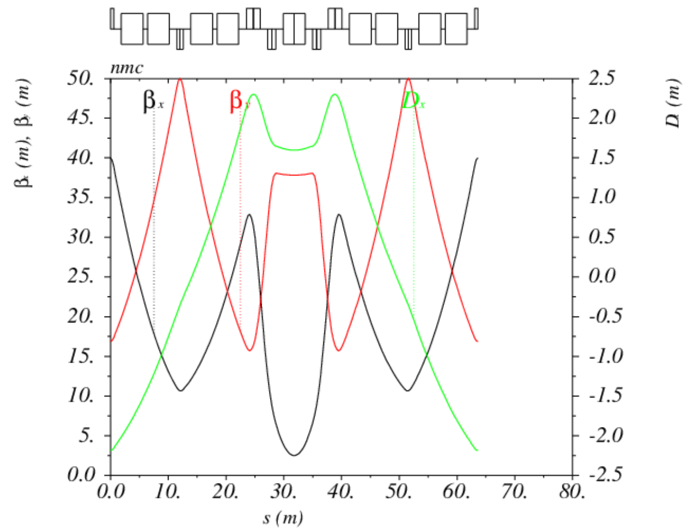
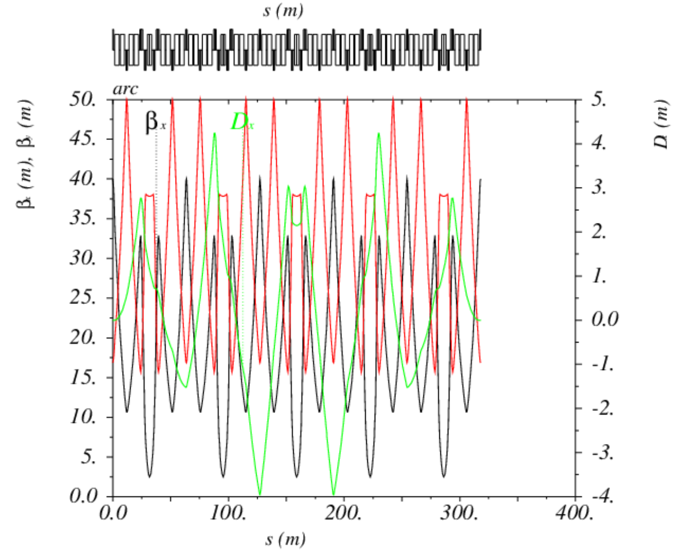
Layout

- De
- 2 c
- **3-** ex
- **N** tra to ... id ... rcs
- **D** eq

- **5 resonant NMC arc cells** with horizontal phase advance tuned to **8π** for disp. suppression
- High dispersion reduce strength of sextupoles. 2 sext families. **Very good non-linear dynamics performance**
- Due to space constraints can only achieve 41GeV for dipole field of 1.7T (limit for iron dominated magnets)
  - **Need super-ferric magnets @ 2.1T to reach 50GeV**





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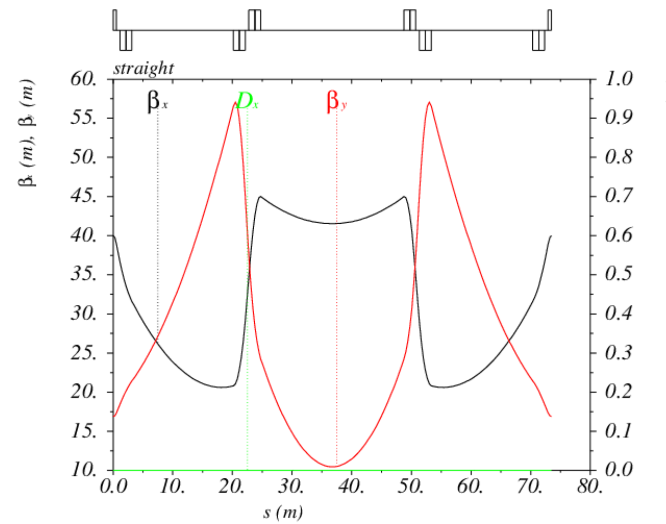
Layout

- Design based and adapted from PS2

- **Quadrupole doublet LSS leave more space** for beam transfer (BT) equipment, collimation and RF
- 73.5 m length per straight section
- **4 quad families are used for achieving horizontal optics constraints** (BT equipment, collimator,…) and general tuning
- **Horizontal tunability provided only by LSS, vertical is flexible**



equipment, collimation and RF

Parameters

Parameter	HP-PS	
Circumference [m]	1174	
Symmetry	3-fold	
Beam Power [MW]	2	
Repetion Rate [Hz]	1	
Kinetic Energy @ inj. [GeV]	4	
Kinetic Energy @ ext. [GeV]	50	75
Protons/pulse [10^{14}]	2.5	1.7
Dipole ramp rate [T/s]	4.2	5.9
Bending field @ext [T]	2.09	3.13
Max. quad field [T]	1.36	1.82
Dipole gap height [mm]	111	92
Norm. emit H/V [um.rad]	15/12.8	10.6/8.3

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LE scenario:

- more demanding beam dynamics (higher bunch population → more dominated by collective effects)
- more conventional magnet parameters

Parameters

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Repetion Rate [Hz]	1	
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HE scenario:

- required pulse intensity reduces from 2.5 to 1.7E14 p^+
- demanding magnet technology (high field+ramp rate)

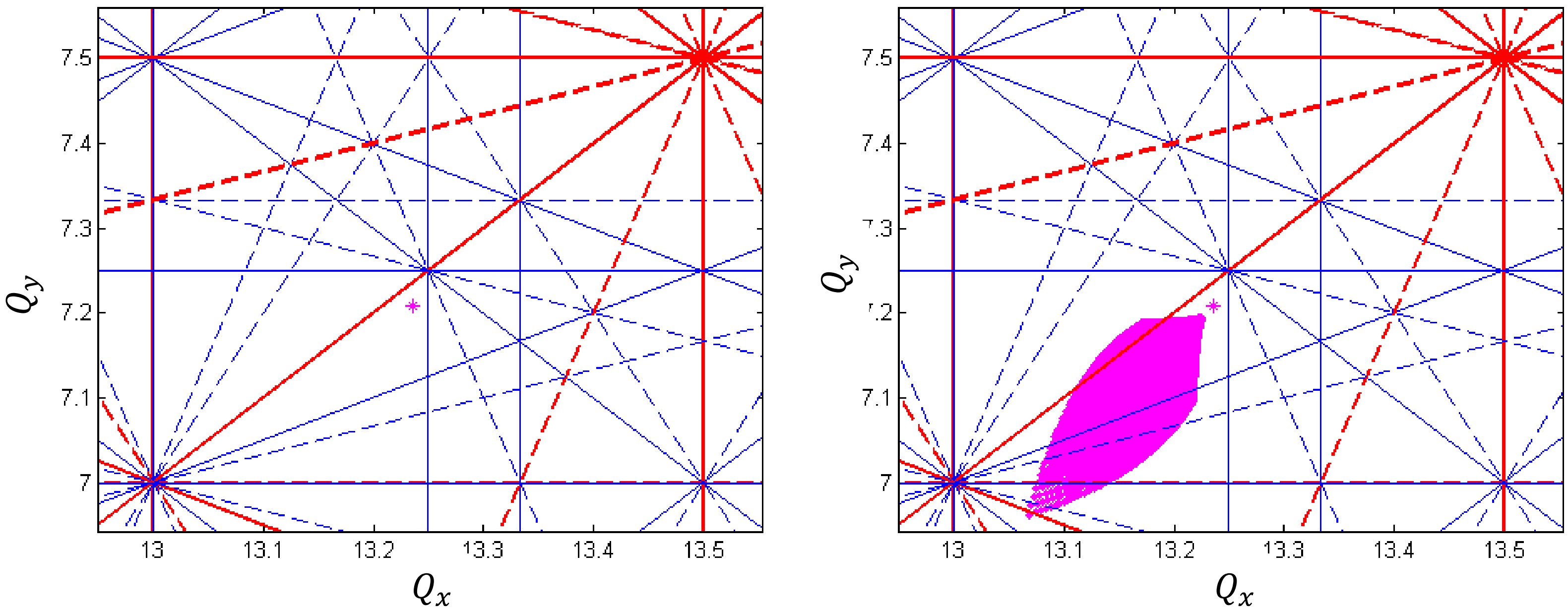
Space Charge detuning

$$\delta Q_{x,y} = -\frac{N_b r_e}{(2\pi)^{3/2}\beta^2\gamma^3\sigma_s}\oint\frac{\beta_{x,y}}{\sigma_{x,y}(\sigma_x+\sigma_y)}ds$$

