Comparison of Zotter's and Henry and Napoly's formalisms in deriving a general expression for longitudinal and Transverse Resistive Wall impedance

LCU meeting June 18, 2007

E. Métral and B. Salvant

Agenda

- Context

- Henry and Napoly approach (1991)
- Zotter approach

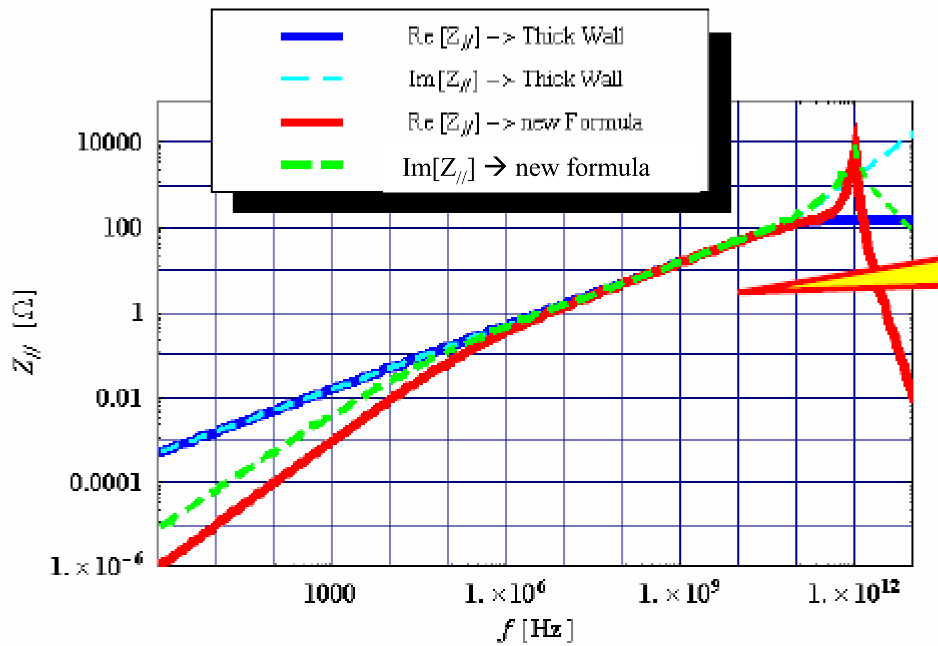
- Comparison for a round infinite layer

LHC IMPEDANCE STATUS AND STRATEGY

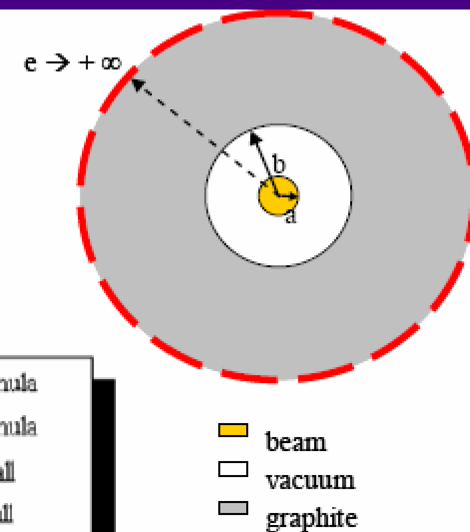
E. Métral with Hubert Medina and Benoit Salvant

