Beta Measurement and Correction at KEKB Rings

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Outline

- On-Momentum Beta Correction (used in operation)
  - Beta Function Measurement by using Steering Kicks
    - Basic Idea
    - Fitting Algorism
    - Normalization of Fitting Results
  - Correction by Quadrupole Fudge Factor
- Off-Momentum Beta Correction (under development)
  - Beta Function Chromaticity Measurement
  - Correction by Sextupole Fudge Factor

On-Momentum Beta Measurement 1/6

- Available Monitor Systems
  - Tune Meter
  - Multi Turn Beam Position Monitor (BPM)
- Available Knobs
  - Magnet Parameters
    - Steering Dipole Kick Angle
    - Quadrupole Strength
    - etc...
- Basic Idea of Beta Measurement
  - Measure Betatron Tune
  - Measure Orbit Response of Single Dipole Kick
  - Fit Measured Orbit Response to Analytic Orbit Response Function via Beta Function and Betatron Phase Advance

On-Momentum Beta Measurement 2/6

- Analytic Response of Single Dipole Kick

$$\triangle\chi(s) = \frac{\sqrt{\beta_\chi(s)}}{2\sin\pi\nu_\chi}\triangle\theta_\chi\sqrt{\beta_\chi(s_{kick})}\cos\left(|\phi_\chi(s)-\phi_\chi(s_{kick})|-\pi\nu_\chi\right)$$

  - Sign of $\phi_\chi(s)-\phi_\chi(s_{kick})$ is given by order of beam line elements because of monotonicity of betatron phase advance

- Assuming ϕ'>0, analytic response is rewritten as:

$$\begin{aligned}\triangle\chi_i^j &= (\cos\pi\nu\ f_j X_j^S - S_{i\,j}\sin\pi\nu\ f_j Y_j^S)X_i^M + (\cos\pi\nu\ f_j Y_j^S + S_{i\,j}\sin\pi\nu\ f_j X_j^S)Y_i^M \\ &= (\cos\pi\nu\ X_i^M + S_{i\,j}\sin\pi\nu\ Y_i^M)f_j X_j^S + (\cos\pi\nu\ Y_i^M - S_{i\,j}\sin\pi\nu\ X_i^M)f_j Y_j^S\end{aligned}$$

Notations:

$$f_j \equiv \frac{\triangle\theta_j}{2\sin\pi\nu},\ S_{i\,j} \equiv \mathrm{sign}\,(\phi_i^M - \phi_j^S),\ X_i^l \equiv \sqrt{\beta_i^l}\cos\phi_i^l,\ Y_i^l \equiv \sqrt{\beta_i^l}\sin\phi_i^l \quad (l = M, S)$$

- i: i-th BPM
- j: j-th Steering Dipole
- M: at Monitor(BPM)
- S: at Steering Dipole

On-Momentum Beta Measurement 3/6

- Fitting Orbit Response
  - By Least Square Method for residual orbit error
  - Minimize polynomial function of degree four
- Reducing Difficulty(Degree) of Fitting Problem
  - By fixing either $X_i^M,\ Y_i^M$ or $f_j X_j^S,\ f_j Y_j^S$, square error sum seems quadratic form of $f_j X_j^S,\ f_j Y_j^S$ or $X_i^M,\ Y_i^M$, respectively.
- Brute Force Fitting Scheme
  - 1. Give initial $X_i^M,\ Y_i^M$ from the model optics.
  - 2n. Construct quadratic form of $f_j X_j^S,\ f_j Y_j^S$ from $X_i^M,\ Y_i^M$ of previous (2n-1)-th step and solve it by using Singular Value Decomposition(SVD).
  - 2n+1. Construct another quadratic form of $X_i^M,\ Y_i^M$ from $f_j X_j^S,\ f_j Y_j^S$ of previous (2n)-th step and solve it.
  - Iterate until residual error of SVD is converged.

On-Momentum Beta Measurement 4/6

- Brute Force Fitting Scheme(continuing...)
  - Iteration is stopped at local minimum and it is fixed point of X_i^M, Y_i^M
    - Residual error square sum has trivial lower limit 0.
    - Residual errors of each iterations makes a decreasing series.
    - Both 2n-th and 2(n+1)-th X_i^M, Y_i^M is equivalent if residual is converged.
  - Brute Force Fitting works fine, however...
    - Very Slow(needs about 10^4 iterations/Using 10^{-8} for tolerance of relative improvement of residual error)
- Improving Fitting Scheme
  - Using general minimizing algorism
    - Powell's direction set method
    - Conjugate Gradient method
  - Solving fixed point problem
    - Newton-Raphhson method(Very Fast)

On-Momentum Beta Measurement 5/6

- Solving fixed point problem
  - Iteration loop of Brute Force Fitting seems like a map of either $X_i^M,\ Y_i^M$ or $f_j X_j^S,\ f_j Y_j^S$.
  - Finding vector to converge iteration loop is equivalent to solve the fixed point of the map described by the pair of the least square fit.
  - We can quickly solve the fixed point problem about $f_j X_j^S,\ f_j Y_j^S$ by using multi-dimensional Newton-Raphson method.
    - Main fitting algorism for KEKB beta measurement system.

