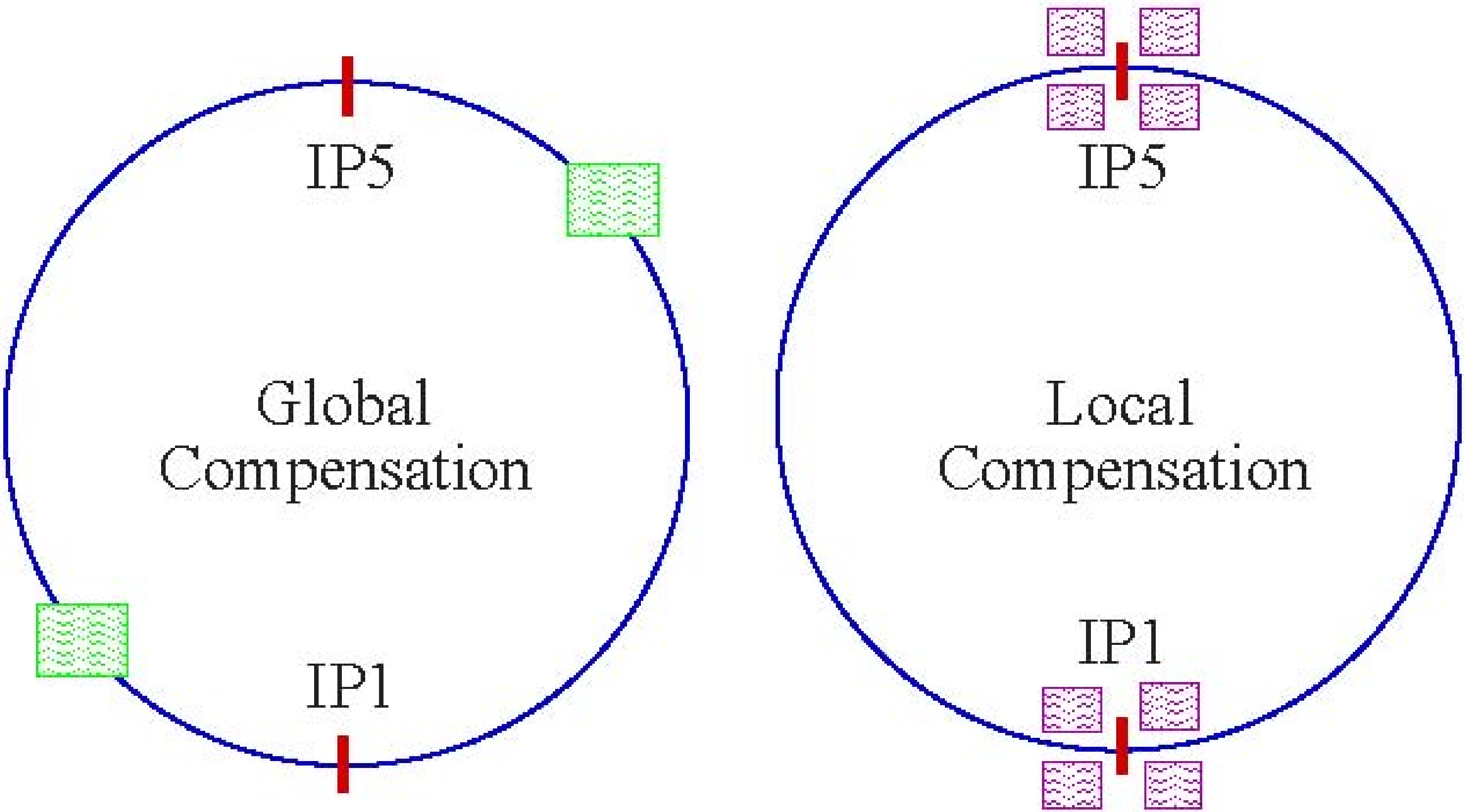
crab cavity studies for early separation scheme