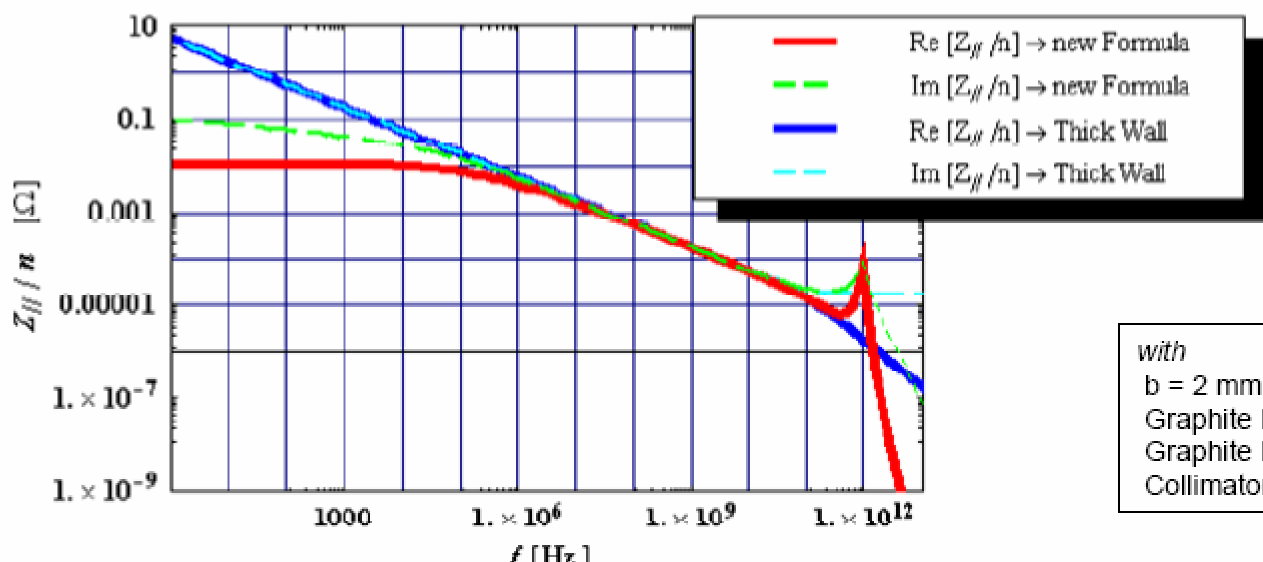
- **The LHC impedance is dominated by the collimators**
- **Current work**
  - **Link the Mathematica program rewall to ZBASE $\Longrightarrow$ Hubert Medina**
  - **General program for the resistive-wall impedance (upgrade of rewall) $\Longrightarrow$ Benoit Salvant**
- **LHC collimators Phase II $\Longrightarrow$ Video conference on LHC collimation (07/03/07)**
- **Studies on TCT, TCLI, TCDQ, TCDS and TDI**
- **More details for the injection and dump septum**
- **Next important work**
- **Stability diagram at top energy before the squeeze**
- **Conclusion**

GENERAL PROGRAM FOR RESISTIVE-WALL IMPEDANCE (2/4)



3 regimes found
(as for the transverse)





with
b = 2 mm
Graphite Relaxation Time = 0.8 10^{-12} s
Graphite DC conductivity = 10^{5} $S.m^{-1}$
Collimator Length = 1 m

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Henry and Napoly approach (CLIC Note 142 – 1991)

- Longitudinal Impedance Z_z /L

$$Z_z(r,\theta,\omega) = \sum_{m=0}^{\infty} \cos(m\theta)\ r^m\ Z_z^{(m)}(\omega)$$

where

$$Z_z^{(m)}(\omega) = -\frac{i\omega Z_0}{\pi c}\left(\frac{r_0}{a^2}\right)^m (m+1)\ \mathrm{H}_m(Ka)\left[\left(m(m+1) - \frac{\omega^2 a^2}{c^2}\right)\ \mathrm{H}_m(Ka) + 2(m+1)\frac{\omega^2 a}{Kc^2}\mathrm{H}_{m+1}(Ka) + \frac{\alpha_m}{2}Ka\left(\mathrm{H}_{m+1}(Ka) - \mathrm{H}_{m-1}(Ka)\right)\right]^{-1}$$

with the following notations

$$K = (\pm 1 + i)\sqrt{\frac{|\omega| Z_0 \sigma}{2c}} \qquad \text{for} \quad \pm\omega \geq 0$$

and

$$\alpha_m = \begin{cases} 2 & \text{for } m = 0 \\ m+1 & \text{for } m \geq 1 \end{cases}$$