On-Momentum Beta Measurement 6/6

- Reconstructing Integer Part of Betatron Phase Advance
  - The assumption ϕ'>0 leads to $\phi_{i+1}^M - \phi_i^M > 0$.
  - If maximum phase advance between neighborhood BPMs is less than 2π, the unique phase advance between BPMs is decided.
    - KEKB rings have about 450 BPMs and its betatron tunes are less than 50.

- Normalization
  - Single dipole kick response is conserved by following transformation:

$$\beta_i^M \to \lambda\beta_i^M,\ \beta_j^S \to \lambda^{-1}\beta_j^S,\ \phi_i^M \to \phi_i^M + \theta,\ \phi_j^S \to \phi_j^S + \theta$$

  - Scaling factor of β and origin of ϕ MUST be normalized.
  - Our normalization:

$$\sum_i \frac{1}{\beta_i^M} = \sum_i \frac{1}{\beta_{model}(s_i^M)},\ \sum_i \phi_i^M = \sum_i \phi_{model}(s_i^M),$$

On-Momentum Beta Correction 1/4

- Basic Idea of Beta Correction
  - Correct disagreement between measured and model optics by using quadrupole fields around ring.
  - Use analytic response functions: Beta Function, Betatron Phase Advance, and Betatron Tune
- Available Correction Knobs
  - Fudge Factor of Quadrupole Power Supply

$$B^{(n)} = \frac{a_f\, B\rho\, K_n + b_f}{L}$$ (we use Amplitude Fudge: a_f)

  - Horizontal Cosine like Bump around Sextupole Pair
    - Depending non-interleaved chromaticity correction system
    - Transfer Matrix between sextupole pair is -I'

On-Momentum Beta Correction 2/4

- Response Functions for Quadrupole Fudge Factor

$$\frac{\triangle\beta_\chi(s_i)}{\beta_\chi(s_i)} = -\frac{1}{2\sin 2\pi\nu_\chi}\triangle K_\chi^j \beta_\chi(s_j)\cos\left(2|\phi_\chi(s_i)-\phi_\chi(s_j)|-2\pi\nu_\chi\right)$$

$$\triangle\phi_\chi(s_i) = \int_0^{s_i}\left(\frac{1}{\beta_\chi(s')+\triangle\beta_\chi(s')}-\frac{1}{\beta_\chi(s')}\right)ds'$$

$$=\frac{1}{2\sin 2\pi\nu_\chi}\triangle K_\chi^j \beta_\chi(s_j)\left(2\sin 2\pi\nu_\chi\ \sin^2 \min(\phi_\chi(s_i)-\phi_\chi(s_j),0)\right.$$

$$\left. +\sin\phi_\chi(s_i)\ \cos\left(2\phi_\chi(s_j)-\phi_\chi(s_i)-2\pi\nu_\chi\right)\right)$$

$$\triangle\nu_\chi = \frac{1}{2\pi}\left(\triangle\phi(C)-\triangle\phi(0)\right) = \frac{1}{4\pi}\triangle K_\chi^j\beta_\chi(s_j)$$

$$\triangle K_x^j \equiv \triangle K_1^j,\ \triangle K_y^j \equiv -\triangle K_1^j,\ \triangle K_1^i \equiv K_1^i\triangle a_f^i,\ \triangle a_f^i \equiv a_f^i - 1$$

On-Momentum Beta Correction 3/4

Square Error Sum to Minimize

$$e^2 \equiv \left(\frac{\pi}{\sin 2\pi\nu}\right)^2 \left|\triangle\nu - (\nu^{measured} - \nu^{model})\right|^2$$
$$+ \sum_i \left(\left| \frac{\triangle\beta(s_i)}{\beta(s_i)} - \frac{\beta^{measured}(s_i) - \beta^{model}(s_i)}{\beta^{model}(s_i)} \right|^2 \right.$$
$$\left. + \left|\triangle\phi(s_i) - (\phi^{measured}(s_i) - \phi^{model}(s_i))\right|^2 \right)$$

- Correction fudge factor Δa_f to minimize e^2 is obtained by SVD.
- In practical correction, we apply about 10 or 50 times extra weighing for betatron tune.