- **Intensity limited by** space-charge and **other collective effects**, especially at injection flat bottom
- **Calculation of the linear part of the space charge detuning** (Laslett) for Gaussian bunches (**pessimistic consideration**)
- Beam considered as for the PS2 with a **25 ns bunch structure, 17.8 ns bunch length and 6.43e-3 energy spread.**

- **For keeping space-charge tune-shift below -0.2, the vertical emittance has to be increased accordingly and transverse acceptance reduced**

Space Charge detuning



- Nominal working point: Qx=13.24, Qy=7.21
- The Lasslett tune shift up to 4σ and for emittance values of εx=10 um.rad and εy=8 um.rad
- Crossing of the (2, -2) resonance

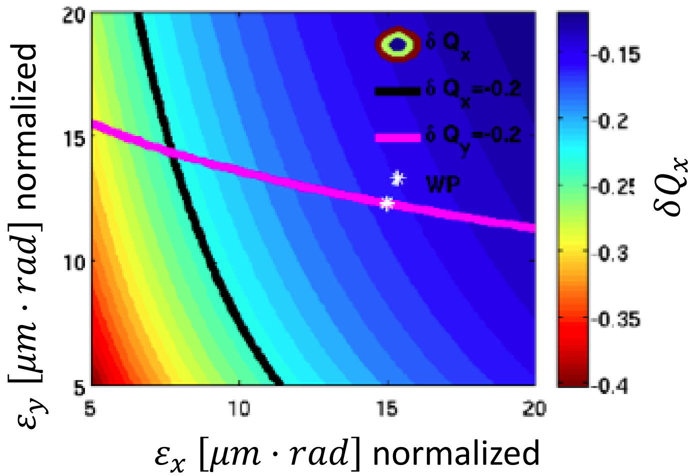
Space Charge detuning

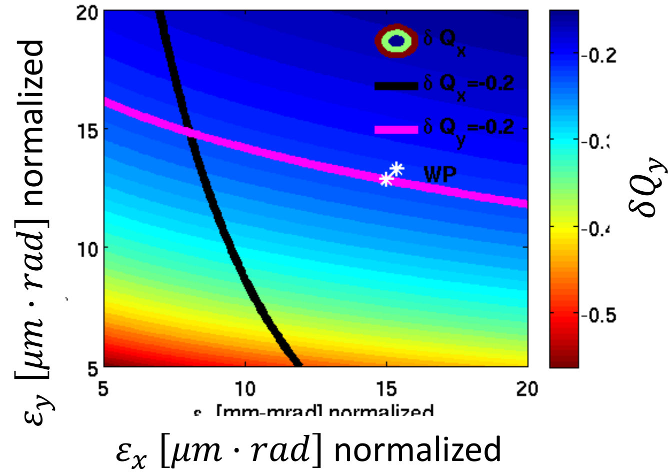
Emittance area parametrized with H and V space-charge tune shift

50 GeV

$\varepsilon_x = 15\ \mu m \cdot rad$

$\varepsilon_y = 12.3\ \mu m \cdot rad$

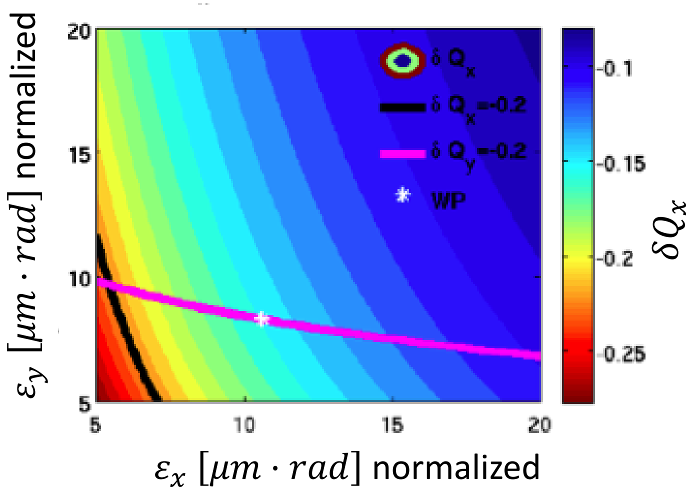


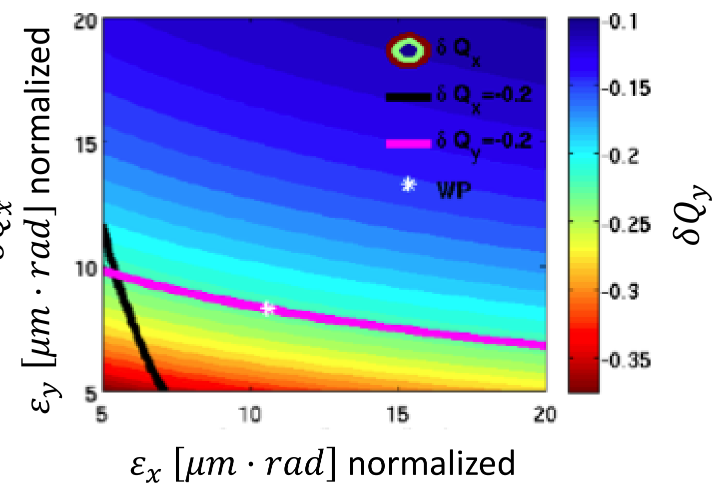


75 GeV

$\varepsilon_x = 10.7\ \mu m \cdot rad$

$\varepsilon_y = 8.3\ \mu m \cdot rad$





Small vertical beam size chosen since vertical acceptance more critical.

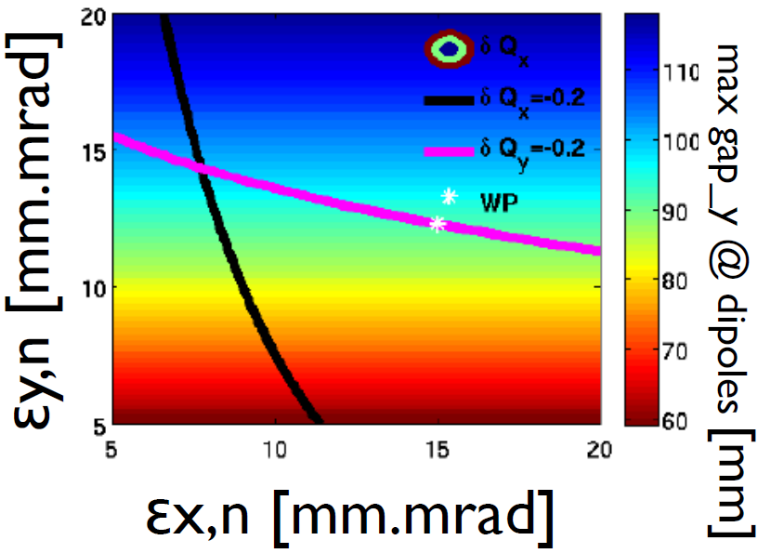
Geometrical acceptance

Emittance area parametrized with dipole gap height

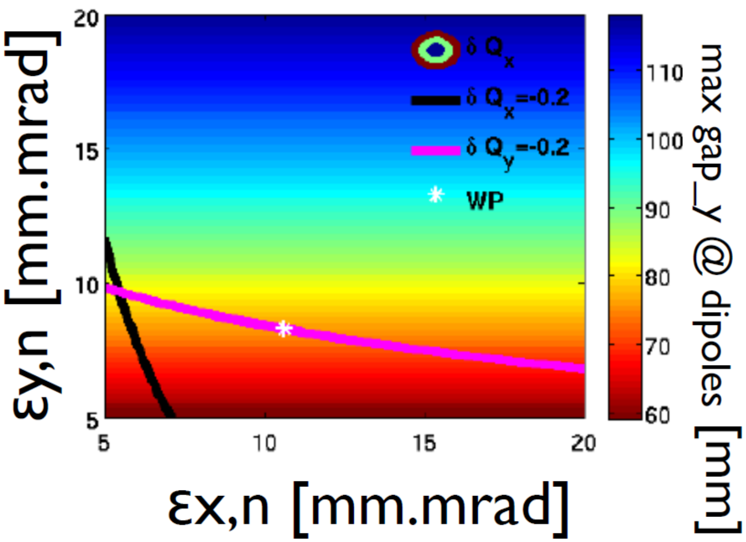
$$R^{\min}{}_{x,y} = n_{\sigma_{x,y}}\sqrt{\beta_{x,y}\varepsilon_{x,y}} + \eta_{x,y}\left(\frac{\delta p}{p_0}\right)_{\max}$$

- R^{min}: the minimum beam pipe radius to fit all the particles
- **Calculations done at 4.5 σ in both planes**

