Simulation of transverse multi-bunch instabilities of proton beams in LHC

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Outline

- Motivation
- Simulation Techniques & Approximations
- Resistive Wall Impedance Models
- Measurements in CERN SPS & Comparison to Simulation
- LHC simulated
- Summary

Motivation

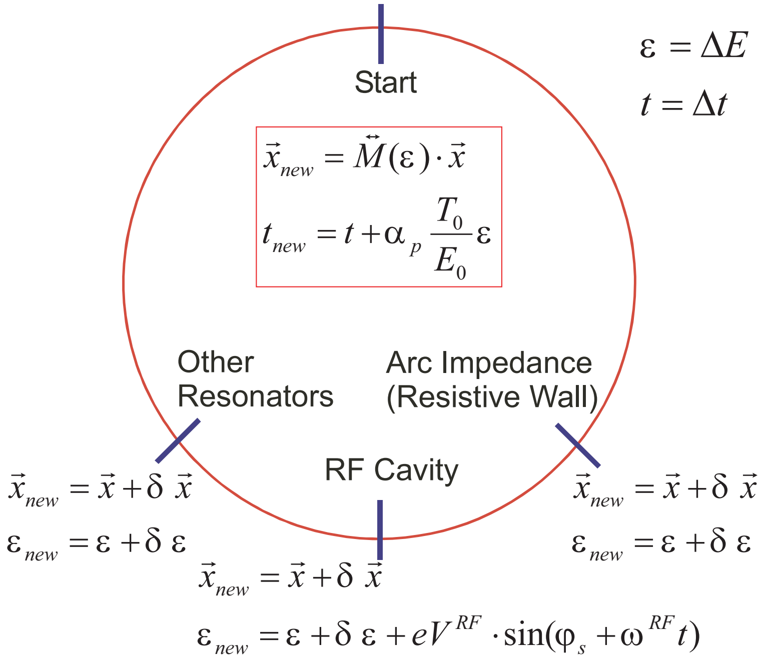
Stability analysis normally done in frequency domain.

Simulation additionally allows:

- Non-equidistant filling schemes
- Investigate Transition Effects
- Interplay between different effects (impedances)

- Long-Range Effects
- Correct and efficient implementation of corresponding impedances

The Simulation



$\varepsilon = \Delta E$

$t = \Delta t$

Start

$$\vec{x}_{new} = \overleftrightarrow{M}(\varepsilon)\cdot\vec{x}$$

$$t_{new} = t + \alpha_p \frac{T_0}{E_0}\varepsilon$$

Other Resonators

$$\vec{x}_{new} = \vec{x} + \delta\,\vec{x}$$
$$\varepsilon_{new} = \varepsilon + \delta\,\varepsilon$$

Arc Impedance (Resistive Wall)

$$\vec{x}_{new} = \vec{x} + \delta\,\vec{x}$$
$$\varepsilon_{new} = \varepsilon + \delta\,\varepsilon$$

RF Cavity

$$\vec{x}_{new} = \vec{x} + \delta\,\vec{x}$$
$$\varepsilon_{new} = \varepsilon + \delta\,\varepsilon + eV^{RF}\cdot\sin(\varphi_s + \omega^{RF} t)$$

$$M = \begin{pmatrix} \sqrt{\frac{\beta_{i+1}}{\beta_i}}\cos(\mu(\varepsilon)) & \sqrt{\beta_{i+1}\beta_i}\,\sin(\mu(\varepsilon)) \\ -\frac{1}{\sqrt{\beta_{i+1}\beta_i}}\sin(\mu(\varepsilon)) & \sqrt{\frac{\beta_i}{\beta_{i+1}}}\cos(\mu(\varepsilon)) \end{pmatrix}$$

$$\mu(\varepsilon) = \mu_0\left[1 + \frac{\xi - \eta}{E_0}\cdot\varepsilon\right] \qquad \delta\,\vec{x} = \begin{pmatrix} 0 \\ \Delta x' \end{pmatrix} \qquad \delta\varepsilon$$

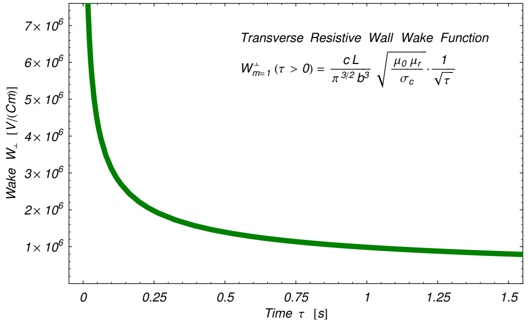
Transverse Kick — Longitudinal Kick

“Classical” Tracking Code
Linear Transfer Matrices + Kicks

Long-Range regime:	$\tau \gg \sigma_\tau$ σ_τ . . . bunch length τ . . . time interval for wake calc.

Impedances with correspondingly long-lasting wake fields:

- Resistive Wall Impedance (with ‘inductive bypass’)
- Narrow-Band Impedances
  (HOMs of cavities, wakes of cavity-like structures)

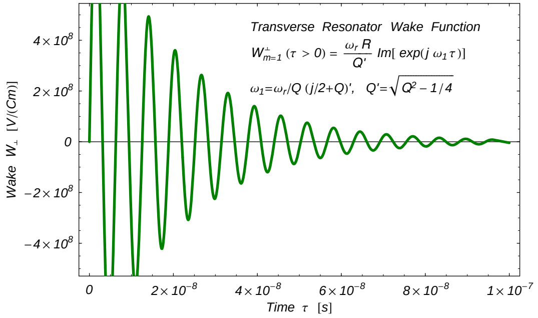


$$W_\perp(\tau) \propto 1/\sqrt{\tau}$$

$$W_\perp(\tau + \Delta\tau) = ?\cdot W_\perp(\tau)$$

$\Rightarrow$ Fast summation via

FFT Convolution



$$W_\perp(\tau) \propto \exp(j\,\omega_1\tau)$$

$$W_\perp(\tau + \Delta\tau) = \exp(j\,\omega_1\Delta\tau)\cdot W_\perp(\tau)$$

$\Rightarrow$ Time evolution and summation

using Phasors

= Resonator Model $(R,\,Q,\,\omega_r)$

The Simulation / Approximations

- 1 Detailed Bunch
- Multiple Bunches represented by 1 super-particle

rigid bunch approximation

$$W_{pot}(\tau) \approx W(\tau)$$

wake function approximation

$$W_{pot}^{\parallel}(\tau) = \int_0^\infty dt\; W_{\parallel}(t)\; \lambda(\tau - t) \qquad \approx W_{\parallel}(\tau) \text{ for } \tau \gg \sigma_t$$

$$W_{pot}^{\perp}(\tau) = \frac{1}{\bar{\xi}} \int_0^\infty dt\; W_{\perp}(t)\; \xi(\tau - t)\; \lambda(\tau - t) \qquad \approx W_{\perp}(\tau) \text{ for } \tau \gg \sigma_t$$

Impedance modelled by 1 kick per turn[a]

lumped impedance approximation

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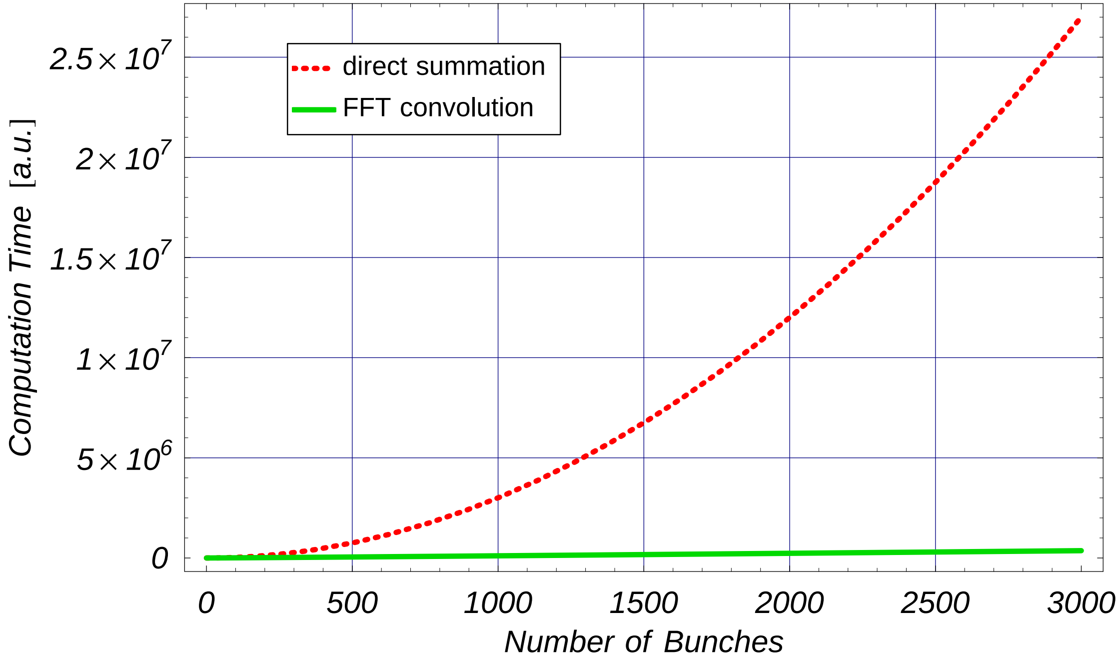
[a] K. Thompson and R. D. Ruth. *Transverse coupled bunch instabilities in damping rings of high-energy linear colliders.* Phys. Rev., D43:3049-3062, 1991.

The Simulation / Wake Summation Problem

The kick $\Delta x'$ on bunch j at turn n, in the case of the resistive wall impedance:

$$\underbrace{\Delta {x'}_n^j = \underbrace{\sum_{i=0}^{j-1} \frac{\langle x \rangle_n^i}{\sqrt{(j-i)\cdot \tau_{buc.}}}}_{\text{sum over preceding bunches at current turn}} + \underbrace{\sum_{k=n-n_{\text{mem}}}^{n-1} \sum_{i=0}^{N_b-1} \frac{\langle x \rangle_k^i}{\sqrt{(n-k)\cdot \tau_{rev.} + (j-i)\cdot \tau_{buc.}}}}_{\text{sum over preceding bunches over previous turns}}}_{\mathcal{O}(N_b^2)}$$



FFT Convolution

Analogy between wake sum and (discrete) convolution:

$$\Delta x'^{0}_{n} = \sum_{k=0}^{n-1} \frac{\langle x \rangle_k}{\sqrt{(n-k)\cdot \tau_{rev.}}} = \sum_{k=0}^{n-1} g(k)\cdot f(n-k)$$ (single bunch case)

Continuous Convolution $h(t) = g * f \equiv \int_{-\infty}^{\infty} d\tau \; g(\tau) f(t-\tau)$

Discrete Convolution $h(n) = g * f = \sum_{k=0}^{N-1} g(k)\, f(n-k) = \sum_{k=0}^{N-1} g_k \, f_{n-k}$