Update Fudge Factor of Real Machine

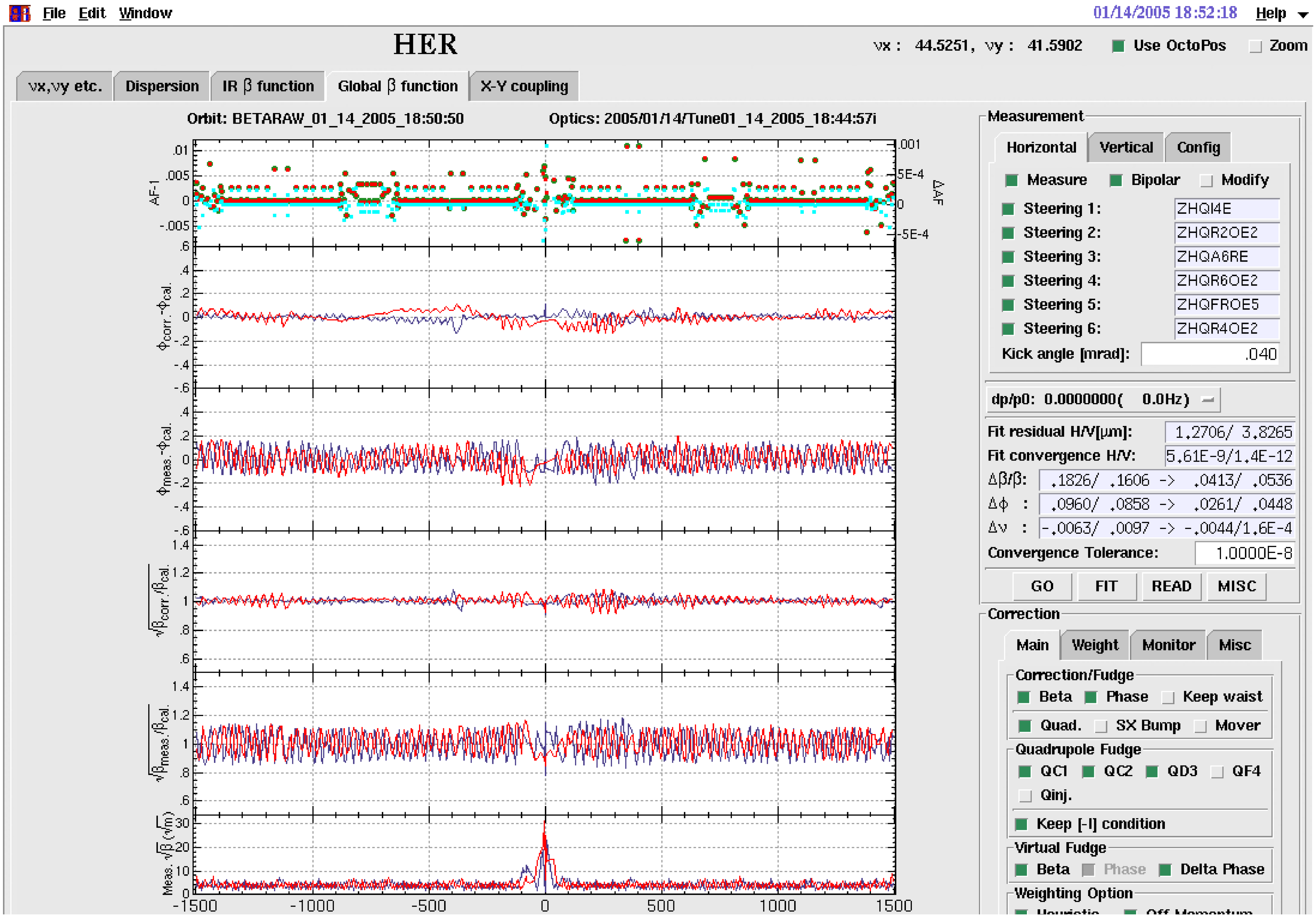
$$a^i_{f\ new} = \frac{a^i_{f\ old}}{1 + \triangle a^i_f}$$

- Real machine is corrected toward model optics.