50 GeV



75 GeV



Magnet parameters

50 GeV HPPS

Type	Length [m]	Strength [m^{-2}]	B_{pt} [T]	R_{xy} [mm]*
Type1 (LSS)	2	-0.0832	-0.89	63
Type2 (ARC)	1.4	-0.0657	-0.55	49
Type3 (ARC)	1.1	-0.0424	-0.42	58
Type1 (LSS)	2	-0.0136	-0.15	63
Type3 (ARC)	1.1	0.0494	0.49	58
Type1 (LSS)	2	0.07	0.75	63
Type4 (ARC)	2.4	0.1142	1.36	70

Number of type 1: 18
Number of type 2: 30
Number of type 3: 48
Number of type 4: 30

Total number: 126

Dipole characteristics (135 dipoles):
B = 2.1 T with
Ramp rate: 3.82 T/s
Max gap_x = 145.7 mm
Max gap_y = 110.9 mm

*Beam pipe thickness considered: 8mm

Magnet parameters

75 GeV HPPS

Type	Length [m]	Strength [m^{-2}]	B_{pt} [T]	R_{xy} [mm]*
type1	2	-0.0832	-1.1	52
type2	1.4	-0.0657	-0.73	44
type3	1.1	-0.0424	-0.54	50
type1	2	-0.0136	-0.18	52
type3	1.1	0.0494	0.63	50
type1	2	0.07	0.92	52
type4	2.4	0.1142	1.82	63

Number of type 1: 18
Number of type 2: 30
Number of type 3: 48
Number of type 4: 30

Total number: 126

Dipole characteristics (135 dipoles):
B = 3.13 T with
Ramp rate: 5.9 T/s
Max gap_x = 132.6 mm
Max gap_y = 92.3 mm

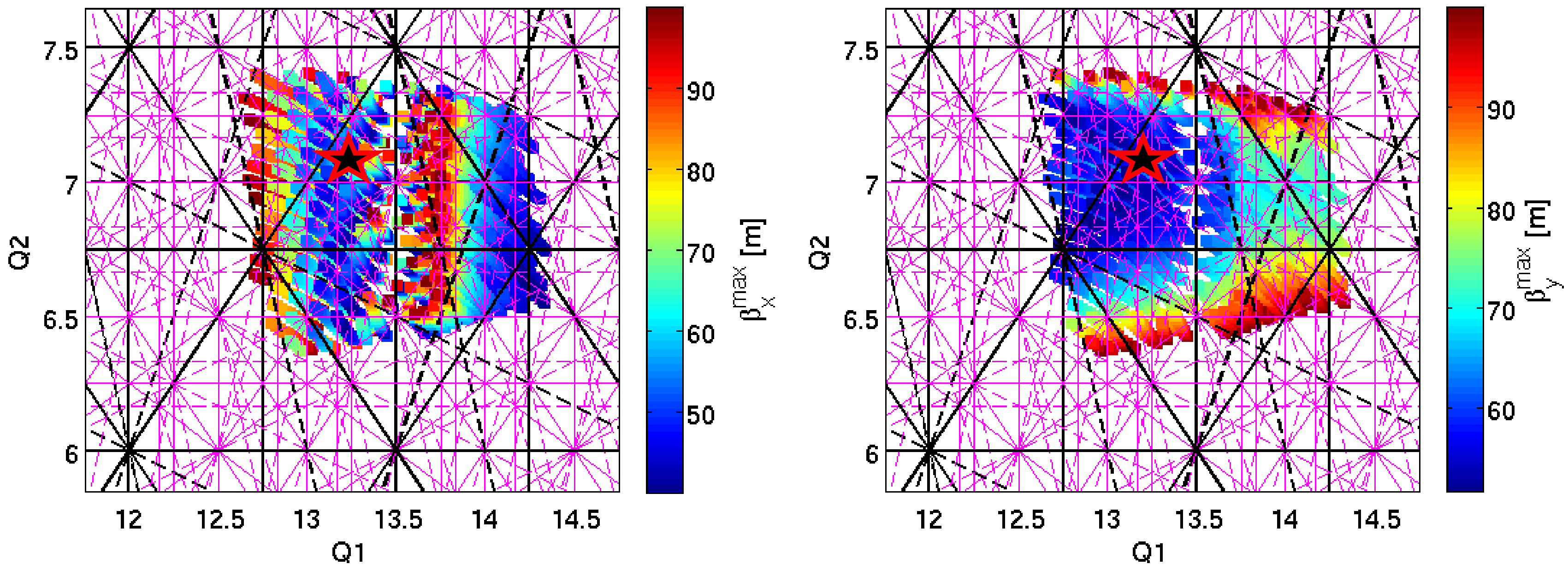
Challenging

*Beam pipe thickness considered: 8mm

Tunability studies

Long straight section tunability

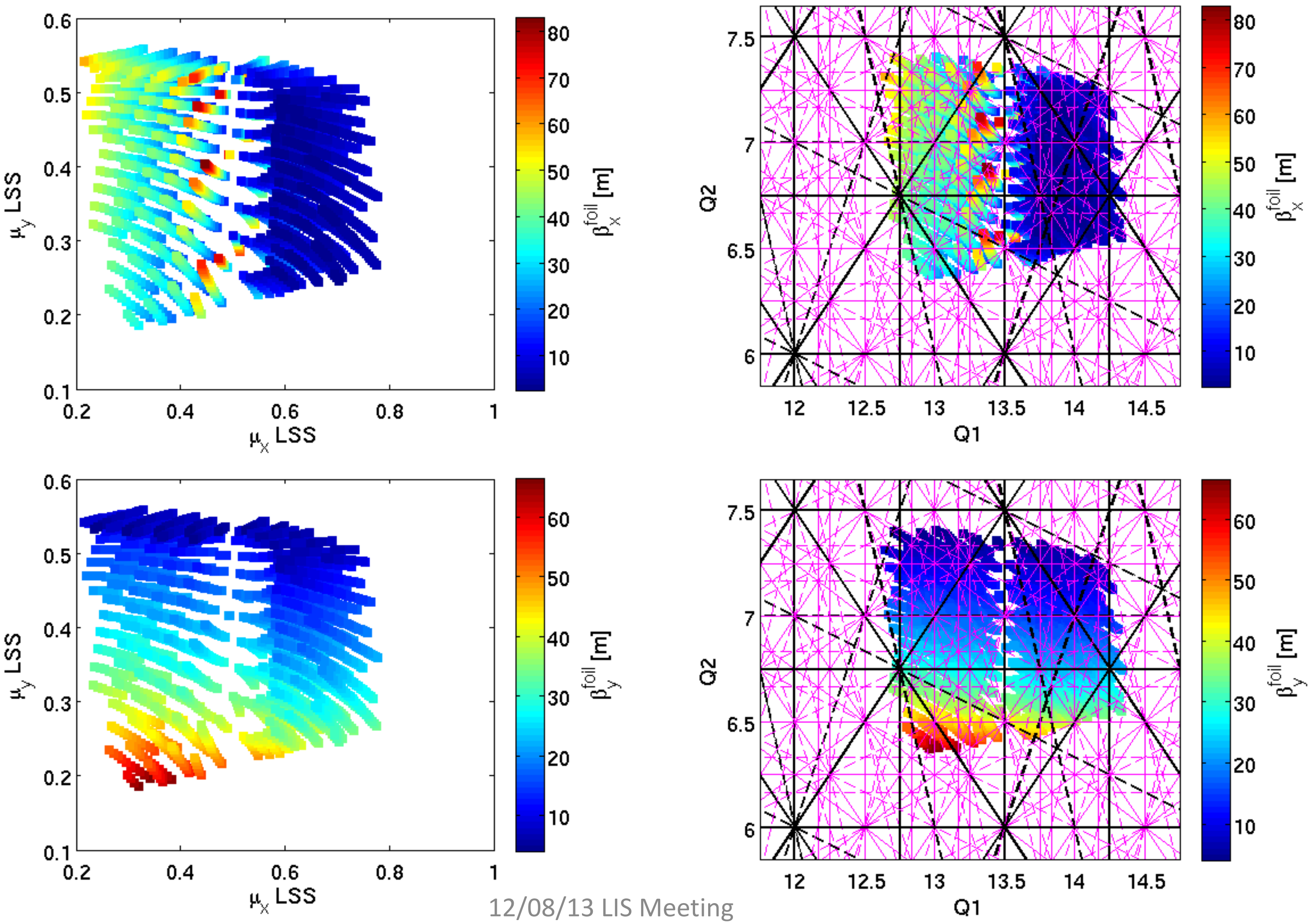
- **Arc quadrupoles fixed, LSS quadrupoles varied and only stable solutions are kept** (around 30k cases studied)



- Parameterization of the **horizontal and vertical tunes of the ring with the maximum** horizontal (left) and vertical (right) **beta functions**
- **Resonances up to 4th order** are shown
  - Systematic resonances are shown in black while non systematic in pink. The normal resonances are shown with solid lines while the skew ones with dashed lines.