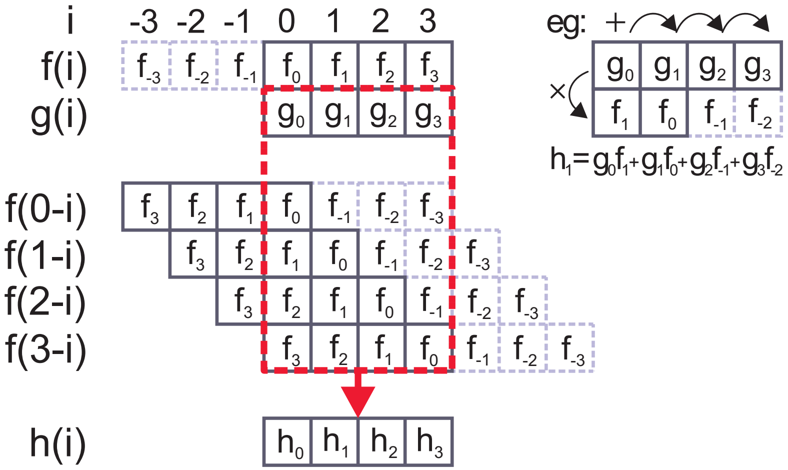
Convolution Theorem $\mathcal{F}\left[f * g\right] = \mathcal{F}[f]\mathcal{F}[g]$ or $f * g = \mathcal{F}^{-1}\left[\mathcal{F}[f]\mathcal{F}[g]\right]$

Compute wake sum via the FFT convolution in frequency domain:

$$\Delta x'^{0}_{n} = h_n = (g * f)_n = \sum_{k=0}^{N-1} g_k \, f_{n-k} \quad \overset{FFT}{\underset{IFFT}{\Longleftrightarrow}} \quad G_j F_j = H_j$$

FFT Convolution

Circular Convolution



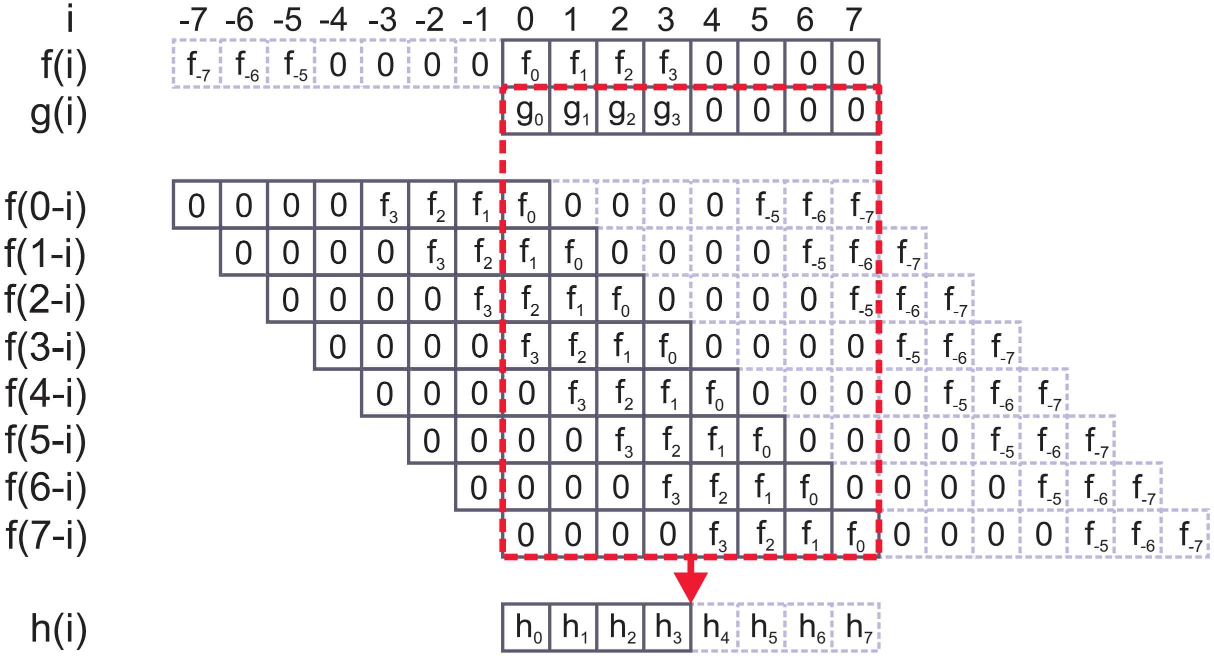
i	-3	-2	-1	0	1	2	3			
f(i)	f_{-3}	f_{-2}	f_{-1}	f_0	f_1	f_2	f_3			
g(i)				g_0	g_1	g_2	g_3			
f(0-i)	f_3	f_2	f_1	f_0	f_{-1}	f_{-2}	f_{-3}			
f(1-i)		f_3	f_2	f_1	f_0	f_{-1}	f_{-2}	f_{-3}		
f(2-i)			f_3	f_2	f_1	f_0	f_{-1}	f_{-2}	f_{-3}	
f(3-i)				f_3	f_2	f_1	f_0	f_{-1}	f_{-2}	f_{-3}
h(i)				h_0	h_1	h_2	h_3			

eg: +

×	g_0	g_1	g_2	g_3
	f_1	f_0	f_{-1}	f_{-2}

$h_1=g_0f_1+g_1f_0+g_2f_{-1}+g_3f_{-2}$

Linear Convolution

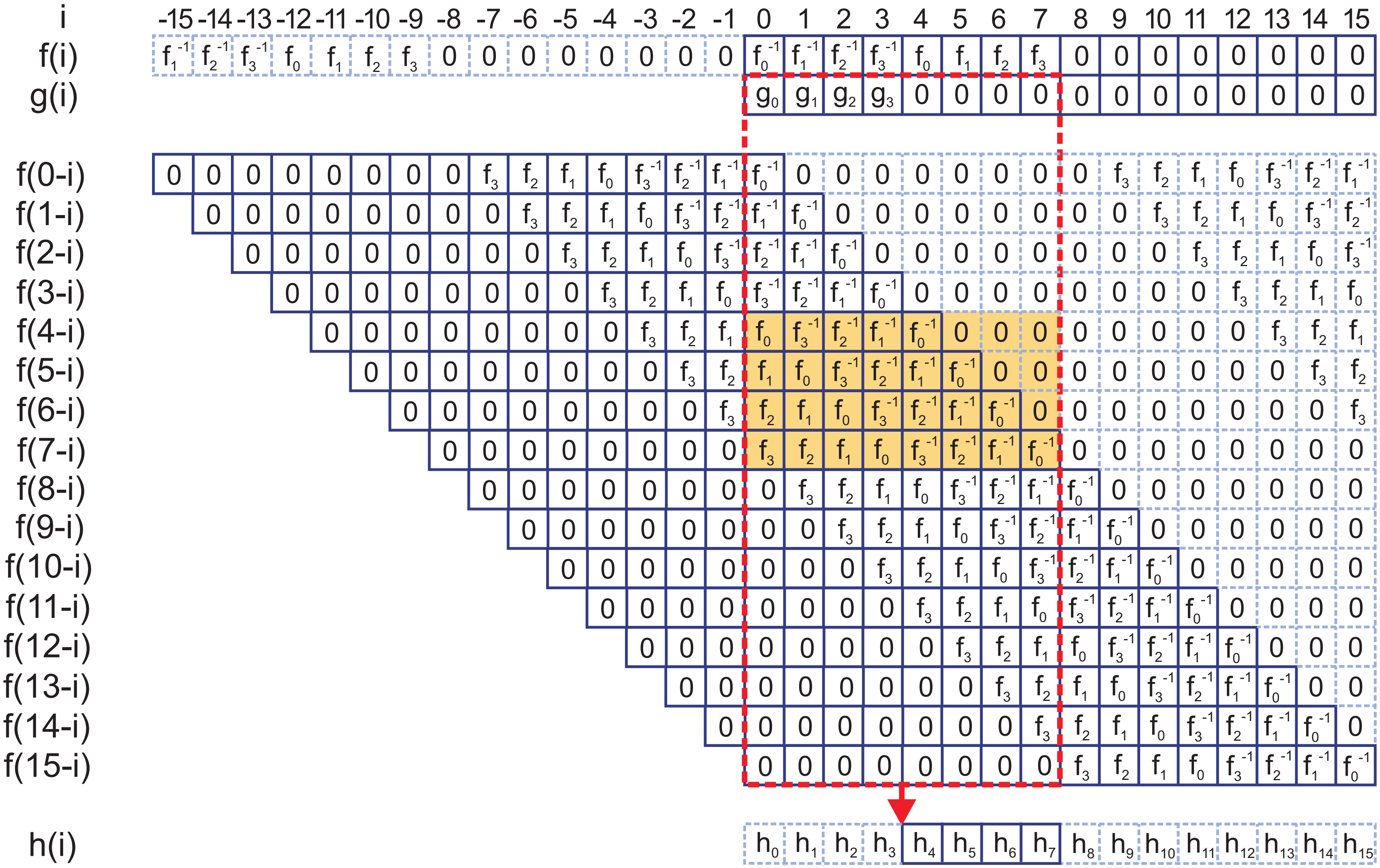


i	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7							
f(i)	f_{-7}	f_{-6}	f_{-5}	0	0	0	0	f_0	f_1	f_2	f_3	0	0	0	0							
g(i)								g_0	g_1	g_2	g_3	0	0	0	0							
f(0-i)	0	0	0	0	f_3	f_2	f_1	f_0	0	0	0	0	f_{-5}	f_{-6}	f_{-7}							
f(1-i)		0	0	0	0	f_3	f_2	f_1	f_0	0	0	0	0	f_{-5}	f_{-6}	f_{-7}						
f(2-i)			0	0	0	0	f_3	f_2	f_1	f_0	0	0	0	0	f_{-5}	f_{-6}	f_{-7}					
f(3-i)				0	0	0	0	f_3	f_2	f_1	f_0	0	0	0	0	f_{-5}	f_{-6}	f_{-7}				
f(4-i)					0	0	0	0	f_3	f_2	f_1	f_0	0	0	0	0	f_{-5}	f_{-6}	f_{-7}			
f(5-i)						0	0	0	0	f_3	f_2	f_1	f_0	0	0	0	0	f_{-5}	f_{-6}	f_{-7}		
f(6-i)							0	0	0	0	f_3	f_2	f_1	f_0	0	0	0	0	f_{-5}	f_{-6}	f_{-7}	
f(7-i)								0	0	0	0	f_3	f_2	f_1	f_0	0	0	0	0	f_{-5}	f_{-6}	f_{-7}
h(i)								h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7							

To get the correct wake sums, linear convolution has to be realized by zero padding.

FFT Convolution

Multi-Bunch Multi-Turn Convolution Scheme



i	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
f(i)	f_1^{-1}	f_2^{-1}	f_3^{-1}	f_0	f_1	f_2	f_3	0	0	0	0	0	0	0	0	f_0^{-1}	f_1^{-1}	f_2^{-1}	f_3^{-1}	f_0	f_1	f_2	f_3	0	0	0	0	0	0	0	0
g(i)																g_0	g_1	g_2	g_3	0	0	0	0	0	0	0	0	0	0	0	0
f(0-i)	0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}
f(1-i)		0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}
f(2-i)			0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}
f(3-i)				0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0
f(4-i)					0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0	0	0	0	0	f_3	f_2	f_1
f(5-i)						0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0	0	0	0	0	f_3	f_2
f(6-i)							0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0	0	0	0	0	f_3
f(7-i)								0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0	0	0	0	0
f(8-i)									0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0	0	0	0
f(9-i)										0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0	0	0
f(10-i)											0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0	0
f(11-i)												0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0	0
f(12-i)													0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0	0
f(13-i)														0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0	0
f(14-i)															0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}	0
f(15-i)																0	0	0	0	0	0	0	0	f_3	f_2	f_1	f_0	f_3^{-1}	f_2^{-1}	f_1^{-1}	f_0^{-1}
h(i)																h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}

FFT Convolution

Wake Summation Algorithm

Search GCD (greatest common divisor) of given bucket layout. Defines new, equidistant bunch pattern. `mask`.