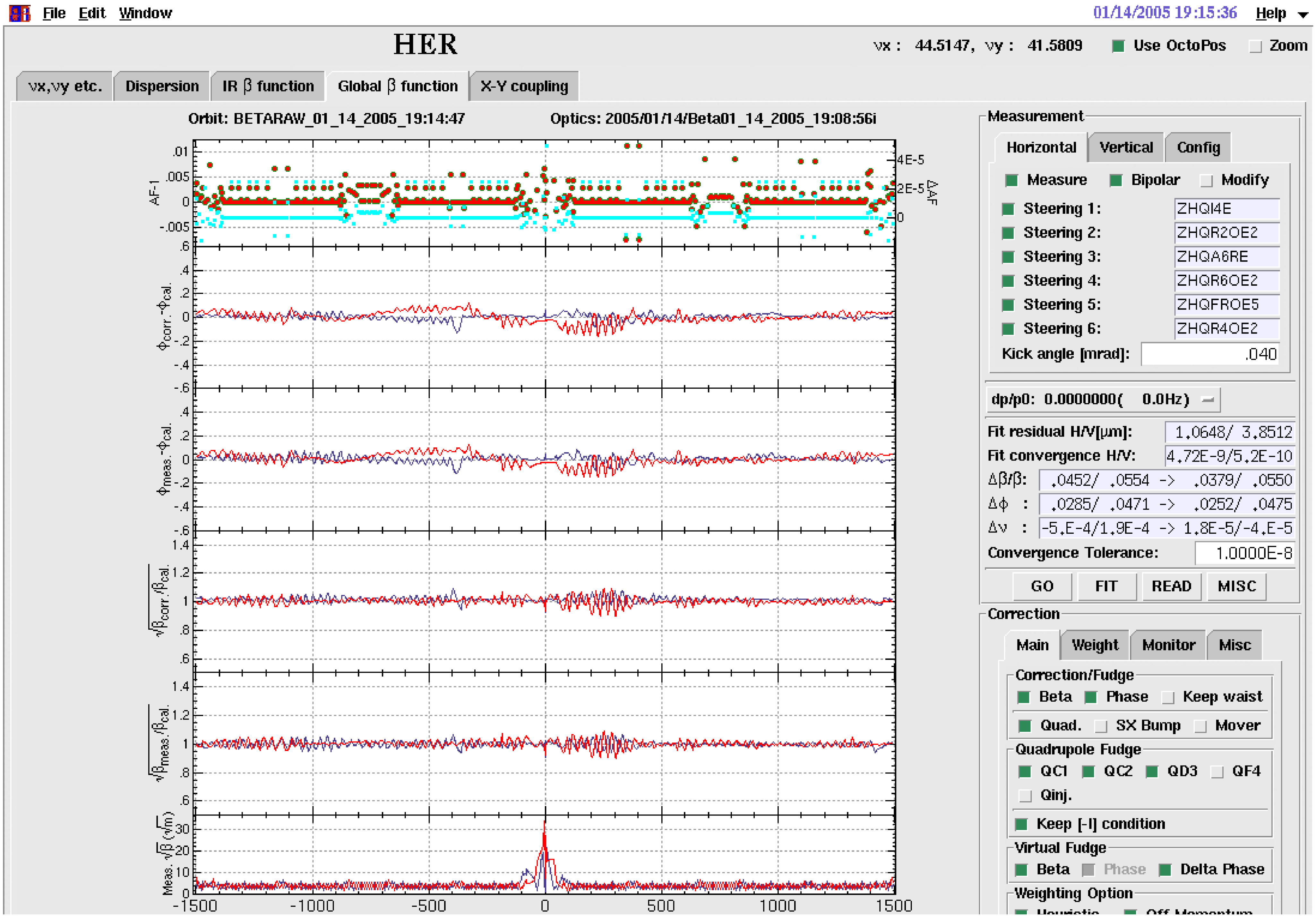
On-Momentum Beta Correction 4/4

- On-Momentum Beta Correction System in KEKB
  - Performed as a part of regular optics correction.
    - One correction per two weeks (after regular maintenance shutdown)
  - Implemented on SAD (See http://acc-physics.kek.jp/SAD/).
    - GUI: X11 + SAD/Tkinter
    - Hardware Access: EPICS Channel Access
    - Automatic tool(We only push GO and SET button)
  - Beta function(horizontal and vertical) measurement is completed within 7 minutes.
    - Time for usual correction (xy-coupling, dispersion, and beta function) is 30 ～ 60 minutes per ring.
  - Correction performance in typical case:
    - Relative error of beta function $\Delta\beta/\beta$ is adjusted within 10%.
    - Tune shift $\Delta\nu$ is adjusted within 3×10^{-4}

Before Beta Correction



After Beta Correction



Off-Momentum Beta Correction 1/4

- Motivation
  - To resolve disagreement between measured and model optics
    - Betatron Tune Chromaticity Curve
    - Relationship: Beam Life Time/Strength of Synchro-Beta Resonance vs Sextupole Parameters
- Basic Idea
  - Measure Off-Momentum Beta Function
  - Correct disagreement of momentum dependency between measured and model optics by using sextupole fields around ring.
    - KEKB rings have 54 sextupole families for LER and 52 families for HER.
  - Side effect from orbit offset at sextupole COULD be cured by on-momentum corrections: xy-coupling, dispersion, and beta function.