Frank Zimmermann & Ulrich Dorda

global vs local crab crossing



even 1 crab cavity / beam could suffice with proper phase advance between IP1 and IP5 (Ulrich)

Rama Calaga
LUMI'06

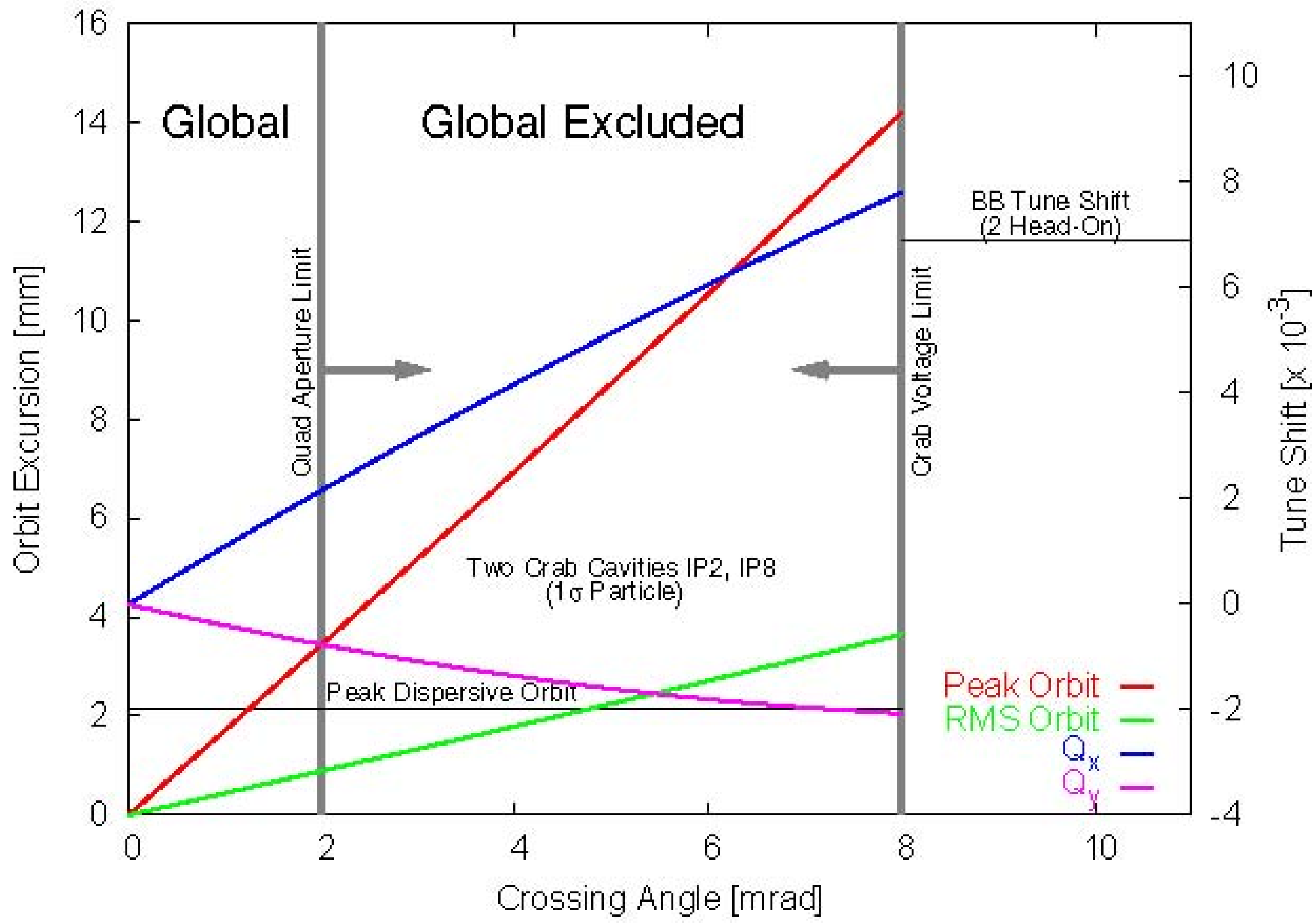


Figure 4: Orbit excursion and tune shift induced by two global crab cavities for a particle that is 1σ away from the center of the bunch.