Mode m

Henry and Napoly approach (CLIC Note 142 – 1991)

- Transverse Impedance $Z_\perp$ obtained through the Panofsky-Wenzel Theorem

$$\mathbf{Z}_\perp(\omega) = \sum_{m=1}^{\infty} m\, r^{m-1}(\cos(m\theta)\ \mathbf{e}_r - \sin(m\theta)\ \mathbf{e}_\theta)\ \frac{ic}{\omega}\ Z_z^{(m)}(\omega)$$

- Warning: conventions are different from Henry and Napoly's to Zotter's formalisms:
  - Henry and Napoly chose not to divide the transverse impedance by i in their definition
  - Henry and Napoly did not normalize the transverse impedance with the offset r_0 of the beam to the center of the pipe
  - Henry and Napoly used the exp(-iωt) convention while Zotter uses the exp(jωt) convention

- Therefore, we have the following relations:

$$Z_{//}^{(Zotter)} = Re[Z_z^{(0)}] - jIm[Z_z^{(0)}]$$

$$Z_\perp^{(Zotter)} = \frac{1}{r_0}\left(Im\left[Z_\perp^{(1)}\right] + jRe\left[Z_\perp^{(1)}\right]\right)$$

Henry and Napoly approximation for intermediate and high frequencies (CLIC Note 142 – 1991)

- Approximated longitudinal impedance $Z_{//}$

$$Z_z^{(m)}(\omega) = \frac{1-i}{\alpha_m \pi a}\sqrt{\frac{Z_0\omega}{2\sigma c}}\left(\frac{r_0}{a^2}\right)^m (m+1)\left[1 - \frac{1+i}{\sqrt{2}\alpha_m}\left(\frac{\omega s_0}{c}\right)^{\frac{3}{2}}\right]^{-1}$$

Approximation valid if:
- the skin depth is much lower than the beam pipe radius $\delta \ll a$
- only the first modes *m* are considered

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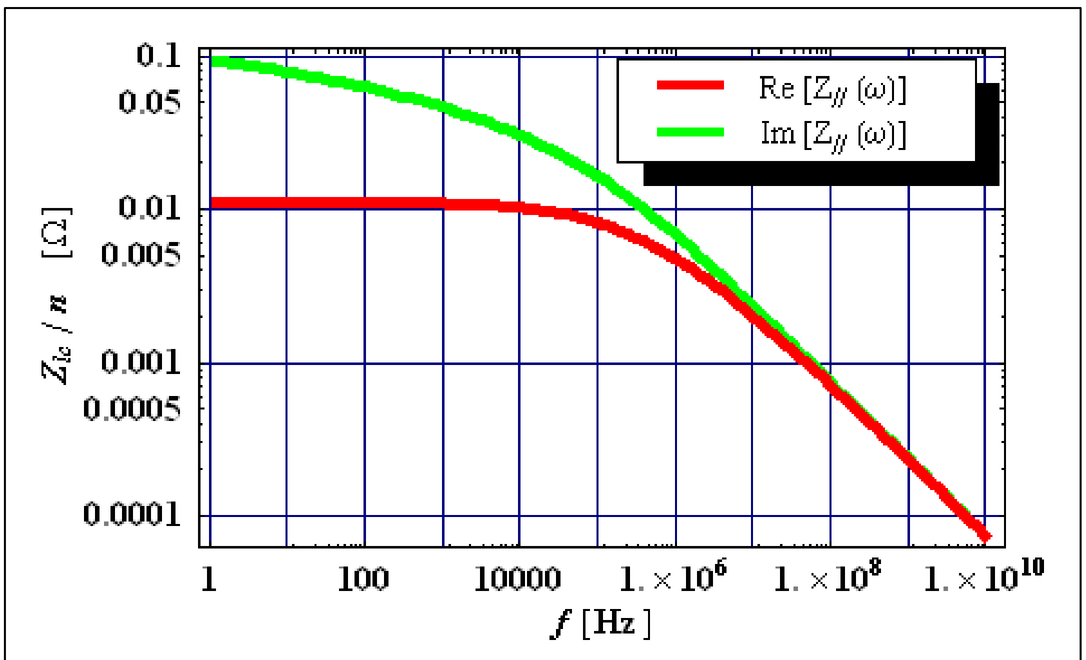
Resistive Wall longitudinal Impedance of…
… a monolayer extending to +∞