Off-Momentum Beta Correction 2/4

- Off-Momentum Beta Measurement
  - Measure both beta function and betatron tune with momentum shift by changing frequency of accelerating cavity.

$$\frac{\triangle f}{f_0} = -\left(\alpha_c - \frac{1}{\gamma^2}\right)\frac{\triangle p}{p_0}$$

    - Typical slippage factor $\alpha_c - \gamma^{-2}$ is about 3×10^{-4}.
  - Beta measurement is performed with five different frequency shifts: -400, -200, ±0, +200, +400Hz
    - The range of $\Delta p/p_0$ is $\pm 2 \times 10^{-3}$ that corresponds to above frequencies.
- Correction Knobs
  - Amplitude fudge a_f of Sextupole Magnet Power Supply

Off-Momentum Beta Correction 3/4

Square Error Sum to Minimize

$$e^2 = \sum_i \left((2\pi)^2 (\triangle\nu^{p_i}_{mes.} - \triangle\nu^{p_i}_{model})^2 \right.$$

$$\left. + \sum_j (\triangle\phi^{p_i}_{mes.}(s_j) - \triangle\phi^{p_i}_{model}(s_j))^2 + \sum_j (\triangle\hat{\beta}^{p_i}_{mes.}(s_j) - \triangle\hat{\beta}^{p_i}_{model}(s_j))^2 \right)$$

$$\triangle\nu^p \equiv \nu^p - \nu^{p_0},\ \triangle\phi^p(s) \equiv \phi^p(s) - \phi^{p_0}(s),\ \triangle\hat{\beta}^p(s) \equiv \frac{\beta^p(s)}{\beta^{p_0}(s)} - 1$$

- p_0: Reference Momentum
- p_i: Momentum of i-th Measurement
- s_j: j-th BPM location

Response Functions for Sextupole Fudge Factor

- Obtained from numerical derivative of model optics.

Off-Momentum Beta Correction 4/4

- Update Fudge Factor of Real Machine

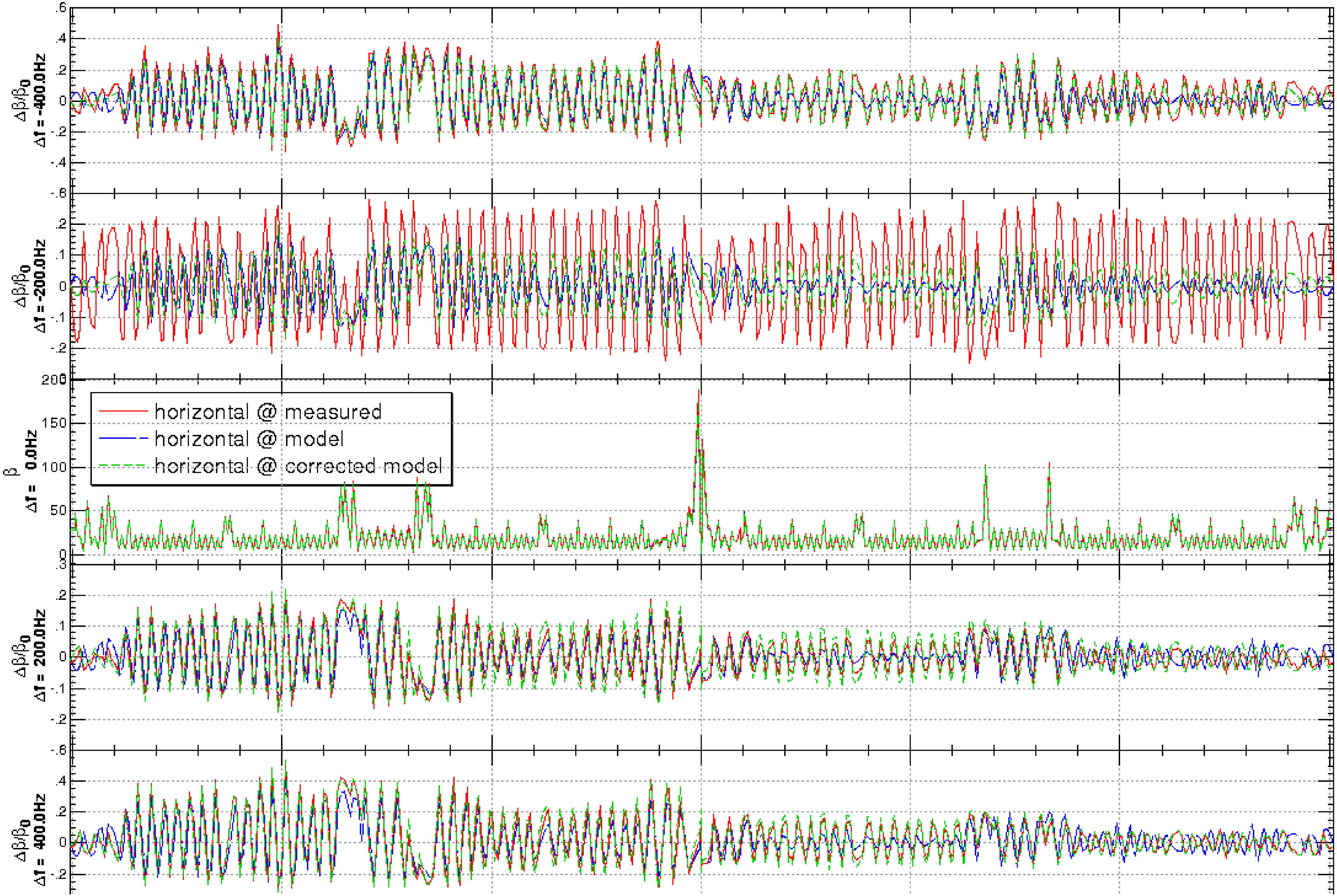
$$a^i_{f\ new} = \frac{a^i_{f\ old}}{1 + \triangle a^i_f}$$