Set up extended zero-padded arrays f'^{c}.

Precalculate FFTs of f^c for $c = 0, \ldots, n_{\text{mem}}$, this gives arrays $F^c = \mathcal{F}\left[f^c\right]$

At turn n do the following:

- Write (signal, offsets) to array g_i use `mask`.
- FFT of g, $G^{c=0} = \mathcal{F}\left[g\right]$
- Multiply $\forall\, c \in \left[0, n_{\text{mem}}\right]: \;\; H^c = F^c \cdot G^c$
- Inv. FFT $\forall\, c \in \left[0, n_{\text{mem}}\right]: \;\; h^c = \mathcal{F}^{-1}\left[H^c\right]$
- Sum over n_{mem} turns to get the kicks, use the `mask`: $\Delta x'^{j}_{n} = \sum_{c=1}^{n_{\text{mem}}} h^c_{j+N_b} + h^{c=0}_j$.

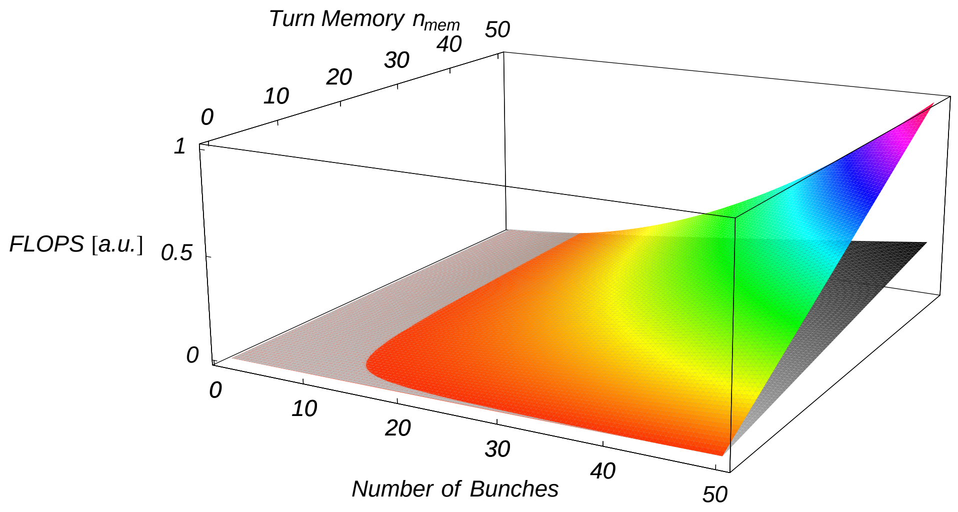
Speed Considerations

Direct Summation

$$\left(2n_{\text{mem}} + 1\right) N_b^2 + N_b$$

FFT Convolution

$$\left(n_{\text{mem}} + 2\right) 4N_b \log 4N_b + \left(5n_{\text{mem}} + 5\right) N_b$$

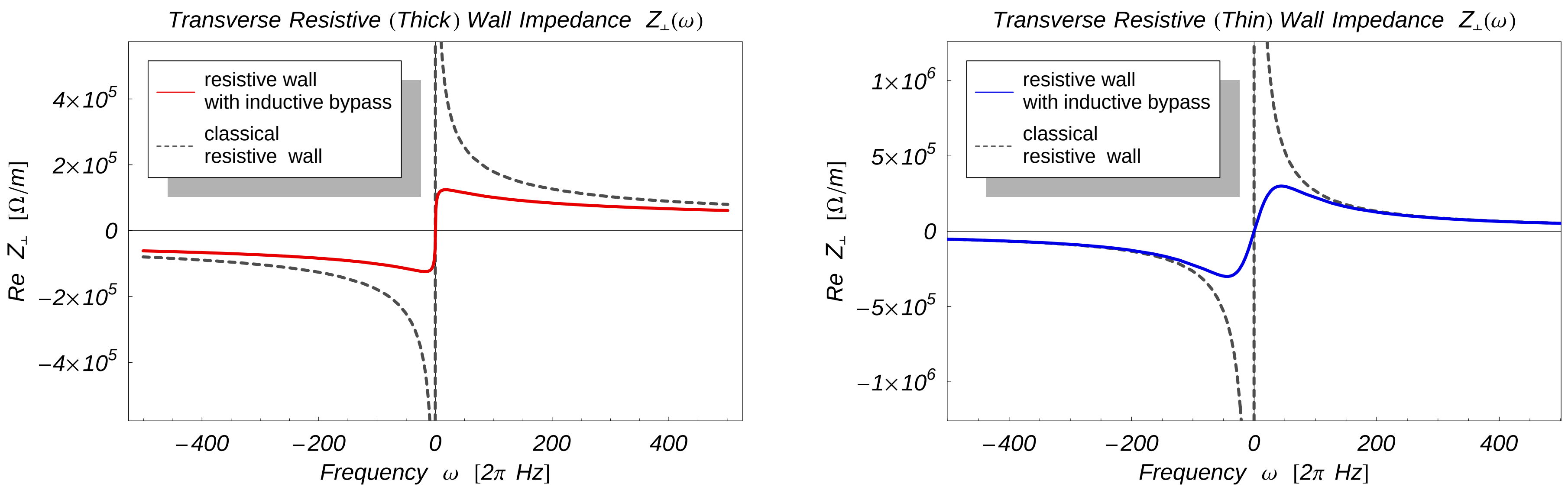


Resistive Wall Impedance Models

Classical thick & thin wall formula known to be incorrect for $\omega \rightarrow 0$.

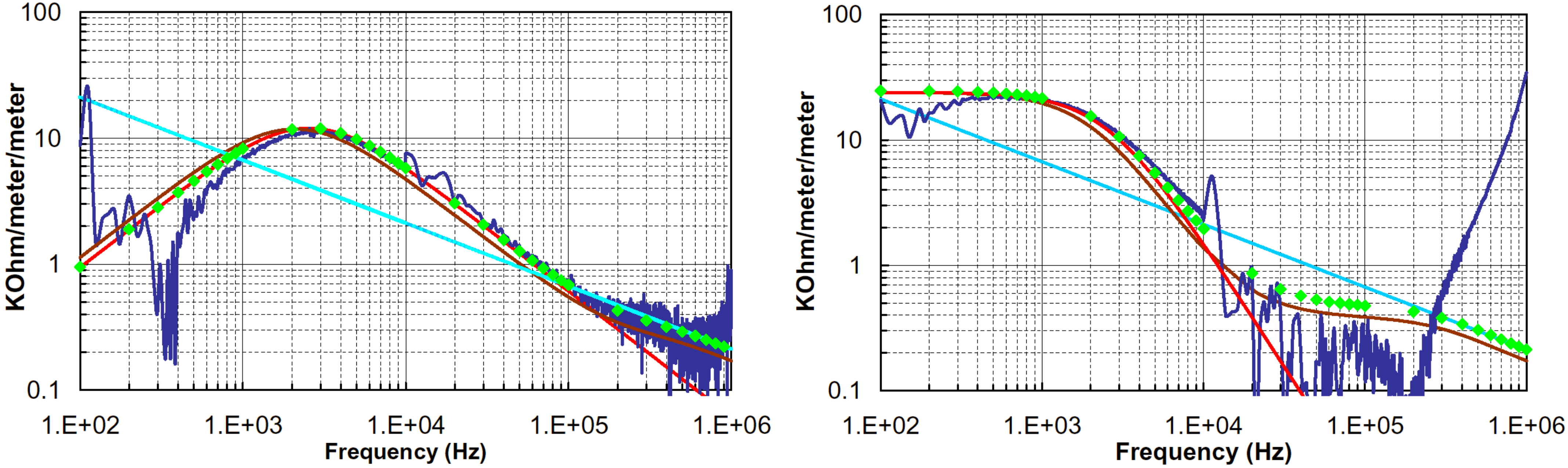
$$Z_{m=1}^{\perp,\,\text{thick}}(\omega) = (\operatorname{sgn}\omega + j)\,\frac{Z_0\,L\,\delta_0\,\mu_r}{2\,\pi\,b^3}\cdot\sqrt{\frac{\omega_0}{|\omega|}}$$

$$Z_{m=1}^{\perp,\,\text{thin}}(\omega) = \frac{c\,L}{\pi\,b^3\,\sigma_c\,d\cdot\omega}$$



Resistive Wall Impedance Models

Measured[a]
Transverse Resistive Wall Impedance
Real (left) and Imaginary (right) part



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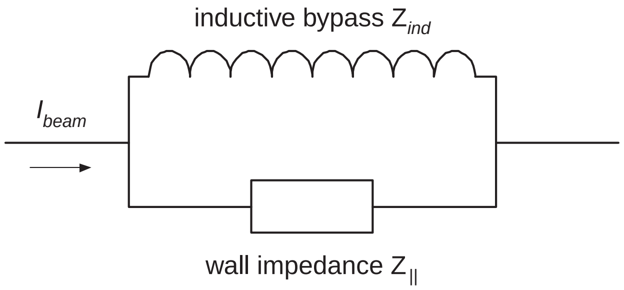
[a] A. Mostacci, F. Caspers, and U. Iriso. *Bench measurements of low frequency transverse impedance*. CERN-AB-2003-051-RF. Proc. of PAC 03, Portland, Oregon, 12-16 May 2003.