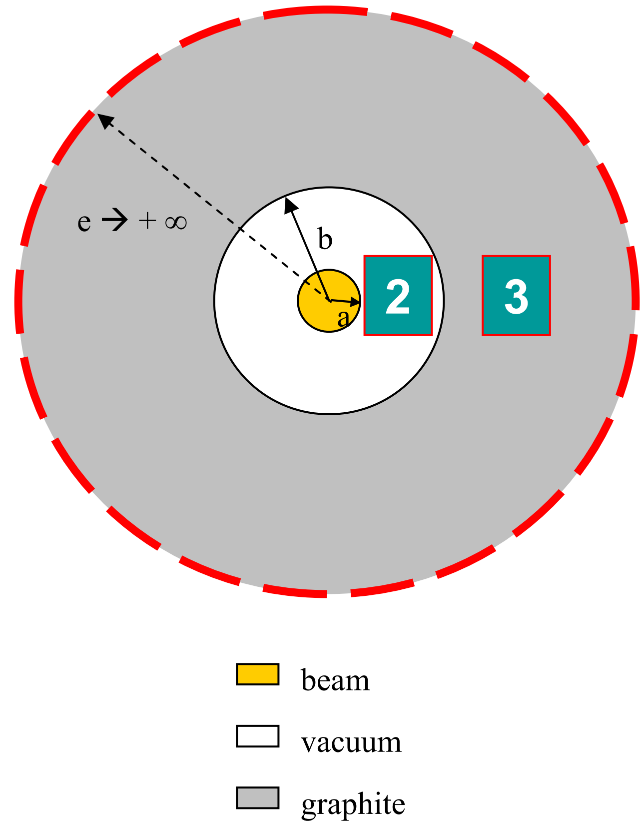
- Solution:

$$Z_{//}^{RW}(\omega) = -\frac{j\omega L\left(1-\beta^2\mu_r\varepsilon_r\right)I_0(s)^2}{2\pi\varepsilon v^2}\left(\frac{K_0(x)}{I_0(x)} - \alpha_2\right)$$

with

$$\alpha_2 = \frac{-\varepsilon_2'\nu_3 K_0(\nu_3 b)K_1(\nu_2 b) + \varepsilon_3'\nu_2 K_0(\nu_2 b)K_1(\nu_3 b)}{\varepsilon_2'\nu_3 I_1(\nu_2 b)K_0(\nu_3 b) + \varepsilon_3'\nu_2 I_0(\nu_2 b)K_1(\nu_3 b)}$$