- Update Sextupole Parameter of Model Optics

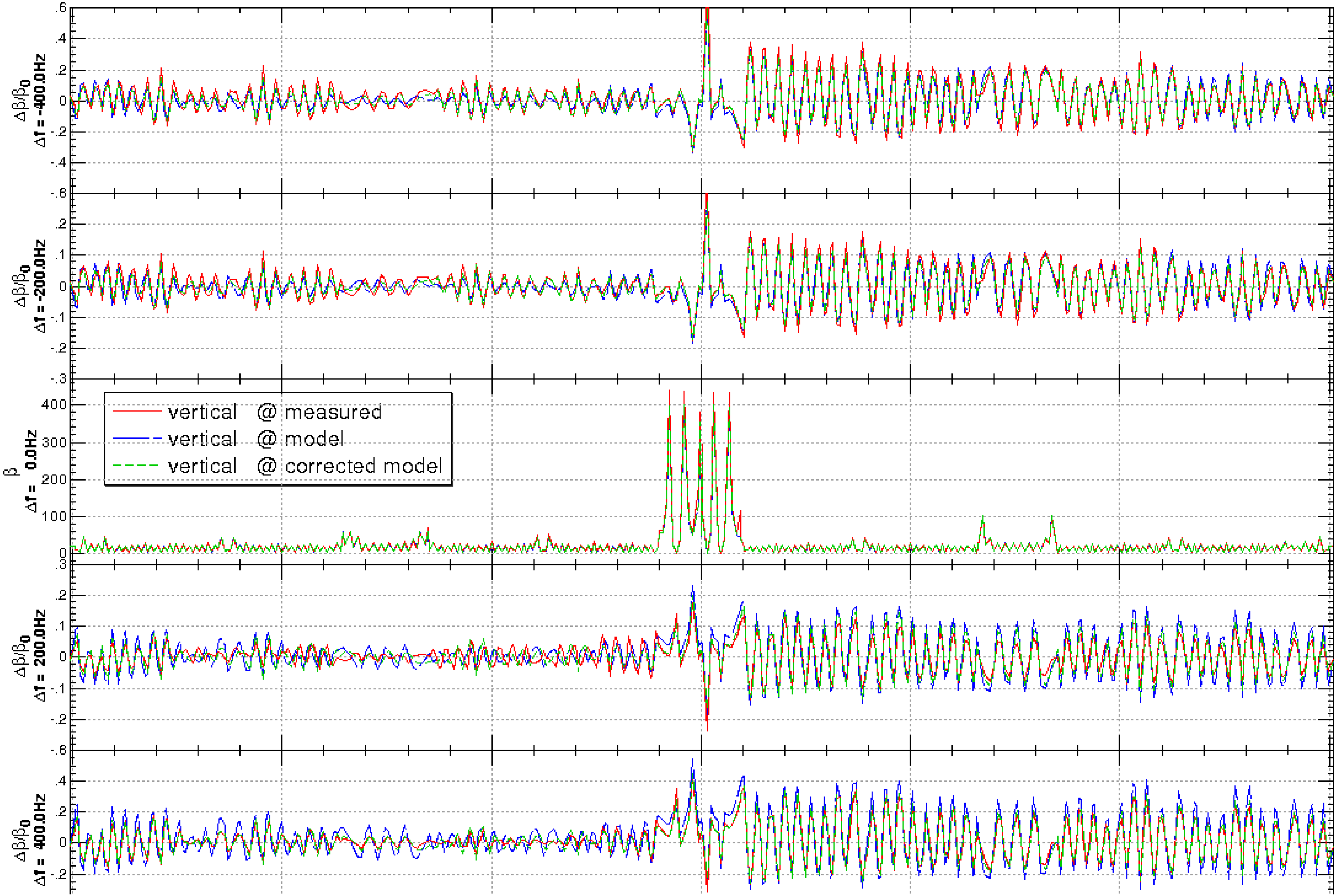
$$K^i_{2\ new} = (1 + \triangle a^i_f) K^i_{2\ old}$$