Resistive Wall Impedance Models

Resistive Wall Impedance with ’inductive bypass’ (L.Vos)

$$Z_{m=1}^{\perp,\,\mathrm{LV}}(\omega) = \frac{Z_0\,L}{2\,\pi\,b^2}\cdot\left[\frac{b\omega\mu_0}{2Z_0}\cdot\frac{1+\frac{Z_0}{Z_1}\tanh\gamma_1 d_1}{1+\frac{Z_1}{Z_0}\tanh\gamma_1 d_1} - j\right]^{-1}$$

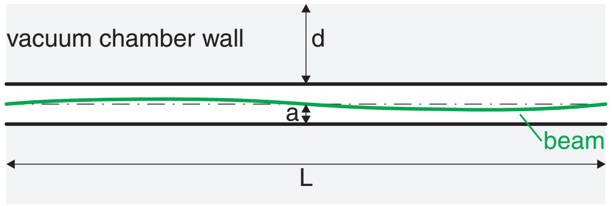


$$Z_{\perp}^{\mathrm{ibp}} = \frac{2c}{b^2\omega}\cdot\frac{Z_{||}\,Z_{\mathrm{ind}}}{Z_{||}+Z_{\mathrm{ind}}}$$

Quasi-Static Beam Model (Burov/Lebedev)

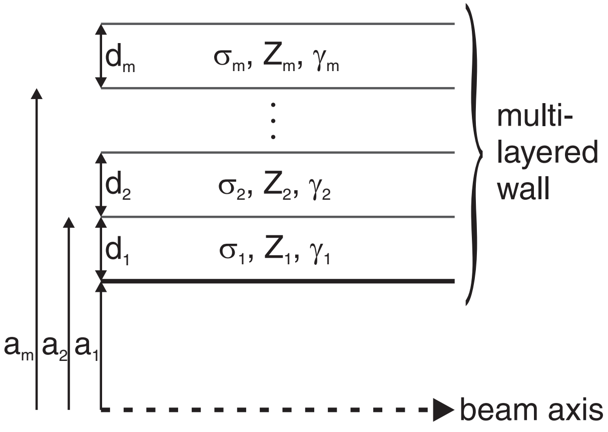
Solving Maxwell Equations, Poisson equation for electric dipole and vector potential for magnetic dipole

$$Z_{m=1}^{\perp,\mathrm{BL}}(\omega) = j\,\frac{Z_0\beta L}{\pi b^2}\,\frac{\mathsf{s}_1'+\tilde{\kappa}_{2,1}\mathsf{s}_1}{\mathsf{s}_1'+\tilde{\kappa}_{2,1}\tilde{\kappa}_{1,0}\mathsf{c}_1+\tilde{\kappa}_{2,1}\mathsf{s}_1+\tilde{\kappa}_{1,0}\mathsf{c}_1'}$$



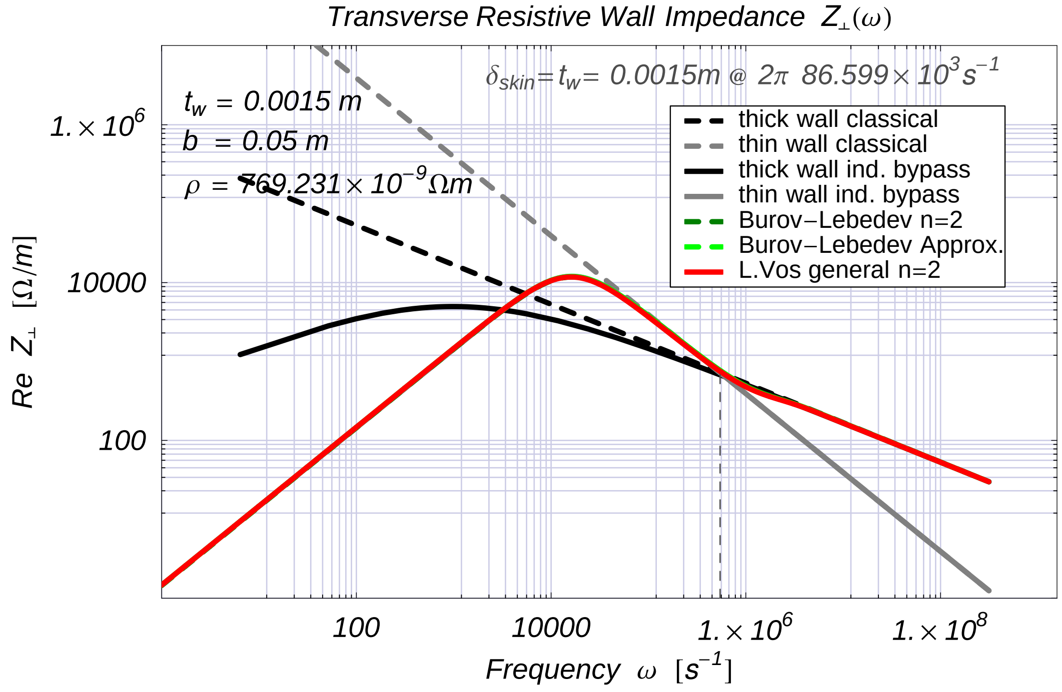
Field Matching (B.Zotter)

Solution of Maxwell’s equation by matching 4 components at every layer. Most rigorous but lengthy or only numeric expressions.

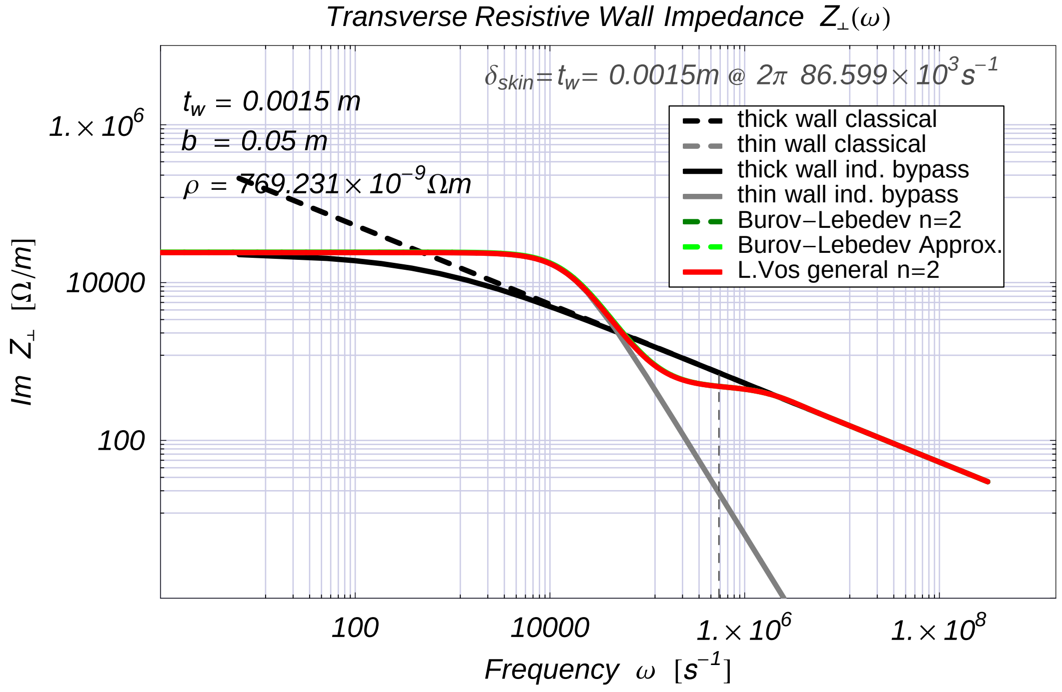


Resistive Wall Impedance Models

Standard Parameter Range: $b \gg d$



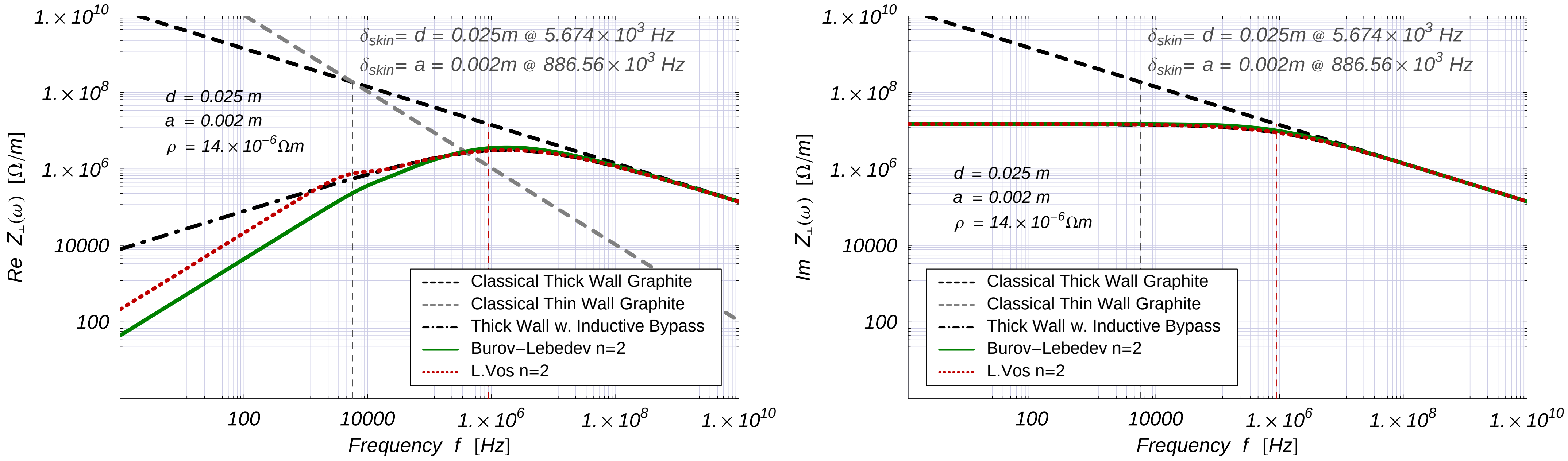
Transverse Resistive Wall Impedance $Z_{\perp}(\omega)$



Transverse Resistive Wall Impedance $Z_{\perp}(\omega)$

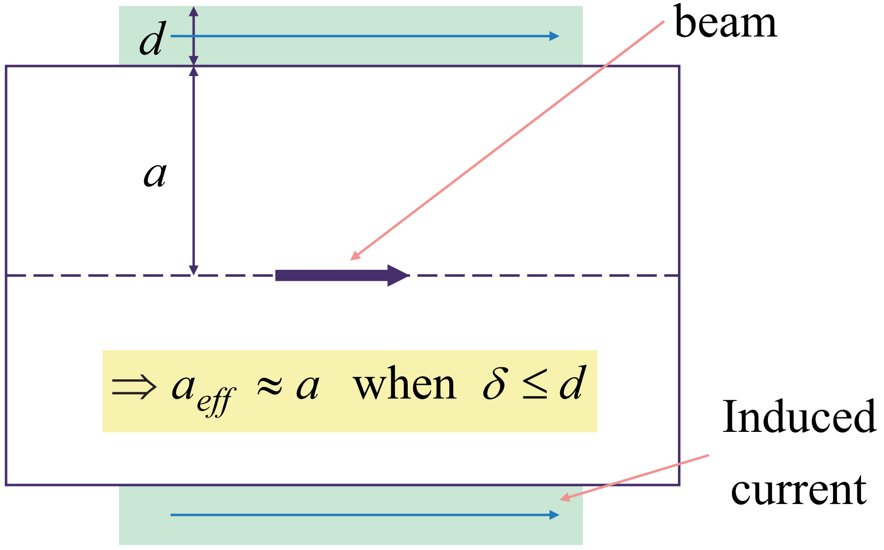
Resistive Wall Impedance Models

LHC Collimators: $b \ll d$

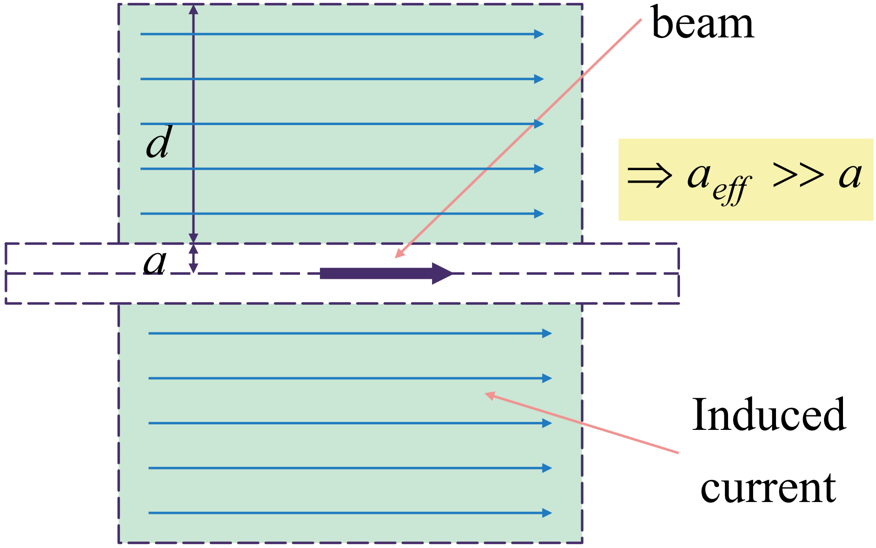


Resistive Wall Impedance Models

Usual regime : $d\,,\delta < a$



New regime : $d >> a\,,\delta \le d$



Standard parameter range: Wall thickness d is smaller than beam pipe radius a, and the skin depth δ for all frequencies under concern is smaller than d.