with b = 1.5 mm

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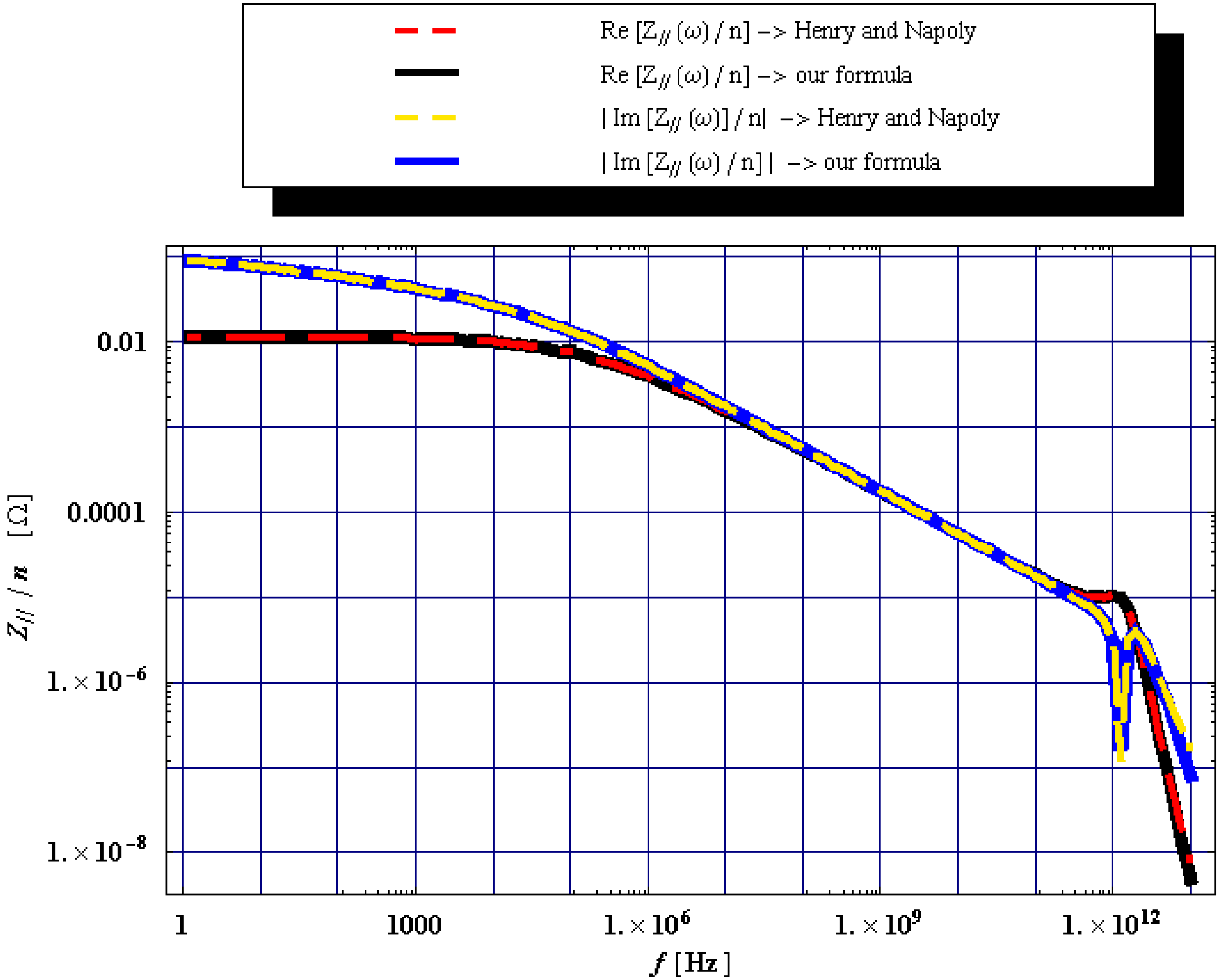
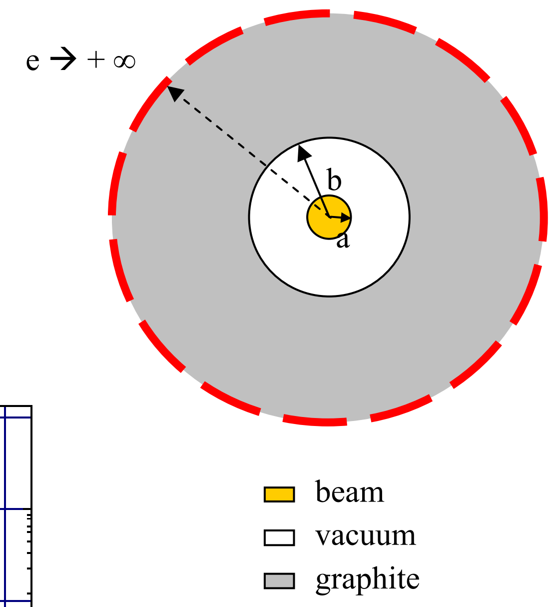
Henry and Napoly approach compared to Zotter approach