Tunability studies

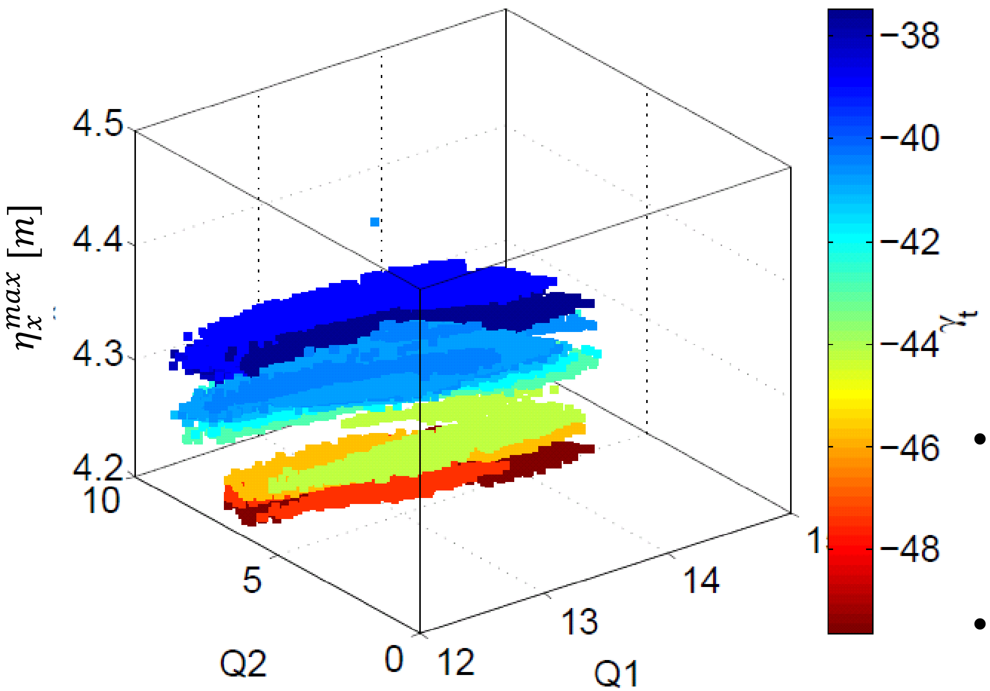
Long straight section tunability - FOIL

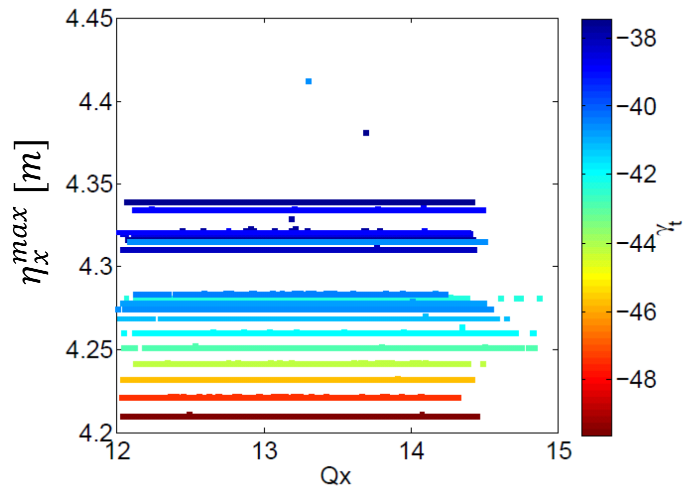


Tunability studies

Global tunability

- **The vertical phase advance μ_y of the arc is rematched. For each μ_y the LSS quad strengths are scanned** (around 30k cases studied for each phase advance)





- **Parameterization of the transition energy with** the horizontal and vertical tunes and max dispersion (projection on top plot)
- Strong dependence on the max dispersion (as expected)
  - Smaller γ_t for larger dispersion

Orbit correction

- Evaluate efficiency and performance of orbit correction system.

Applied random errors (Gaussian cut at 3 sigma)	RMS (following PS2 experience)
Relative dipole field error	5.00e-4
Transverse quadrupole shift	0.2 mm
Longitudinal dipole shift	0.3 mm
Dipole tilt	0.3 mrad

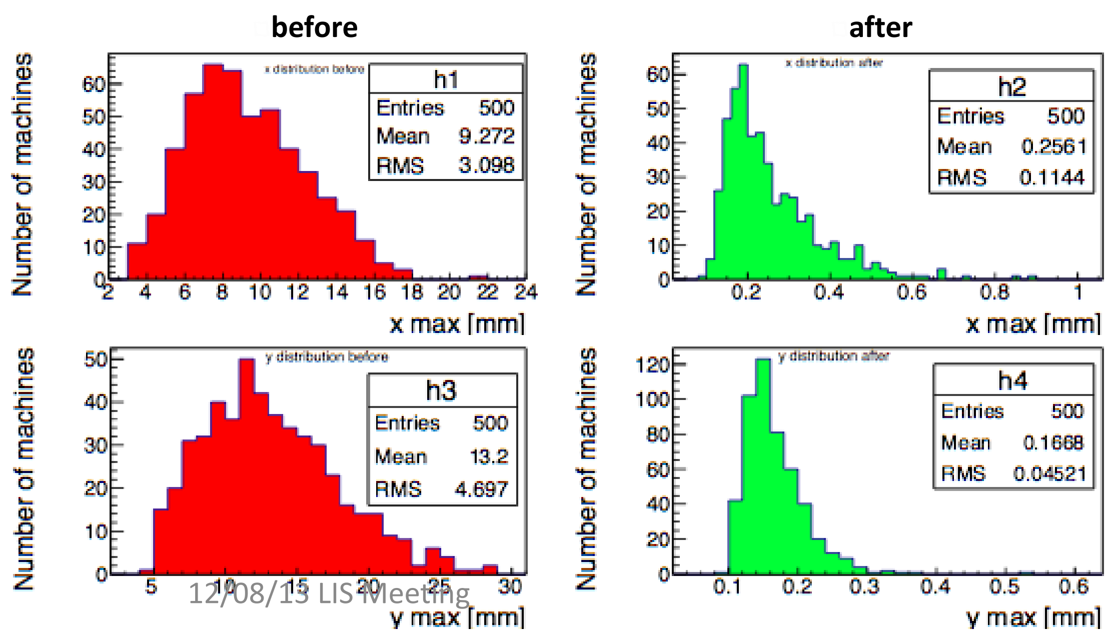
CORRECT module (MADX)
MICADO algorithm
500 random seeds

