Some Notes on:

Continuous LHC Beta-Beat Measurements

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Motivation

- Motivation: System dependence on known/constant beta-function:
  - Machine Protection and Collimation, Physics, Squeeze Diagnostics
  - Classic methods: 'kick'-type excitation & BPMs, K-modulation & Q-PLL, Closed-orbit-response (LOCO)
    → cannot achieve the required precision/time-scales under nominal conditions!

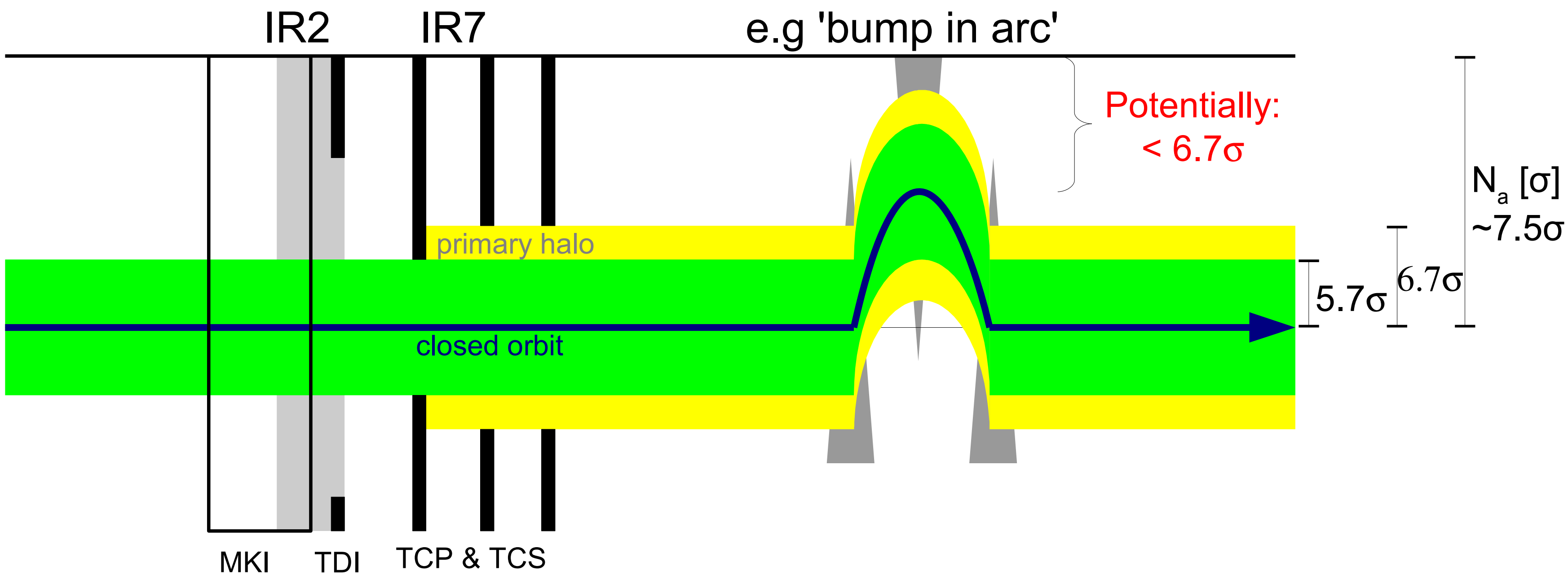
- β-Phase Advance Method - Turn-by-Turn
  - BBQ based Test-Setup in SPS LSS5
  - Systematic and statistical noise contribution
  - Exploitation Examples: SPS lattice drifts & off-momentum beta-beat

- β-Phase Advance Method – Orbit → Report in Preparation

- Next Steps & Control of Betatron-Function:

Compromised Machine Protection via Orbit Bumps

- Combined failure: Local orbit bump and collimation efficiency (/kicker failure):