- longitudinal impedance $Z_{//}$ → mode 0



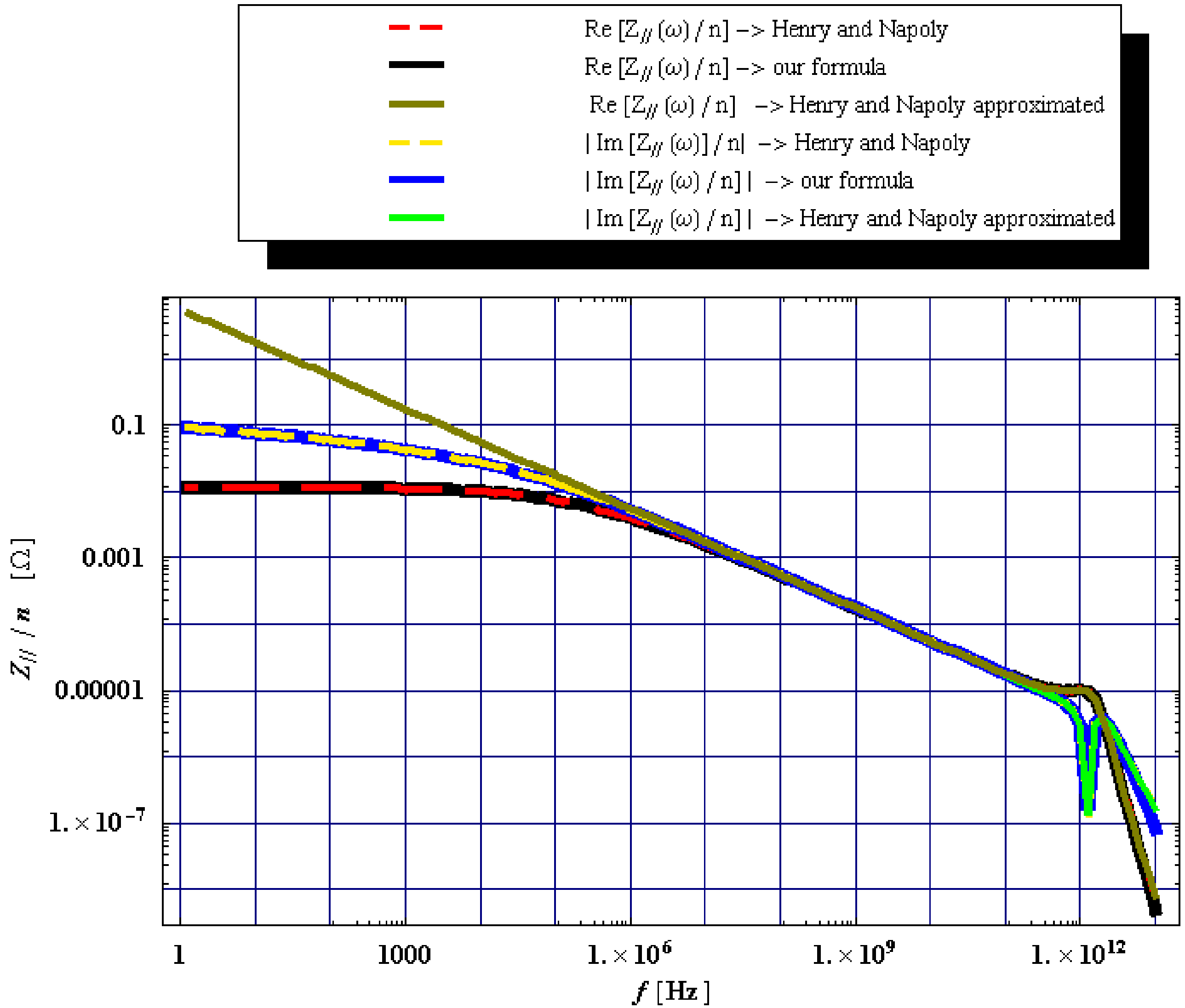


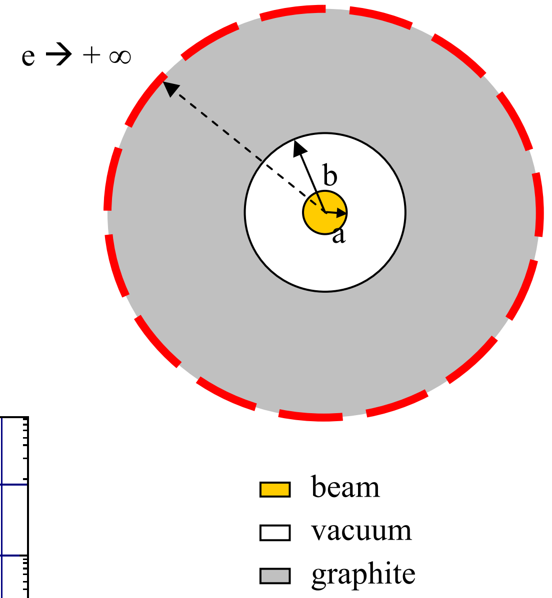
with b = 2 mm

Remarks:
- Perfect agreement until 1THz
- After 1 THz, H&N formula leads CPU precision to its limits.
- H&N formula seems a bit off after 1 THz
  → to be investigated