Orbit corrected by an order of magnitude

Maximum kicks well within limits

x

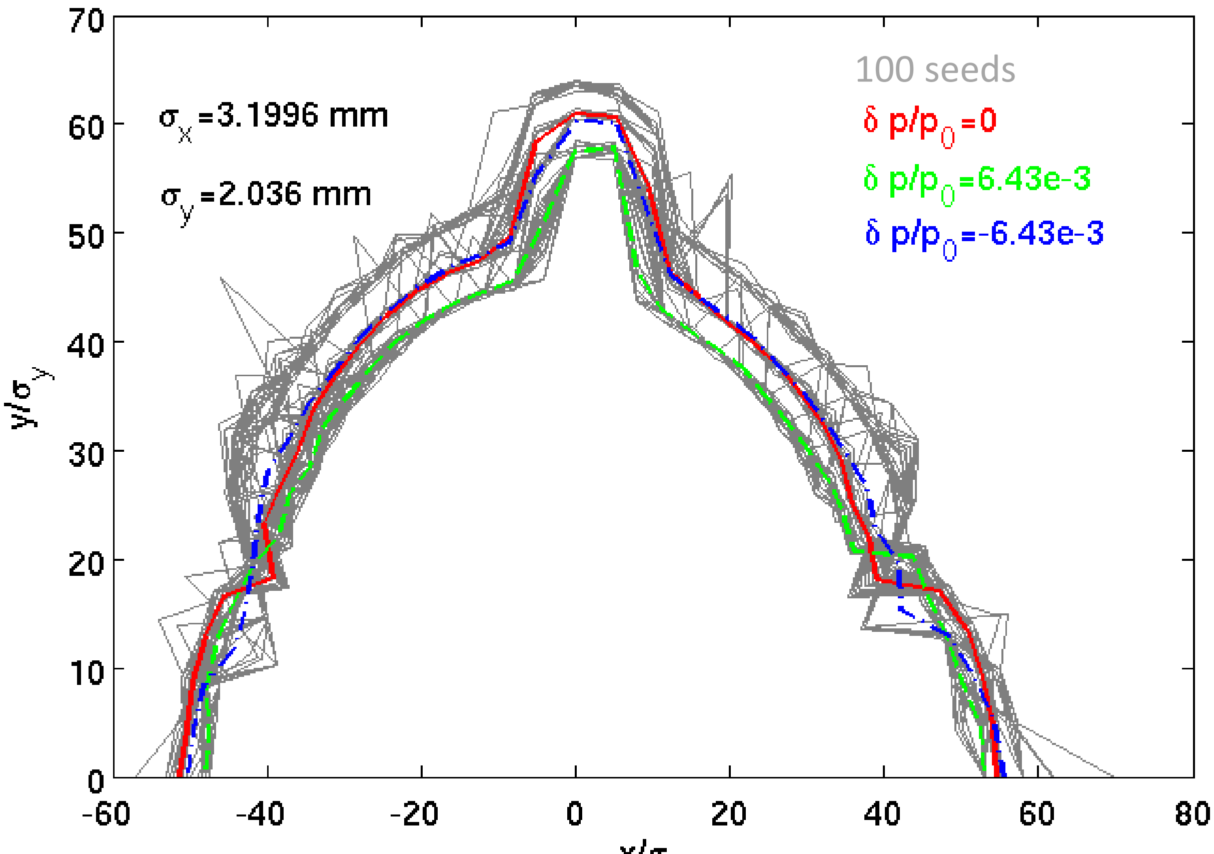
y



Dynamic Aperture studies

Closed Orbit correction

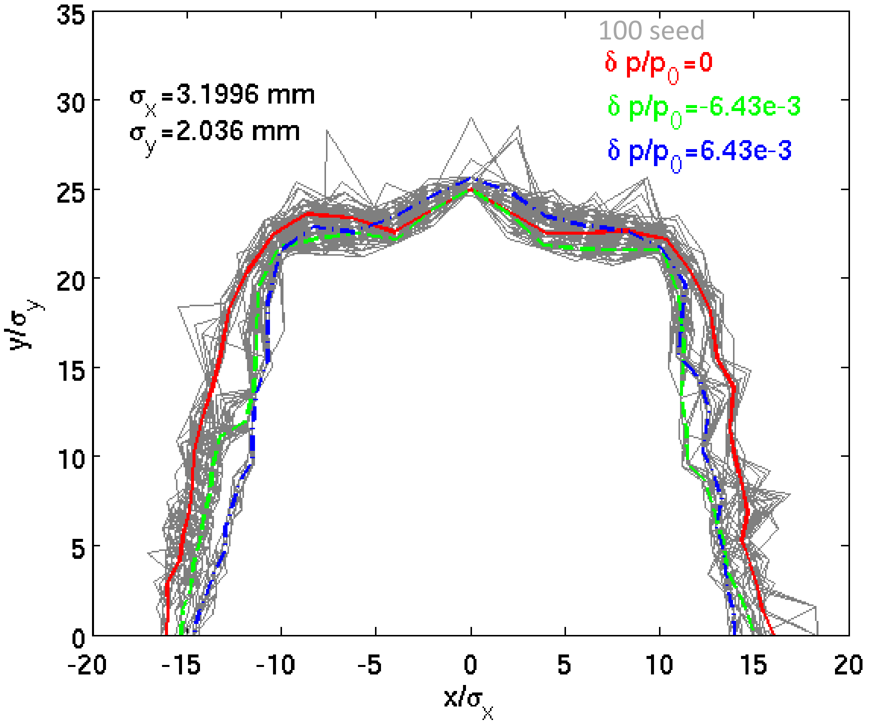
- On and off momentum DA calculations after CO correction (100 seeds (gray))
- **Very large DA**, not limited by orbit errors.



Dynamic Aperture studies

Multipole field errors

- On and off momentum DA calculations including field errors, CO correction, chromaticity correction and rematching of tunes to ideal lattice (100 seeds)
- **DA dominated by multipole field errors but still very comfortable**.



Order	Dipoles (R=3 cm)		Quadrupoles (R=5.95 cm)		Sextupoles (R=5.95 cm)	
	mean	random	mean	random	mean	random
n	b_n/b_1	b_n/b_1	b_n/b_2	b_n/b_2	b_n/b_3	b_n/b_3
1	10^4	5	0	0	0	0
2	0.15	0.1	10^4	5	0	0
3	1	0.5	-2	1	10^4	5
4	0.013	0.064	1	1	-0.5	1.5
5	-0.1	0.064	1	1.5	0.5	1.5
6	-0.003	0.003	3	1	-1	0.5
7	-0.026	0.005	0.5	1	1	0.5
8	0.001	0.001	0.5	0.5	0.5	0.5
9	-0.004	0.001	0.1	0.3	-4	0.3
10	-	-	0.5	0.3	0.1	0.5
11	-	-	0.1	0.3	0.1	0.5

Ref. PS2 CDR (H. Bartosik): Relative multipole components in units of 10-4 at the reference radius R.

Super-ferric dipoles

- For iron dominated dipoles of 1.7 T (limit for this kind), ring could only achieve 41 GeV

- **Necessary to use super-ferric magnets**
  - SF magnet: **Warm iron dominated magnet with super conducting coils**
  - **Proto-type was built and tested successfully** (for PS2, also considered in SIS synchrotron, FAIR project).
  - The **CERN magnet group is interested to further develop this technology** and push the performance with the help of the HP-PS design team.
  - 2.1 T and 3.82 T/s ramp rate for 50 GeV; 3.1 T and 5.95 T/s for 75 GeV

Collimation

- **Collimators to avoid magnet quenching, limit the equip radiation and localise slow losses.**
- **Scrapper-absorber scheme** similar to PS2 one.
- **Optimum position** of absorber wrt scrapper:

$$\cos(\Delta\mu_{a,1,opt}) = {}^{N_s}/_{N_a}$$

$$\Delta\mu_{a,2,opt} = \pi - \Delta\mu_{a,1,opt}$$

→ Ns=2.5, Ns=3 (From PS2) → $\Delta\mu_{a,1}$: 33.5º $\Delta\ \mu_{a,2}$: 146.4º (Ideal position)

Ns, Na: normalised apertures of scrapper and absorber
$\Delta\ \mu_{a,1}$, $\Delta\ \mu_{a,2}$: betatron phases advances of the absorbers wrt scrapper

- **Preliminary available location**:

$\Delta\mu_{a,1,x}$: 25.8º $\Delta\ \mu_{a,2,x}$: 148.6º
$\Delta\mu_{a,1,y}$: 31.6º $\Delta\ \mu_{a,2,y}$: 154.1º

Possible to fit in LSS

- Code has been written to find losses due to aperture.
- Next step: Find optimal thickness and install collimators.

Impedance budget

First parameter estimation:

Parameter	PS2 INJECTION	PS2 EXTRACTION	HPPS-50 INJECTION	HPPS-50 EXTRACTION
Energy[GeV]	4	50	4	50
C[m]	1346.4		1174	
b[mm](pipe radius)	32.5		32.5	
σ_z[m]	1.12	0.30	4.38	4.38
$\overline{\beta_y}$[m]	32		31.61	
$\varepsilon_y[\mu m\ rad]$	1.8	0.17	2.85	0.23
η (slip factor)	-0.037	-0.0012	5.56e-2	8.91e-4
σ_δ	3.2e-3	1e-3	3.8e-3	3.2e-4
N_b	4.2e11		18e11	
N_p	1.1e14		2.5e14	
ν_s (synch. tune)	18e-3	0.8e-3	6.13e-3	7.27e-5
Q_y	6.71		7.21	
I[A]	2.7		11.52	

Impedance budget

Impedance		PS2 Injection	HPPS-50 Injection	PS2 Extraction	HPPS-50 Extraction
Broad band (RW+SC) Longitudinal Impedance	$\frac{Z}{n}[\Omega]$	0.39+49i	0.74-22.80i	0.20+0.13i	0.76-0.38i
Transverse kick factor	k_y[V/pC/m]	24	10.6	47	10.6

PS2 has in account 1500 vacuum flanges

Analytic estimations following: "Impedance considerations for the design of the vacuum system of the CERN PS2 Proton Synchrotron", K.L.F Bane et al.

Instability		PS2 Injection	HPPS-50 Injection	PS2 Extraction	HPPS-50 Extraction
Microwave instability [Single bunch long.]	$\frac{N_{th}}{N_b}$	27	112	59	4.3
TMCI [Single bunch transverse]	$\frac{N_{th}}{N_b}$	10	2.2	2.5	0.3
TCBI [Multi-bunch transverse]	Turns	30	13	294	141

Summary

- 2 ring options for a 2MW proton beam: 50 GeV and 75 GeV.
- Space charge tune shift defines emittances and magnet aperture.
- Good working point from tunability studies.
- Dynamic aperture dominated by multipole field errors but still comfortable.
- Super-ferric dipoles are necessary. Technology already tested. 75 GeV challenging due to both high field and high ramp rate.
- Collimation system under design. First location of collimators given.
- First impedance budget calculations show some challenges to be discussed.