New regime: Wall thickness d is *larger* than beam pipe radius a, and the skin depth δ is in the order of d for low frequencies.

Resistive Wall Wake Function

$$Z_{m=1}^{\perp,\text{ thick,ibp}}(\omega) = (1 + j\ \mathrm{sgn}\,\omega)\ \frac{c\,\mu_0\,L}{2\,\pi\,b^2}\ \frac{1}{-j + \mathrm{sgn}\,\omega\left(1 + b\sqrt{\frac{\sigma_c\,\mu_0}{2\,\mu_r}}\ \sqrt{|\omega|}\right)}$$

$$\Big\downarrow$$

Fourier Transform (not straightforward!)

$$W_{m=1}^{\perp,\text{ thick,ibp}}(t) = \underbrace{+\frac{cL}{\pi^{3/2}b^3}\sqrt{\frac{\mu_o\mu_r}{\sigma_c}}\cdot\frac{1}{\sqrt{|t|}}}_{\text{classic thick wall wake function}}$$

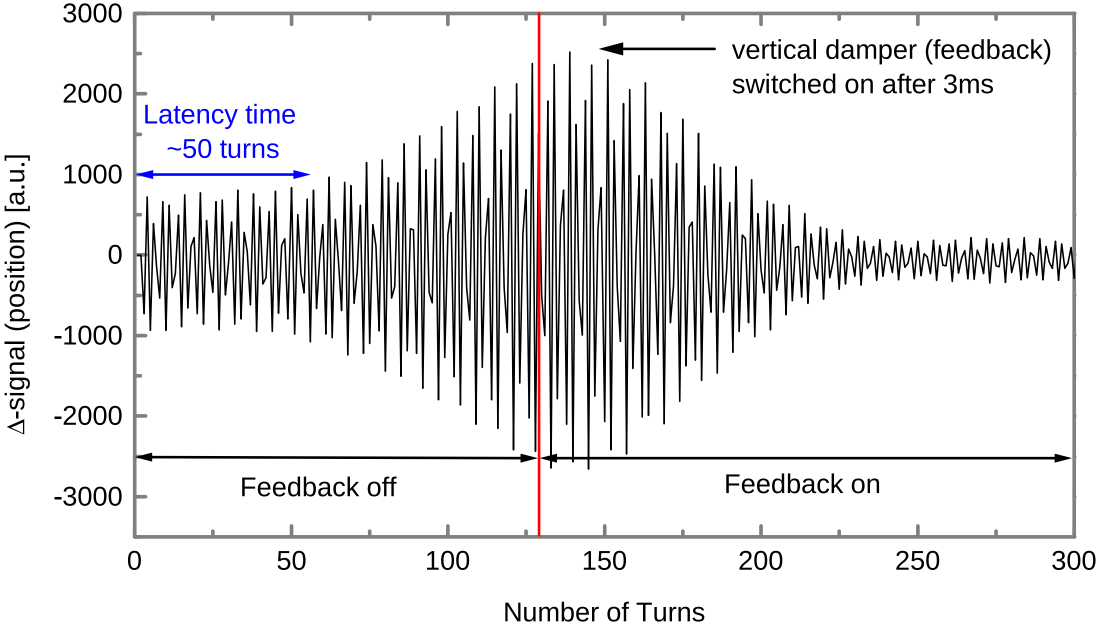
$$\underbrace{-\exp\left[\frac{4\mu_r}{b^2\sigma_c\mu_0}\,|t|\right]\ \frac{2cL\mu_r}{b^4\pi\sigma_c}\ \cdot\left(1-\mathrm{Erf}\sqrt{\frac{4\mu_r}{b^2\sigma_c\mu_0}\,|t|}\right)}_{\text{correction term due to inclusion of inductive bypass}}$$

SPS Measurement & Simulation

Measurement Parameters

Fixed Target Beam	SPS @ inj.
Momentum p [GeV/c]	14
Revolution time $\tau_{\text{rev.}}$ [μs]	23.07
Tunes Q_H / Q_V	26.64 / 26.59
Gamma transition γ_T	23.2
Maximum # of batches	2
# of bunches per batch	2100
Bunch Intensity N_p	$4.8 \cdot 10^9$
Total Intensity $N_{p,tot.}$	$1.0 - 2.0 \cdot 10^{13}$
Batch spacing [ns]	1050
Bunch spacing [ns]	5
Full bunch length [ns]	4
Trans. emittance $\epsilon_{H,V}$ [μm]	<10 / <7.5
Long. emittance ϵ_L [eVs]	0.2

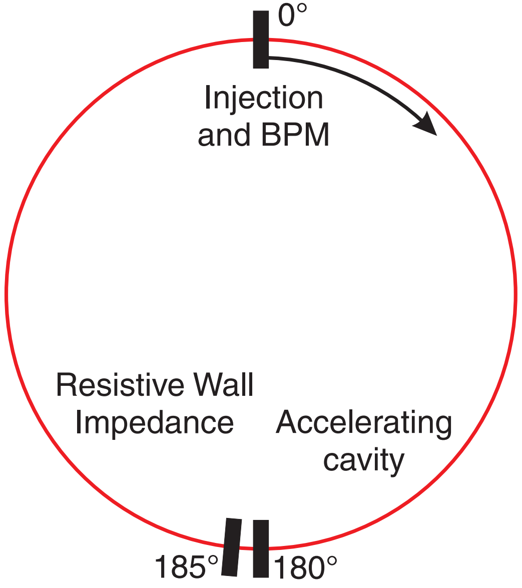
Typical vertical BPM readings for growth rate measurements



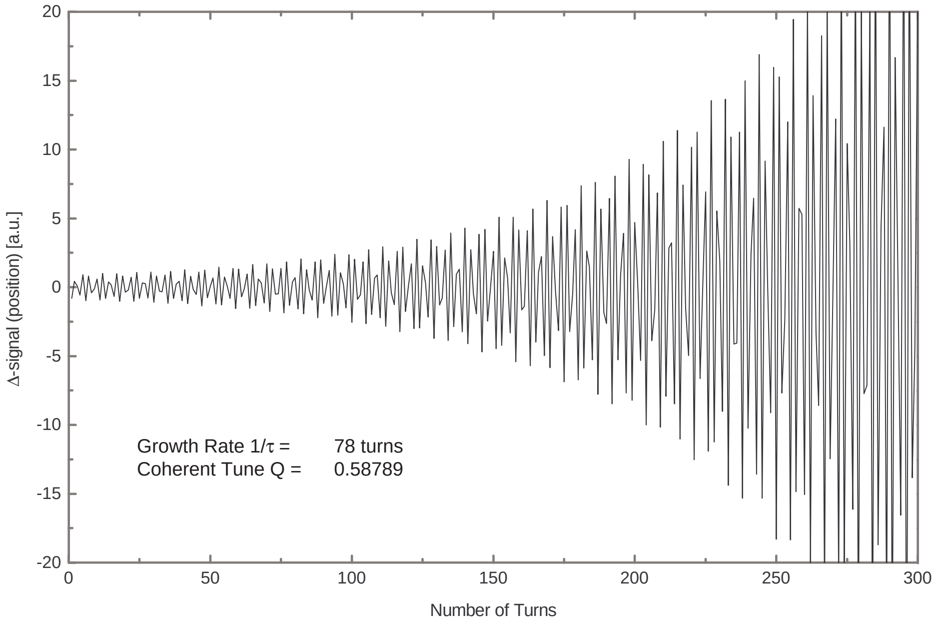
1 Batch (2100 bunches)	Growth rate $2\pi/\tau$ [turns]	Coherent tune Q
Vertical Plane	77 ± 4	0.5927 ± 0.0079
Horizontal Plane	183.5 ± 23.5	0.6180 ± 0.0029

SPS Measurement & Simulation

SPS Simulation Model



Typical simulation result of resistive wall instability



1 Batch (2100 bunches)	Growth rate $2\pi/\tau$ [turns]	Coherent tune Q
Vertical Plane	78 ± 2	0.58708 ± 0.0001
Horizontal Plane	150 ± 5	0.63854 ± 0.0001

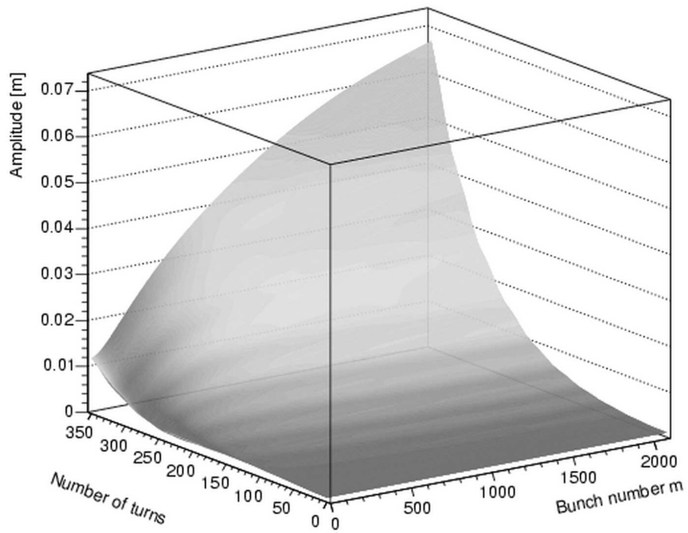
SPS elliptic vacuum chamber geometry → Yokoya factors included!
Wake is not only dependent on exciting charge’s offset, but also on witness particle offset:

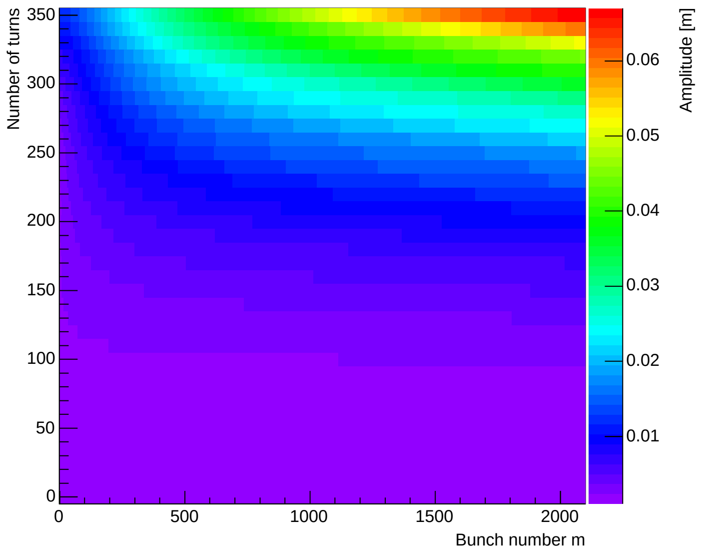
$$W^{\text{pot}}_{\perp,x}(x, y, \bar{x}, \bar{y}, s) \approx x\, W^{\text{pot}}_{\perp}(x, s) + \bar{x}\, W^{\text{pot}}_{\perp}(\bar{x}, s)$$

$$W^{\text{pot}}_{\perp,y}(x, y, \bar{x}, \bar{y}, s) \approx y\, W^{\text{pot}}_{\perp}(x, s) + \bar{y}\, W^{\text{pot}}_{\perp}(\bar{y}, s)$$

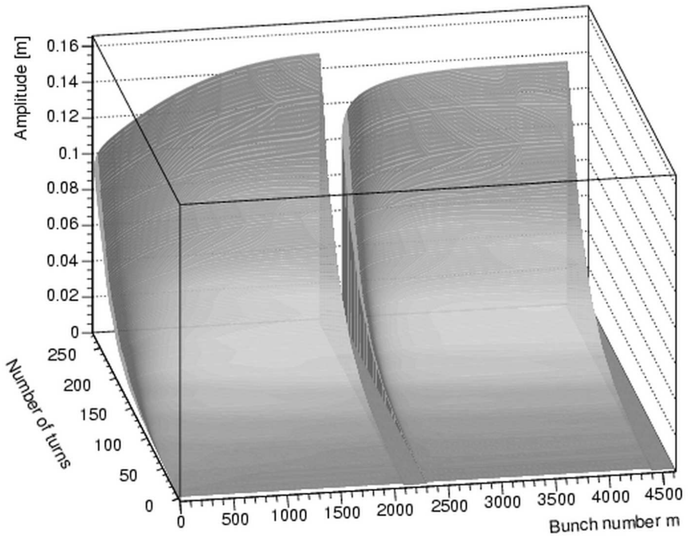
SPS Measurement & Simulation

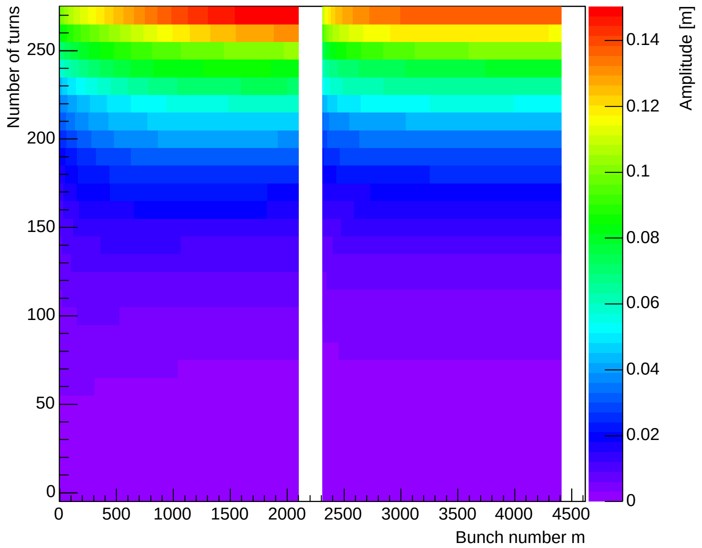
Vertical Amplitude Growth, SPS FT beam, $n_b = 2100$ (1 batch)





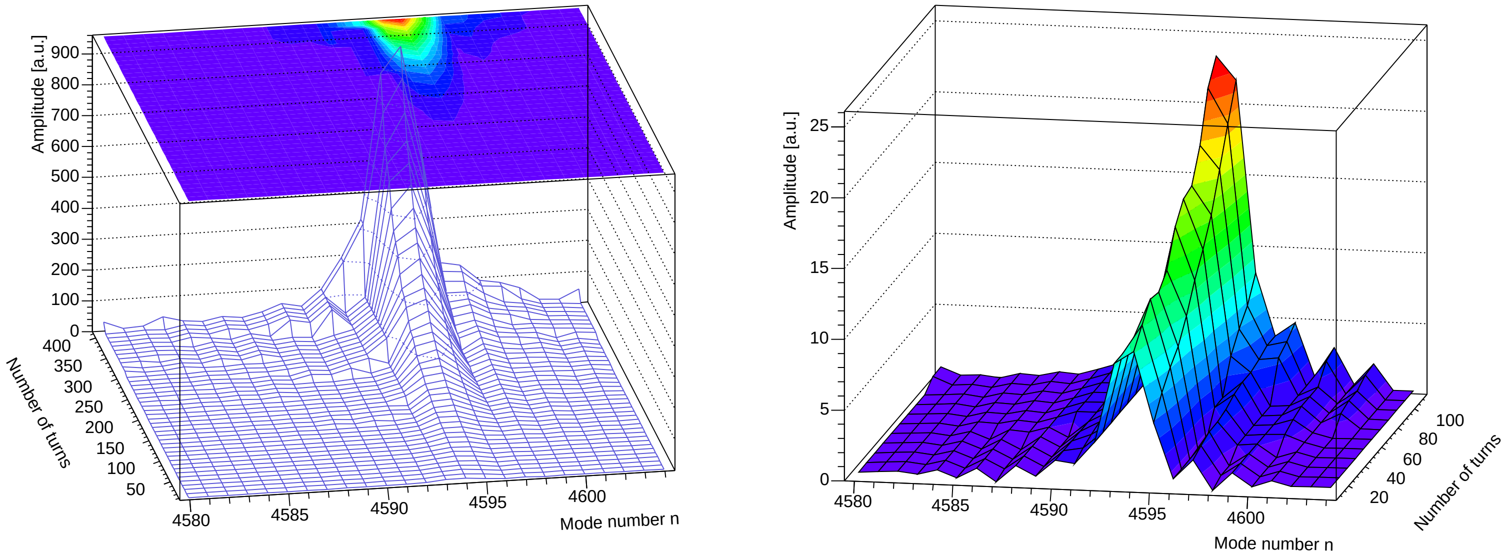
Vertical Amplitude Growth, SPS FT beam, $n_b = 4200$ (2 batches)



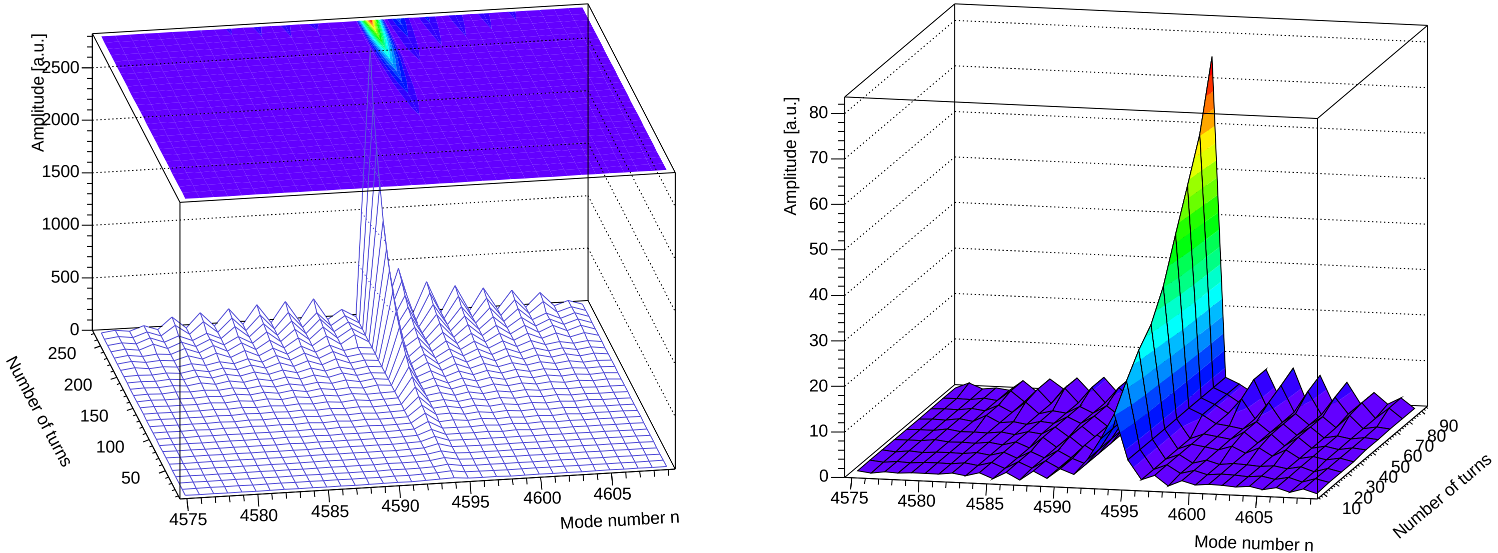


SPS Measurement & Simulation

Coupled Bunch Modes, SPS FT beam, $n_b = 2100$ (1 batch)

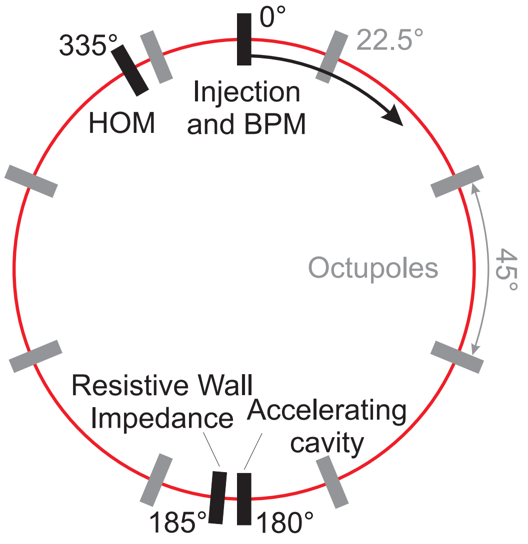


Coupled Bunch Modes, SPS FT beam, $n_b = 4200$ (2 batches)



Simulation of LHC

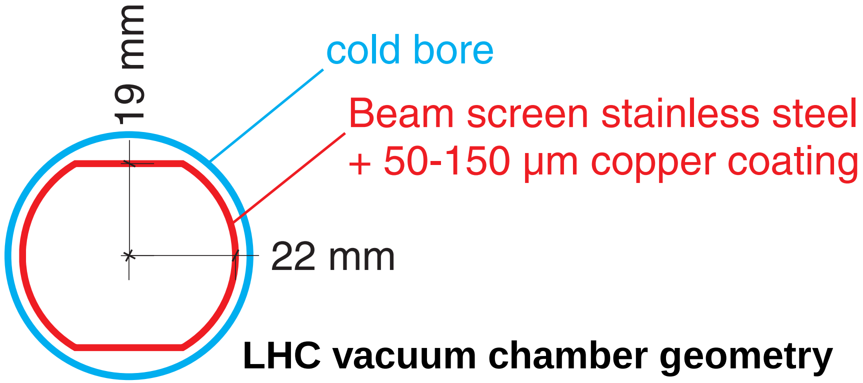
LHC Simulation Model



Parameter	Injection	Collision
Momentum p [GeV/c]	450	7000
Circumference C [m]	26658.883	
Rev. frequency f_0 [Hz]	11245.5	
Dipole field B [T]	0.535	8.33
Hor./Ver. Tune	64.28/59.31	
Harmonic number h	35640	
RF Frequency f_{RF} [MHz]	400.8	
RF Voltage V_{RF} [MV]	8.0	16.0
Particles per bunch N_p $[10^{11}]$	1.15	
Number of bunches n_b	2808	
Bunch spacing τ_{sp} [ns]	25.0	
Total # of particles N_{tot} $[10^{14}]$	3.23	
Total DC beam current I [A]	0.582	
Luminosity L [cm^{-2}s^{-1}]	$1.0 \cdot 10^{34}$	