Henry and Napoly approach compared to Zotter approach

- Longitudinal impedance $Z_{//}$ → mode 0





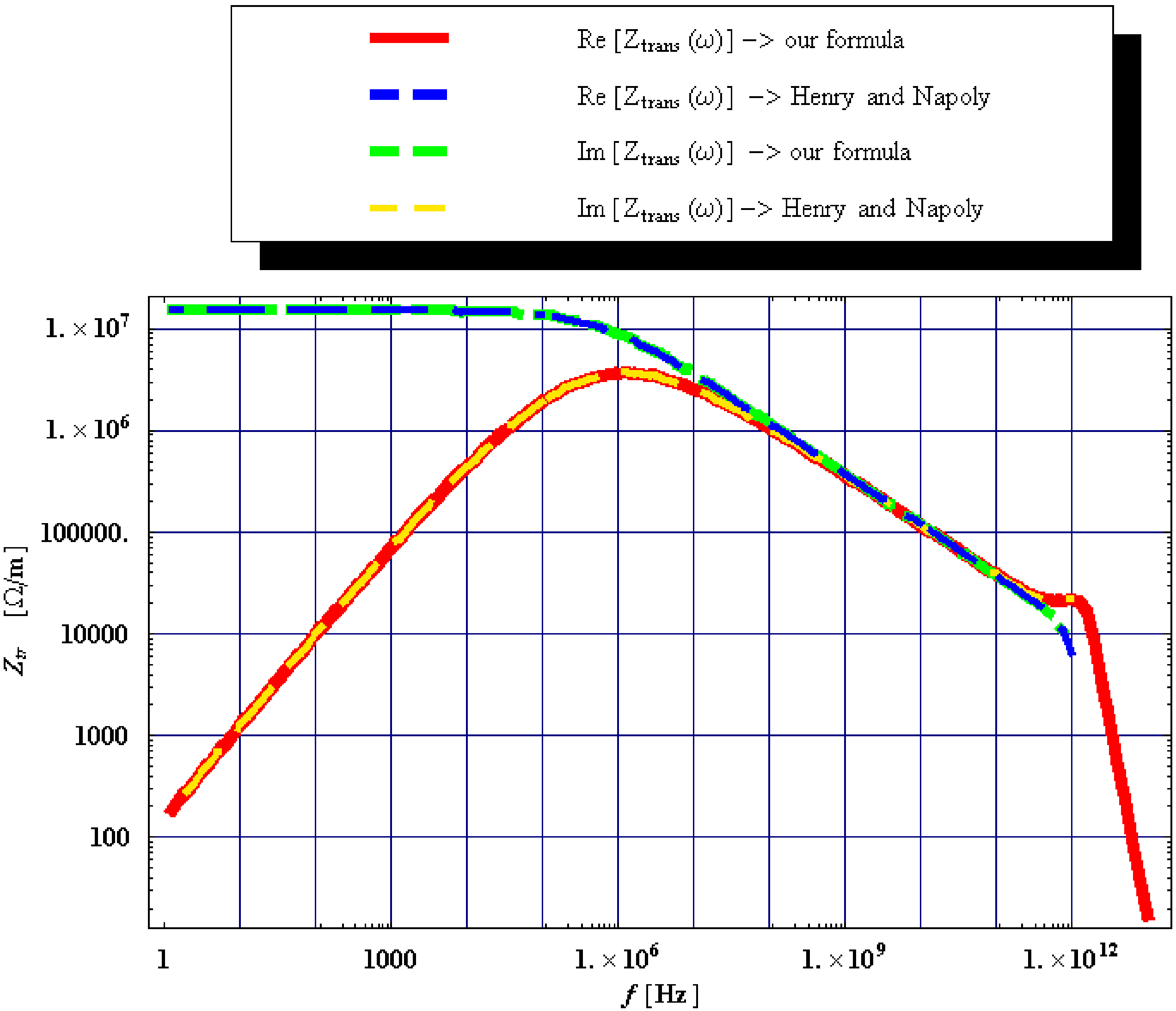
with b = 2 mm

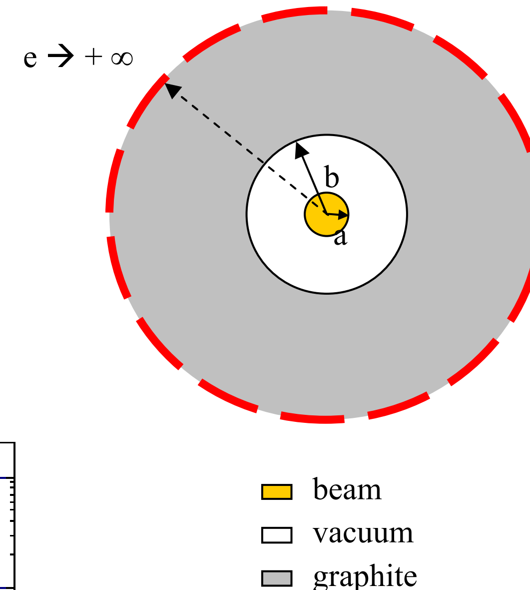
Remarks:
- Perfect agreement until 1THz
- After 1 THz, H&N formula leads CPU precision to its limits.
- Both H&N and Approximated H&N formula seems a bit off after 1 THz

→ to be investigated

Henry and Napoly approach compared to Zotter approach

- Transverse impedance $Z_{\perp}$ → mode 1





with b = 2 mm

<u>Remarks</u>:

- Perfect agreement until 1THz
- After 1 THz, H&N formula leads CPU precision to its limits.

Henry and Napoly approach compared to Zotter approach

- Advantages of Henry and Napoly approach :
  - All modes can be calculated
    - ➔ enable to use the Panofsky-Wenzel theorem to compute the transverse impedance
- Advantages of Zotter approach :
  - Includes non-ultrarelativistic particles
  - Includes Materials with any σ, ε_r and/or μ_r
  - Includes Multi-Layer beam pipe
  - Does not lead to CPU precision issues at high frequencies

All our thanks to:

Stéphane Fartoukh