Next steps

- Collimation system optimisation.
- Space charge studies with realistic beam distribution and including collimation system.
- Magnet design.
- Impedance budget details.
- Beam instrumentation inventory.
- …

THANKS!

Special thanks to H. Bartosik for all the PS2 examples and help

BACKUP SLIDES

What can the LP-SPL deliver ?

- The LP-SPL base line parameters are:

Parameter	Values	Units
Kinetic Energy	4	[GeV]
Beam power	0.144	[MW]
Repetition rate	2	[Hz]
Beam pulse length	0.9	[ms]
Average pulse current	20	[mA]
Peak pulse current	32	[mA]
Protons per pulse	1.13×10^{14}	-
Peak power per cavity	0.5	[MW]

- The present design of the LP-SPL can "only" deliver 1.13×10^{14} ppp.
- We need up to 2.5×10^{14} ppp.

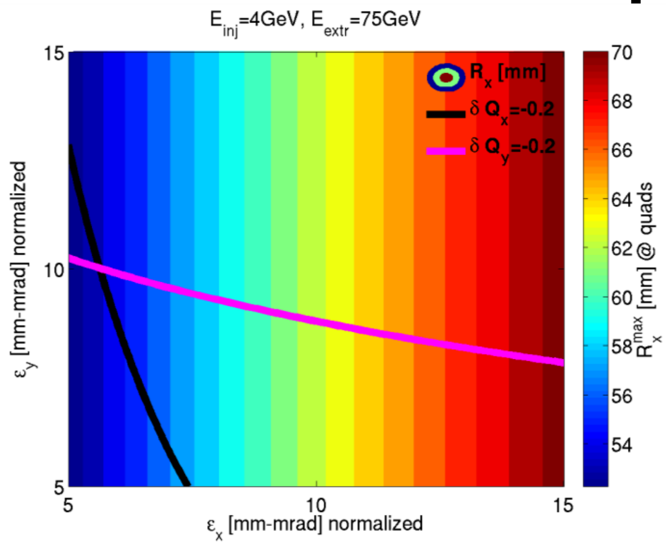
Possible LP-SPL modifications to increase intensity / beam power

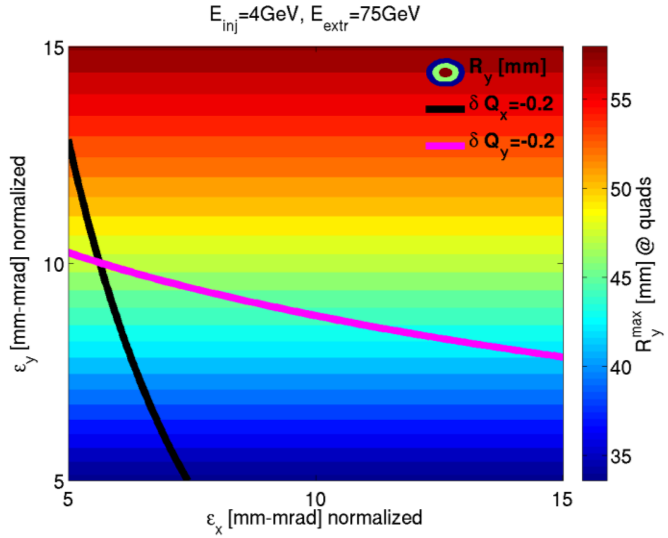
- **Easy & cheap**:
  - Higher repetition rate or longer pulse length.
  - Impact on modulator stored energy, increase of cryogenic load, little difference for klystrons.
- **Expensive**:
  - Higher pulse current.
  - Higher peak power, direct impact on klystron price.
  - If really necessary 40 mA can be made possible, but not higher.
- **Very Expensive:**
  - Higher energy (5 GeV instead of 4 GeV).
  - Longer tunnel more expensive hardware…..

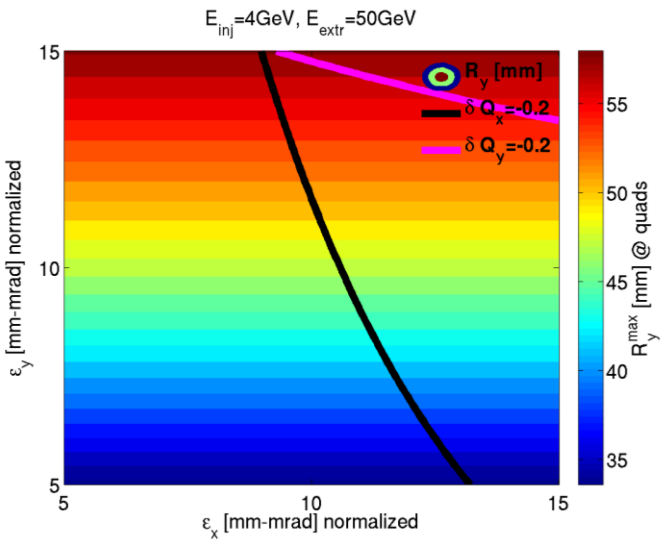
- For the 30 GeV option the repetition rate could be increased to 3 (or 4) Hz:
  - Reduce intensity per pule
  - 2 (3) pulses for HP-PS, 1 pulse for Existing complex
  - HP-PS would need to pulse at 3 (or 4) Hz rate too, with 1 pulse idle.

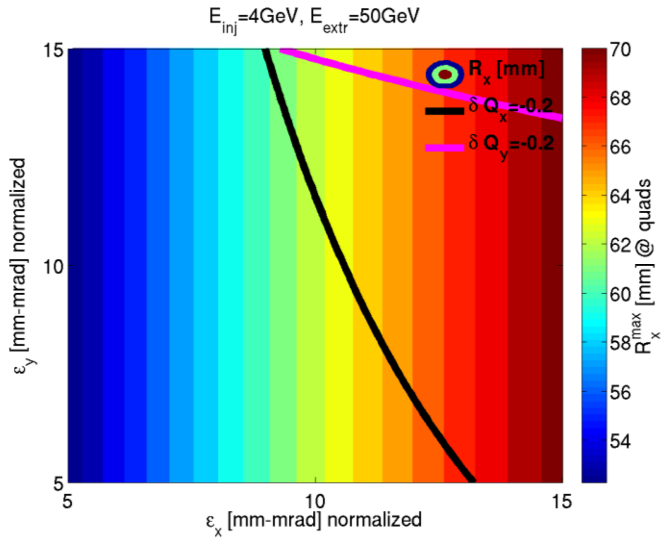
- Increasing the pulse length by a factor 2 to 2.5 is feasible and brings the 50 and 75 GeV requirements in reach.

Quadrupole aperture radius









The maximum quadrupole radius for the 75 GeV (top) and the 50 GeV (bottom) options

Magnet parameters for the 50 GeV option

Type	Length [m]	Strength [m^{-2}]	B_{pt} [T]	R_{xy} [mm]*
Type1 (LSS)	2	-0.0832	-0.89	63
Type2 (ARC)	1.4	-0.0657	-0.55	49
Type3 (ARC)	1.1	-0.0424	-0.42	58
Type1 (LSS)	2	-0.0136	-0.15	63
Type3 (ARC)	1.1	0.0494	0.49	58
Type1 (LSS)	2	0.07	0.75	63
Type4 (ARC)	2.4	0.1142	1.36	70

Dipole characteristics (135 dipoles):

B = 2.1 T with
Ramp rate: 3.82 T/s (500ms ramp up)
Max gap_x = 145.7 mm
Max gap_y = 110.9 mm

Number of type 1: 18
Number of type 2: 30
Number of type 3: 48
Number of type 4: 30

*Beam pipe thickness considered: 8mm

Magnet parameters for the 75 GeV option

Type	Length [m]	Strength [m^{-2}]	B_{pt} [T]	R_{xy} [mm]*
type1	2	-0.0832	-1.1	52
type2	1.4	-0.0657	-0.73	44
type3	1.1	-0.0424	-0.54	50
type1	2	-0.0136	-0.18	52
type3	1.1	0.0494	0.63	50
type1	2	0.07	0.92	52
type4	2.4	0.1142	1.82	63

Dipole characteristics (135 dipoles):