  - Keeping sextupole parameter of real machine
    - Sextupole parameter set for operation is tuned by cut-and-try.
    - We don't want to lose operation stability.
  - Sextupole parameter of model is adjusted toward real machine.

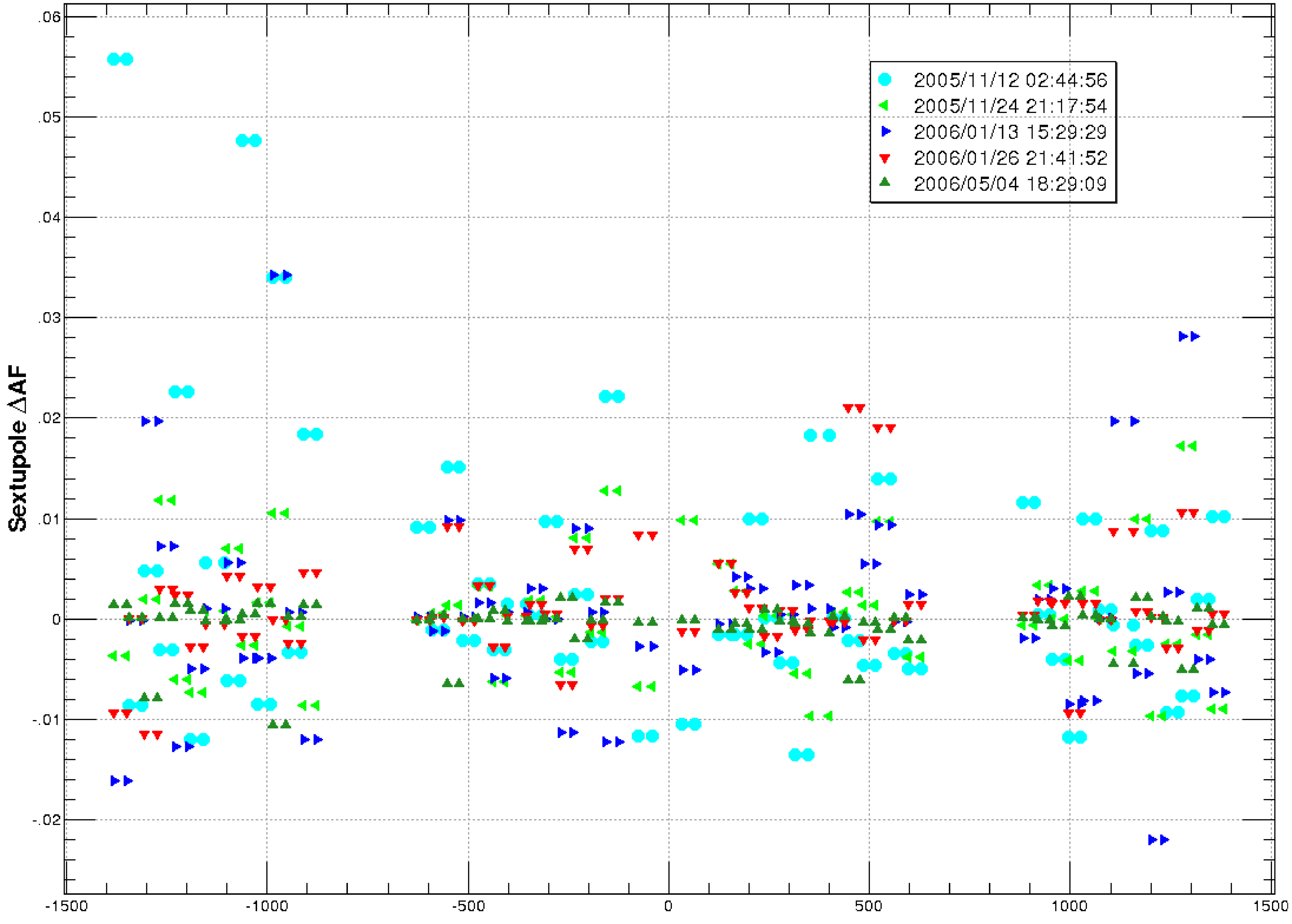
LER Measurement at 2005/11/12 (1)