Impedances

- HOMs
  (undamped modes of the 200MHz cavities)
- Resistive Wall
  (Machine Resistance = mostly beam screen, collimators)



LHC vacuum chamber geometry

Non-Linearities

- Octupoles

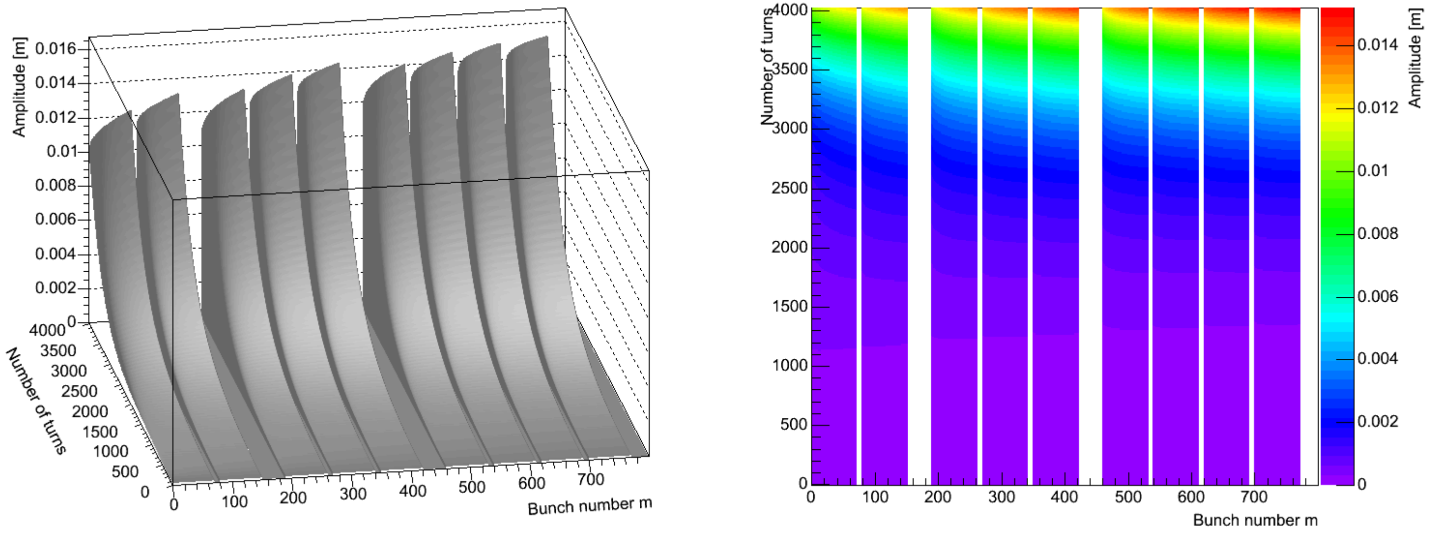
$$\delta x' = \frac{k_3 \cdot l}{3!} \left(x^3 - 3\, x\, y^2 \right)$$

$$\delta y' = \frac{k_3 \cdot l}{3!} \left(3\, x^2\, y - y^3 \right)$$

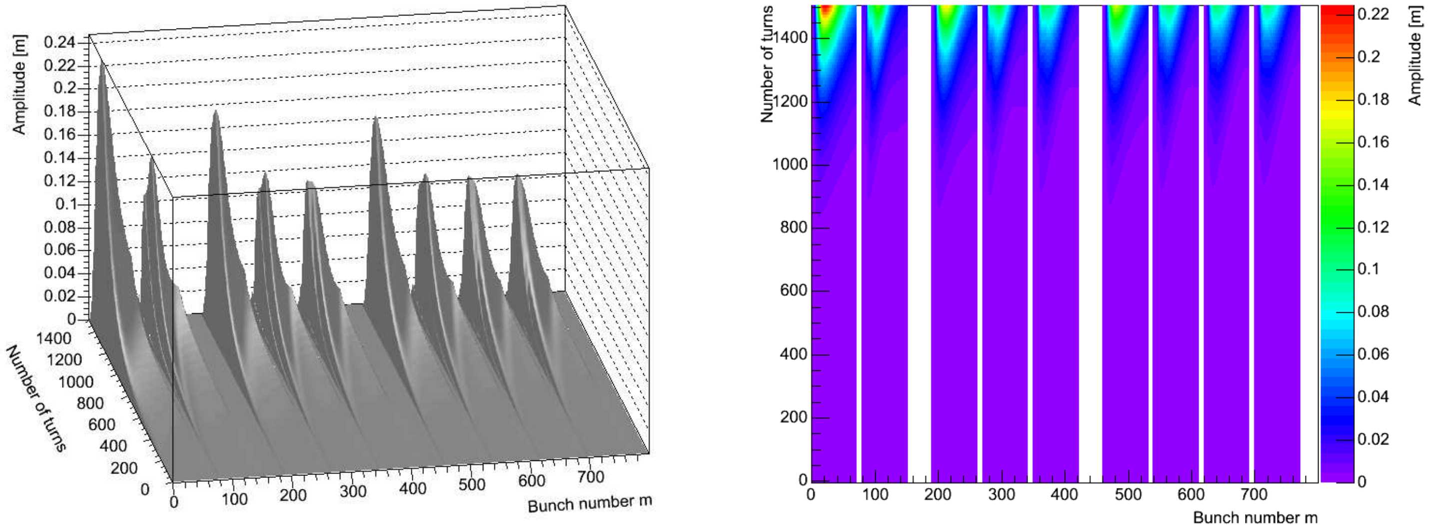
Simulation of LHC – Results

Amplitude growth of individual bunches vs. Turns

Machine Resistance only, LHC Injection Energy, nominal Intensity

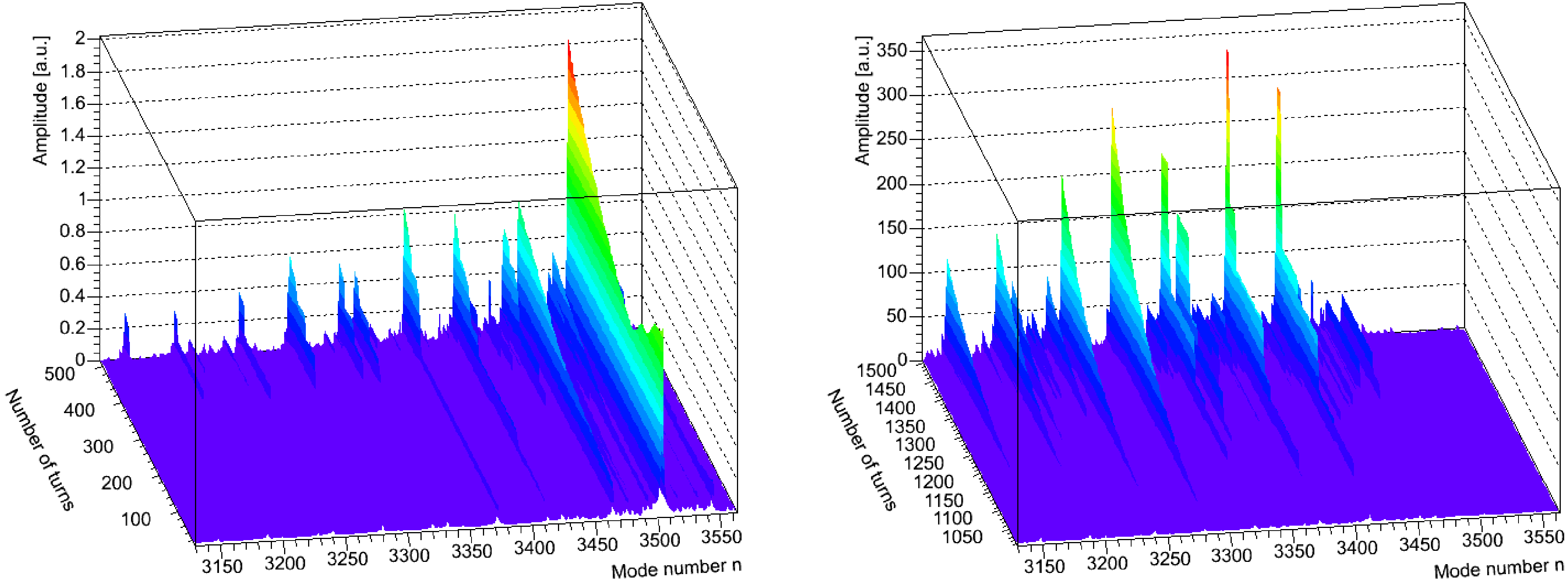


Machine Resistance + Collimators, LHC Injection Energy, nominal Intensity



Simulation of LHC – Results

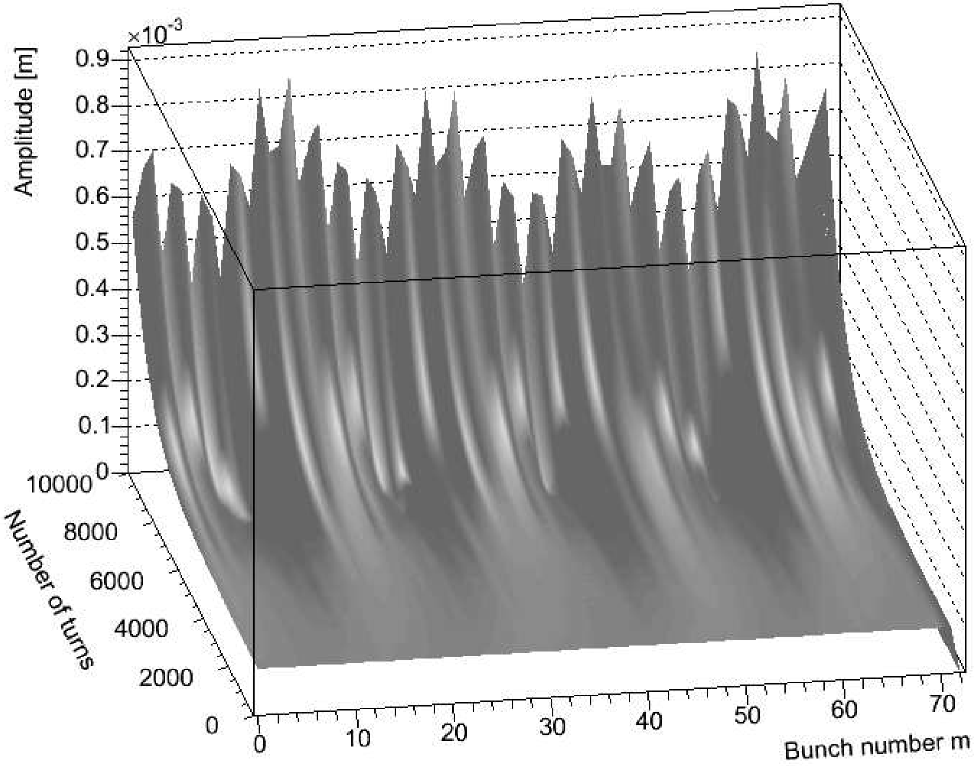
Coupled-bunch mode spectra vs. Turns
at injection energy and nominal intensity
Machine Resistance + Collimators, LHC Injection Energy, nominal Intensity