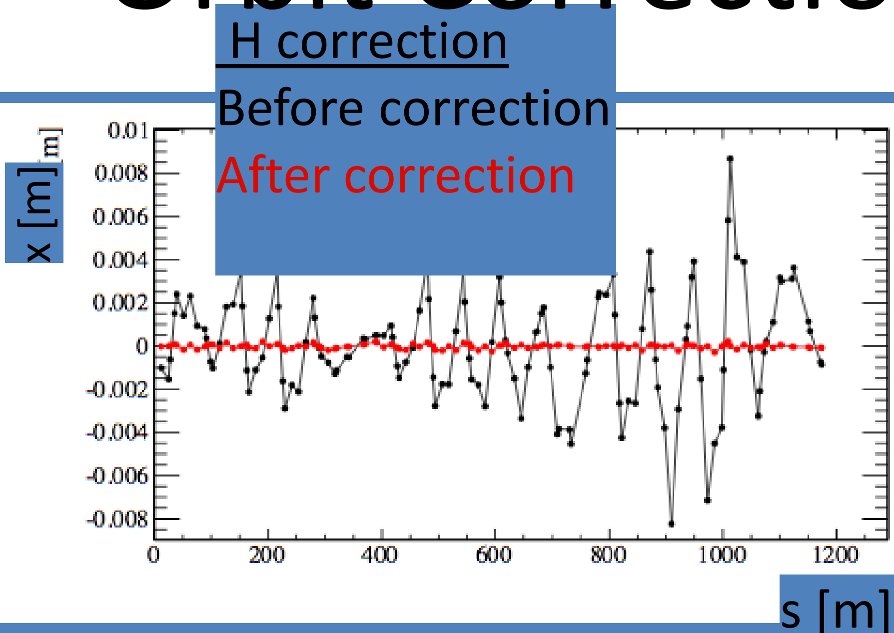
B = 3.13 T with 5.9 T/s ramp rate

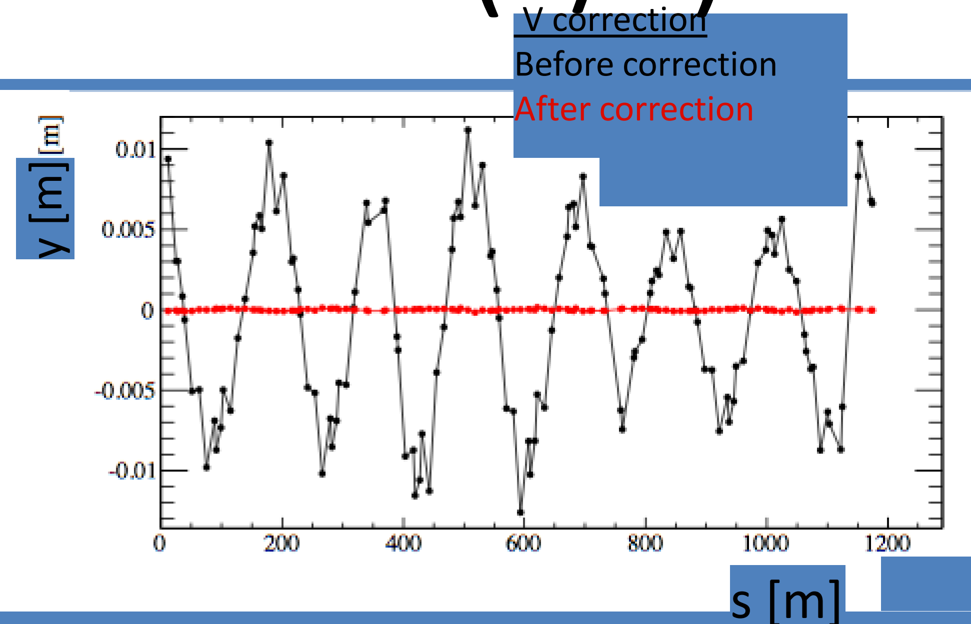
Max gap_x = 132.6 mm
Max gap_y = 92.3 mm

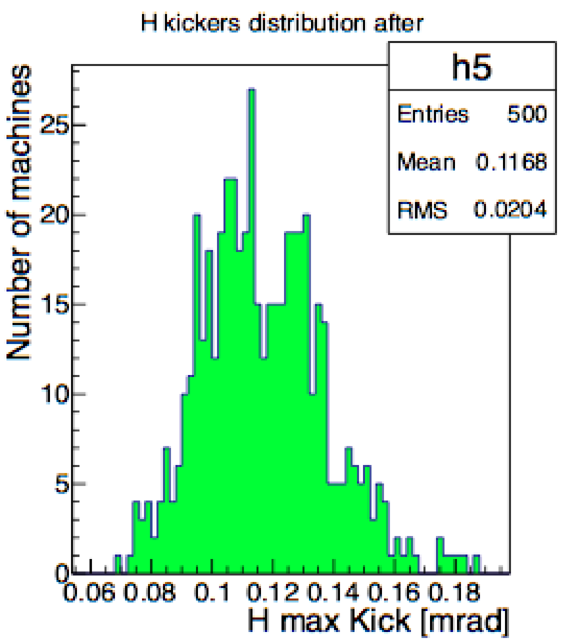
Number of type 1: 18
Number of type 2: 30
Number of type 3: 48
Number of type 4: 30

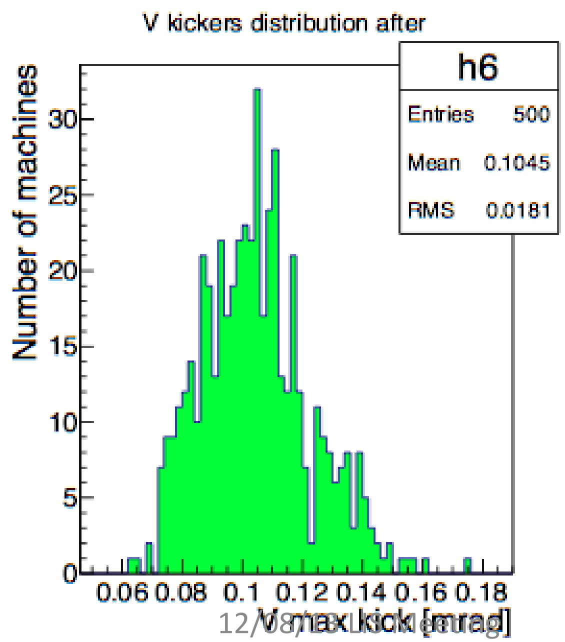
*Beam pipe thickness considered: 8mm

Orbit Correction Studies (3/3)