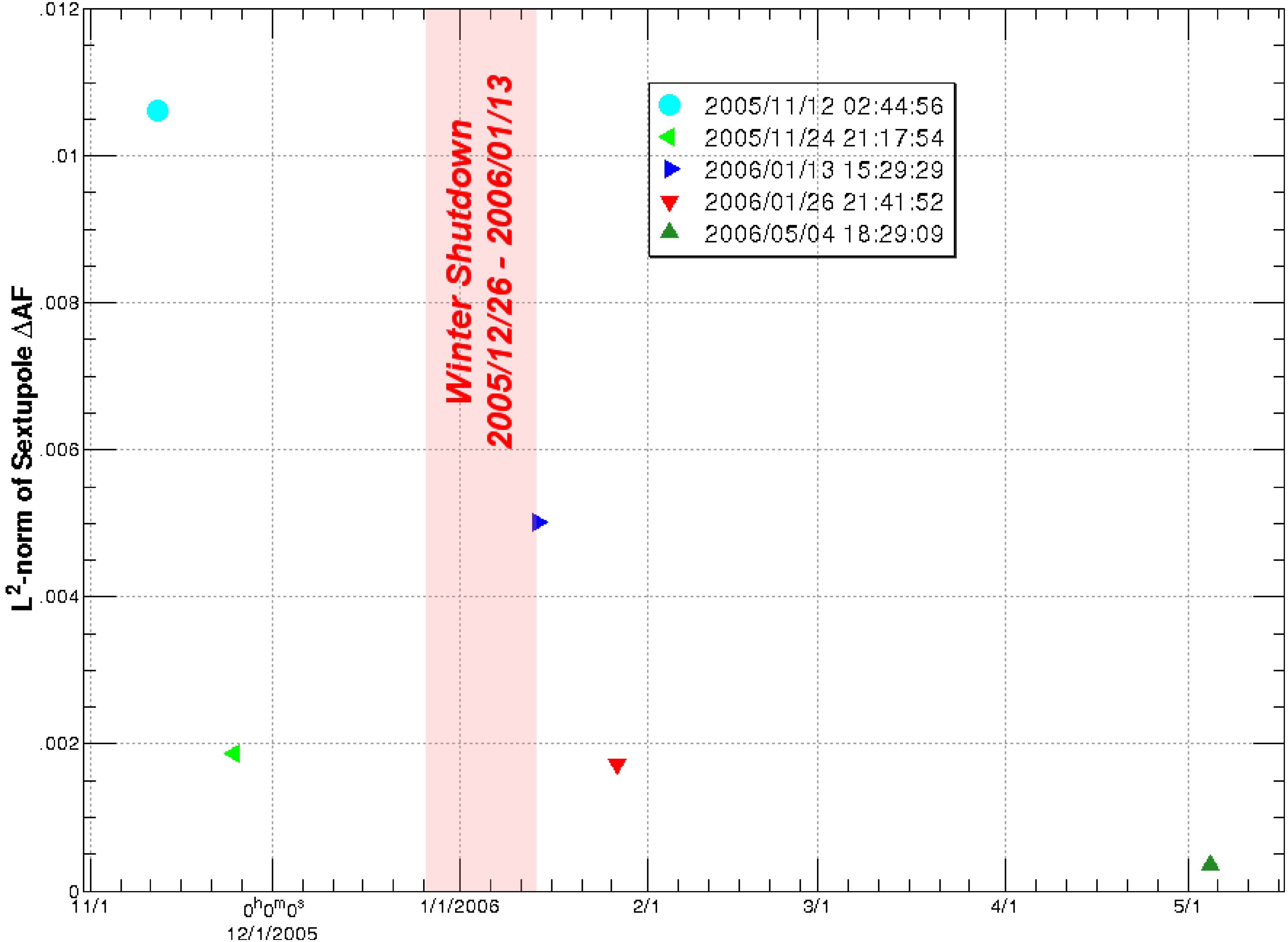
LER Measurement at 2005/11/12 (2)



Distribution of Correction Fudge Factor



L2-norm of Correction Fudge Factor



Summary

- On-Momentum Beta Measurement & Correction
  - Works fine
  - Helpful for operation nearby half-integer resonance line
- Off-Momentum Beta Measurement & Correction
  - Correction is tested on LER.
  - Disagreement is reduced, however, it is not resolved yet.
  - Current measurement time is too long (about 50 minutes).
  - We plan to...
    - Test on HER and compare with LER results
    - Check limit of correction scheme on model optics
    - Check parasitic sextupole term in lattice modeling (wiggler modeling?)