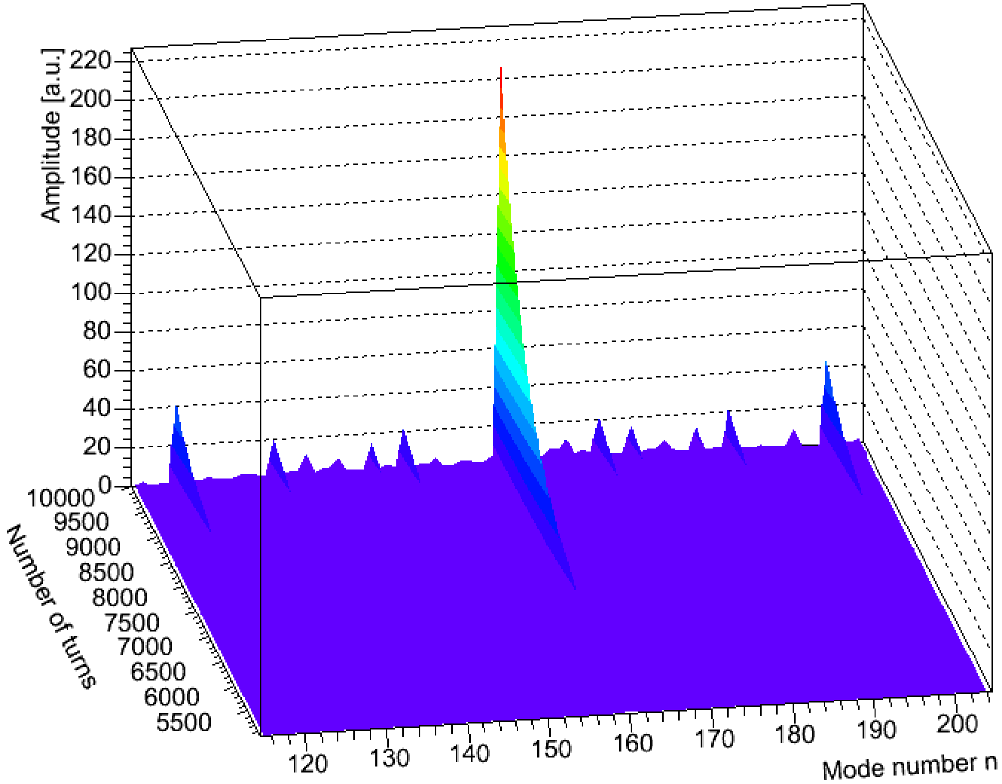


Simulation of LHC – Results

Amplitude growth and coupled-bunch mode spectra vs. Turns

HOMs of 200MHz cavities, LHC Injection Energy, nominal Intensity



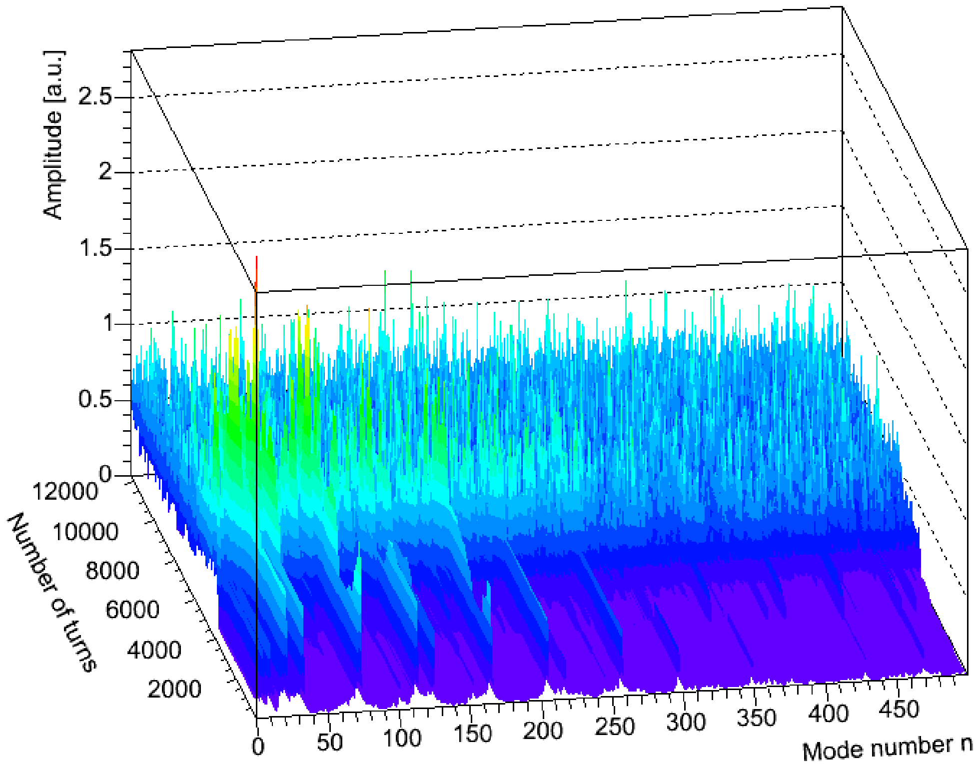


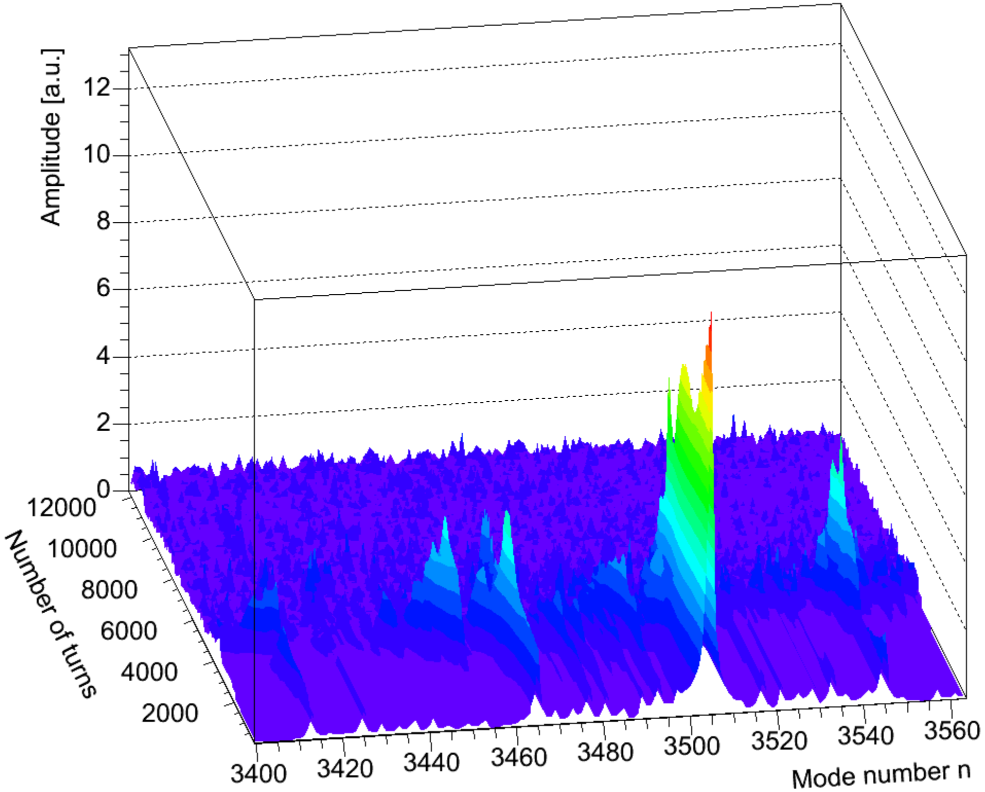
Simulation of LHC – Results

Coupled-bunch mode spectra vs. Turns

Machine Resistance + Collimators + HOMs and Octupoles

LHC Top Energy, ultimate Intensity





Non-linearities through octupoles → tune spread → Landau damping

Simulation of LHC – Results

	LHC injection energy			
	Nominal		Ultimate	
	Mode n	Growth Time τ_{turns} [turns]	Mode n	Growth Time τ_{turns} [turns]
Resistive Wall Classic	3504	659.2 ± 8.7	3504	453.7 ± 6.8
Resistive Wall Classic + Collimator	$<$ 10 turns		$<$ 7 turns	
Resistive Wall Inductive Bypass	3504	677.5 ± 8.7	3504	466.6 ± 7.2
Resistive Wall Inductive Bypass + Collimator	3149	167.8 ± 5.3	3149	115.8 ± 4.1
	3189	160.0 ± 4.6	3189	110.4 ± 3.6
	3241	158.6 ± 3.9	3241	109.7 ± 3.0
	3281	161.6 ± 3.2	3281	112.0 ± 2.7
	3373	162.2 ± 4.3	3373	119.0 ± 3.5
	3413	167.3 ± 4.5	3413	124.9 ± 3.7
	(3504)	(357.8 ± 395.3)	(3504)	(261.5 ± 259.8)
HOMs	160	1441.1 ± 114.9	160	994.0 ± 11.9

	LHC top energy			
	Nominal		Ultimate	
	Mode n	Growth Time τ_{turns} [turns]	Mode n	Growth Time τ_{turns} [turns]
Resistive Wall Classic	3504	10073.5 ± 1029.0	3504	9191.2 ± 523.8
Resistive Wall Classic + Collimator	3504	44.4 ± 40.9	3504	29.3 ± 29.3
Resistive Wall Inductive Bypass	3504	10592.1 ± 1571.8	3504	8340.3 ± 911.2
Resistive Wall Inductive Bypass + Collimator	3464	7057.2 ± 4342.9	3464	4957.9 ± 3753.4
	3465	6451.6 ± 7658.7	3465	4464.3 ± 5719.9
	3504	7352.9 ± 3746.8	3504	4310.3 ± 2285.2
HOMs	no growth visible		no growth visible	
Resistive Wall + Collimator + OCT	damped		damped	

Summary of growth rates.

Summary

- Simulation code *MultiTRISIM* developed
- Efficient implementation of wake summation via **FFT Convolution**
- Resistive wall impedance models in a new parameter regime $\delta_{\text{skin}}, \mathbf{d} > \mathbf{b}$
- Resistive wall **wake function with ’inductive bypass’** computed and used in simulation
- Code **benchmarked** with measurements in CERN SPS
- **Simulation of LHC**. Present octupole design should provide enough Landau damping to stabilize beam at top energy.