Rama Calaga
LUMI'06

global scheme works for small crossing angles equation system for global crab crossing

$$\sum_i \frac{V_{crab,i}\,2\pi c}{E_0 f_{crab,i}} \frac{\sqrt{\beta_{crab,i}\beta_j^*}}{2\sin(\pi Q)} \cos\left(\left|\Delta\phi_{i\to j}\right| - \pi Q\right) = \frac{\theta_{c,j}}{2}$$

equation for local crab crossing

$$V_{crab} = \frac{cE_0 \tan(\theta_c/2)}{e2\pi f_{rf} R_{12}} \approx \frac{cE_0}{e4\pi f_{rf} R_{12}}\theta_c$$

crab voltage required, for R_{12}~30 m

full crossing angle	0.3 mrad	1 mrad	8 mrad
800 MHz	2.1 MV	7.0 MV	56 MV
400 MHz	4.2 MV	13.9 MV	111 MV
200 MHz	8.4 MV	27.9 MV	223 MV

for comparison, the two s.c. crab cavities recently installed at KEKB provide about 1.5 MV crab voltage at 500 MHz each.

rf phase noise tolerance

$$\Delta\phi_{crab} \leq \frac{4\pi\Delta x_{\max}}{\lambda_{rf}\theta_c}$$

relation of crab jitter and beam-beam offset jitter

$$\frac{1}{\varepsilon}\frac{d\varepsilon}{dt} \approx f_{rev} n_{IP} \frac{1-s_0}{4} \frac{(\Delta x)^2}{\sigma_x^2} \frac{1}{\left(1+\frac{g}{2\pi|\xi|}\right)^2}$$

emittance growth due to beam-beam offset jitter [Y. Alexahin]

requiring 1%/hr emittance growth:
$\Delta x<4$ nm with feedback,
$\Delta x<1$ nm w/o feedback

crab-cavity random phase noise tolerance, for 1% emittance growth per hour, IP beam size = 7.4 micron, feedback gain = 0.2

crossing angle	0.3 mrad	1 mrad	8 mrad
800 MHz	0.07^{o} , 0.08 ps	0.02^{o} , 0.025 ps	0.003^{o} , 0.003 ps
400 MHz	0.04^{o} , 0.08 ps	0.01^{o} , 0.025 ps	0.0013^{o} , 0.003ps
200 MHz	0.02^{o} , 0.08 ps	0.005^{o}, 0.025ps	0.0007^{o} , 0.003ps

Note: 0.02 ps is tolerance between different rf systems required for the European X-Ray FEL geometric luminosity loss without crab cavity, due to hourglass and crossing angle

$$R_{geom} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} ds \left(\frac{e^{-\frac{s^2}{\sigma_z^2}}}{\left(1 + \frac{s^2}{\beta^{*2}}\right)} \frac{1}{\sqrt{1 + \Phi^2 \left(1 + \frac{s^2}{\beta^{*2}}\right)}} \right)$$

with $\Phi \equiv \left(\frac{\theta_c \sigma_z}{2\sigma_x^*} \right)$

geometric luminosity loss with crab cavity, finite crab wavelength, hourglass and crossing angle

$$R_{geom}^{crab} = \frac{1}{\sqrt{\pi}\sigma_z}\int_{-\infty}^{\infty} ds \left(\exp\left[-\frac{s^2}{\sigma_z^2} - \frac{\theta_c^2 \left(-k_{crab} s + \sin\left(k_{crab} s\right)\right)^2}{4 k_{crab}^2 \sigma_x^{*2} \left(1 + \frac{s^2}{\beta^{*2}}\right)} \right] \frac{1}{\left(1 + \frac{s^2}{\beta^{*2}}\right)} \right)$$

β*=11 cm, σ_z=7.55 cm, σ_x=7.4 μm