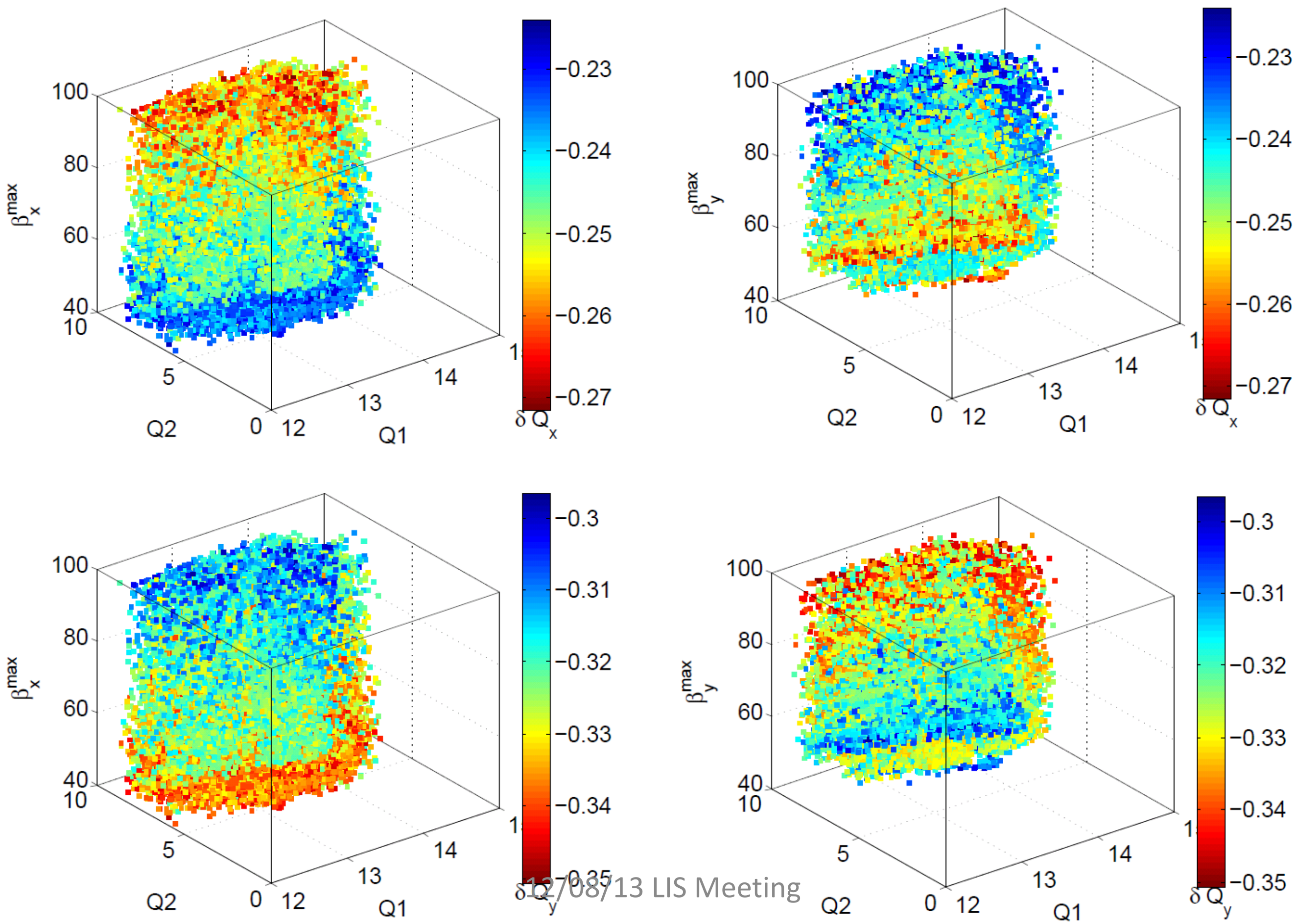


Obtained corrector kicks
<0.2 mrad
(~0.11 T for E=50 GeV)

Tunability studies

Global tunability

- **The vertical phase advance μ_y of the arc is rematched. For each μ_y the LSS quad strengths are scanned** (around 30k cases studied for each phase advance)



Tunability studies

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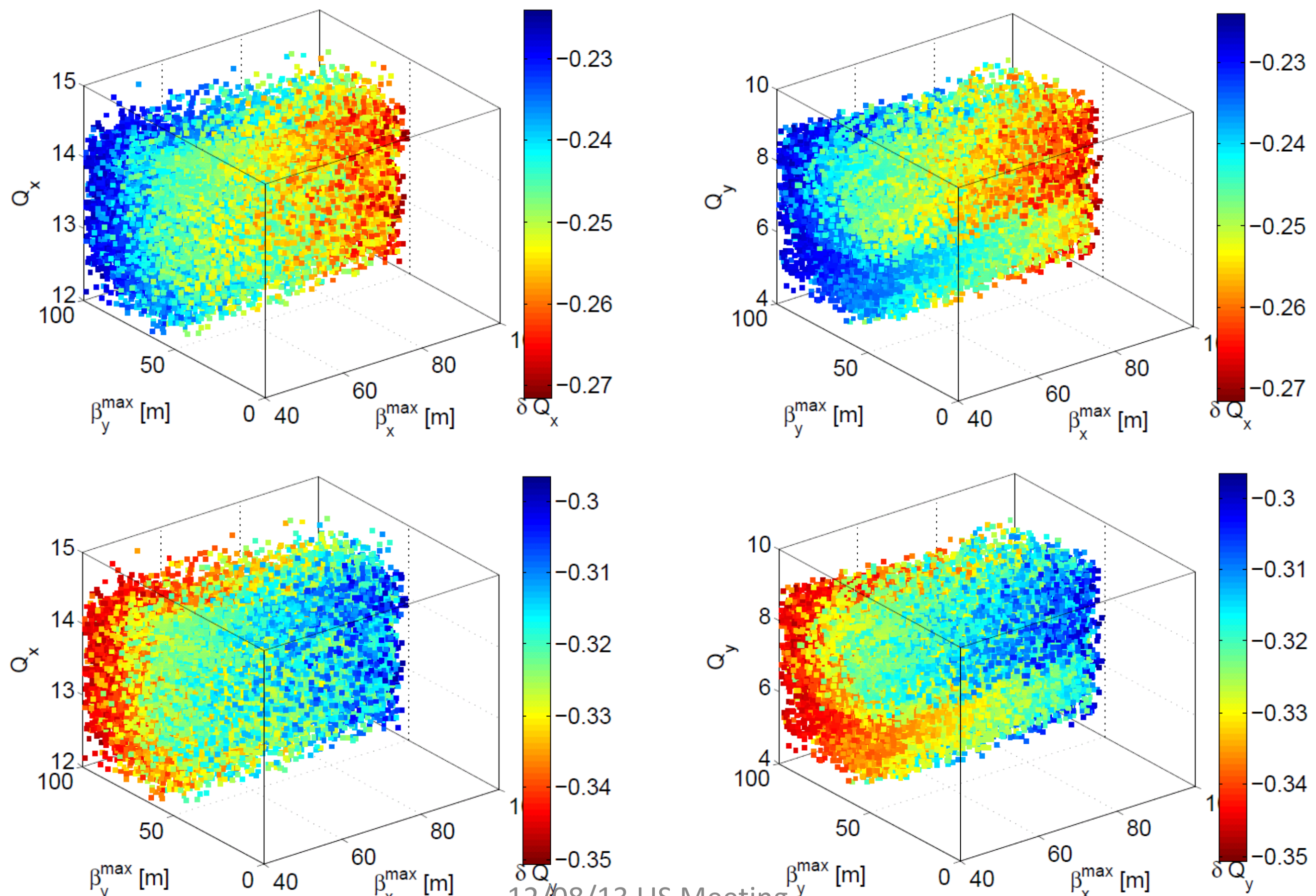

- 2 RF frequencies considered for HP-PS:
  - a) High frequency: 200 MHz (similar to SPS) allows much lower bunch peak current due to the large number of bunches
  - b) Low frequency: 10 MHz (as in the PS) much more flexible tuning range
- For now: 40 MHz system considered (as in PS2) can be tunable in limited range

more about RF's from: Dr. Antoine Lachaize, "Longitudinal aspects of injection and acceleration"

Longitudinal

$$Z = (1-i)\frac{l}{2\pi b}\frac{1}{\delta_s \sigma_c}$$

Round pipe
Thick wall

- l pipe length
- b pipe radius
- $\sigma_c = 1.35 \cdot 10^6 \Omega^{-1} m^{-1}$ metal conductivity (assumed SS)
- δ_s skin depth
  - $\delta_s = \sqrt{2c/Z_0 \sigma_c \omega}$ with $Z_0 = 377\Omega$ and $\omega = c/\sigma_z$ typical frequency of the bunch, σ_z the bunch length

- Important quantity for longitudinal stability

  Z/n with $n = \omega/\omega_0$ where ω_0 is revolution frequency

Transversal

$$k_y = -\langle W_y \rangle = \frac{3.63}{2^{3/2}\pi^2}\frac{cl}{b^3\sigma_z^{1/2}}\sqrt{\frac{Z_0}{\sigma_c}}$$

Kick factor

- l pipe length
- b pipe radius
- $\sigma_c = 1.35 \cdot 10^6 \Omega^{-1} m^{-1}$ metal conductivity (assumed SS)
- $Z_0 = 377\Omega$
- σ_z bunch length

Longitudinal

$$\frac{Z}{n} \approx i\frac{Z_0}{2\gamma^2}\left(1 + 2ln\frac{b_y}{\sigma_y}\right)$$

- γ Lorentz factor
- b_y vertical half-gap
- $\sigma_y = \sqrt{\varepsilon_y \langle \beta_y \rangle}$ typical beam size in the ring
- $Z_0 = 377\Omega$

- Purely reactive (capacitive).

Longitudinal

$$\frac{N_{th}}{N_b} \leq (2\pi)^{3/2} \frac{|\eta|\sigma_z E \sigma_\delta^2}{e^2 c N_b |z/n|}$$

"Microwave instability"
Boussard criterion

- N_{th} number of bunch particles at threshold (very rough estimate)

- η slip factor
- σ_δ the energy spread
- σ_z the bunch length
- N_b the bunch population

Transversal

$$\frac{N_{th}}{N_b} \sim (0.7)\,\frac{4\pi E \nu_s}{e^2 N_b \overline{\beta_y} k_y}$$

"Transverse Mode Coupling Instability TMCI"

- ν_s synchrotron tune
- $\overline{\beta_y}$ average beta function

Transversal

$$\Gamma = \frac{c}{4\gamma}\frac{m_e I}{m_p I_A}\sqrt{\frac{l}{1-[\nu_y]}}\langle\overline{\beta_y}A_y\rangle$$

<u>"Transverse coupled bunch Instability</u>
Assuming only RW

- Γ the growth rate of the instability
- I average current assuming full ring filled with bunches
- $I_A = 17kA$
- $[\nu_y]$ fractional part of vertical tune
- $\overline{\beta_y}$ average beta function
- $A_y = \frac{4}{\pi^{1/2}b^3}\sqrt{\frac{1}{Z_0\sigma_c}}$