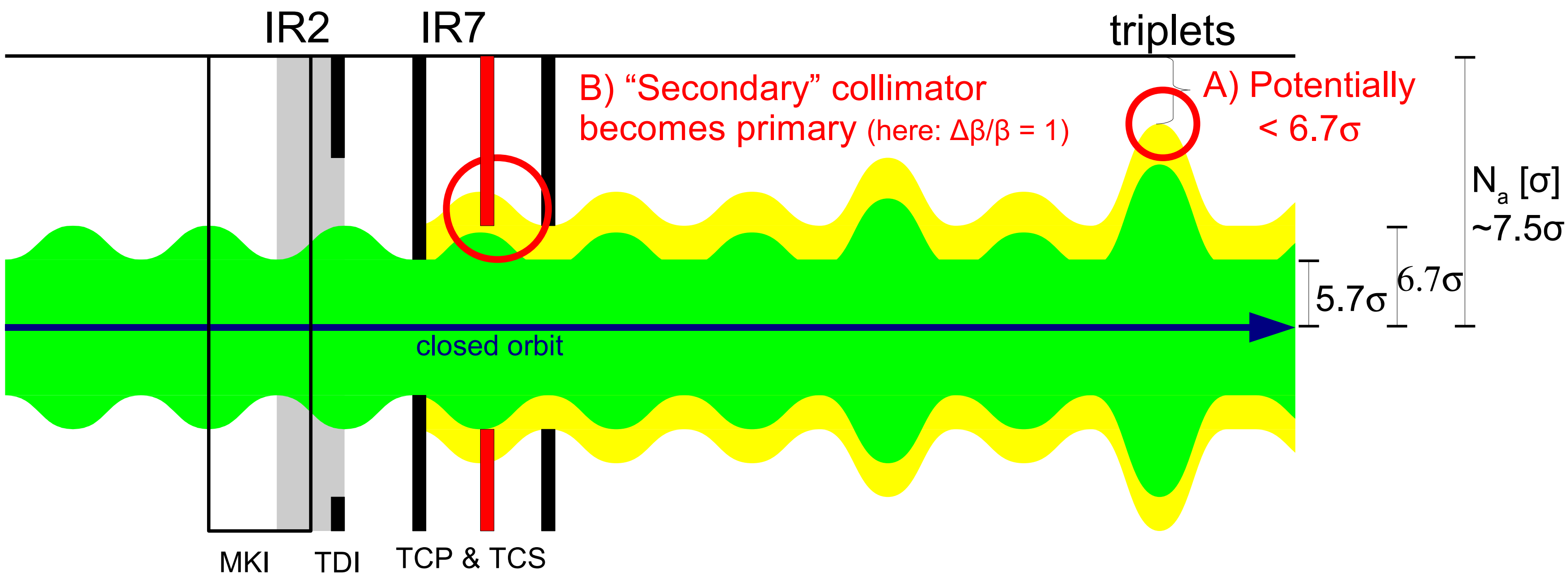
- Primary collimator (TCP) limits $|x_\beta(s)|_{max}$ locally to <5.7σ, secondary collimator (TCS) at~ 6.7σ
- To guarantee two stage cleaning efficiency/machine protection:
  - Local: TCP must be >0.7σ closer than TCS w.r.t. the beam → Orbit FB
  - Global: no other object (except TCP) closer to beam than TCS

→ Orbit bumps may compromise function of machine protection/collimation

→ tackled by LHC Orbit Feedback

Compromised Machine Protection through Beta-Beat

- Combined failure: beta-beat and collimation efficiency



- “Collimator gap must be 10 times smaller than available triplet aperture!”[1]

$$a_{coll} \leqslant a_{triplet} \cdot \overbrace{\sqrt{\frac{\beta_{coll}}{\beta_{triplet}}}}^{\sim 0.15} \cdot \overbrace{\left(\frac{A^{max}_{primary}}{A^{max}_{secondary}}\right)}^{\sim 0.6}$$

A) β-Beat reduces required protection: Δβ/β ≈ 20 % → 20% tighter collimator settings

B) β-Beat reduces cleaning performance

[1] R. Assmann, “Collimation and Cleaning: Could this limit the LHC Performance?”, Chamonix XII, 2003

Performance Limitations & Constraints on β*

If retraction is adjusted such to allow some maximum transient beta beat and orbit error, then **constraint of β^*:**

N.B. $C = \beta_{trip} \cdot \beta^*$

$$\beta^* \geq \frac{C^2}{a^2_{triplet} \cdot \beta_{coll}} \cdot \left(n_{prim} + \Delta A_{max} + 1.7 \cdot \left[n_{prim} \cdot \sqrt{\frac{\Delta\beta_{max}}{\beta_0}} + \frac{\Delta x^{max}_{orbit}}{\sigma_x} \right] \right)^2$$

Increase triplet aperture

IR Upgrade

Increase beta at collimators

Small primary gap

Sufficient number of secondaries at specific phases

Minimize any transient beta beat → **β-beat meas. + RT-Feedback?**

Minimize transient orbit changes → **LHC Orbit Feedback**

Larger β^* - A way to relax operational collimator tolerances!

(However, loose passive protection)

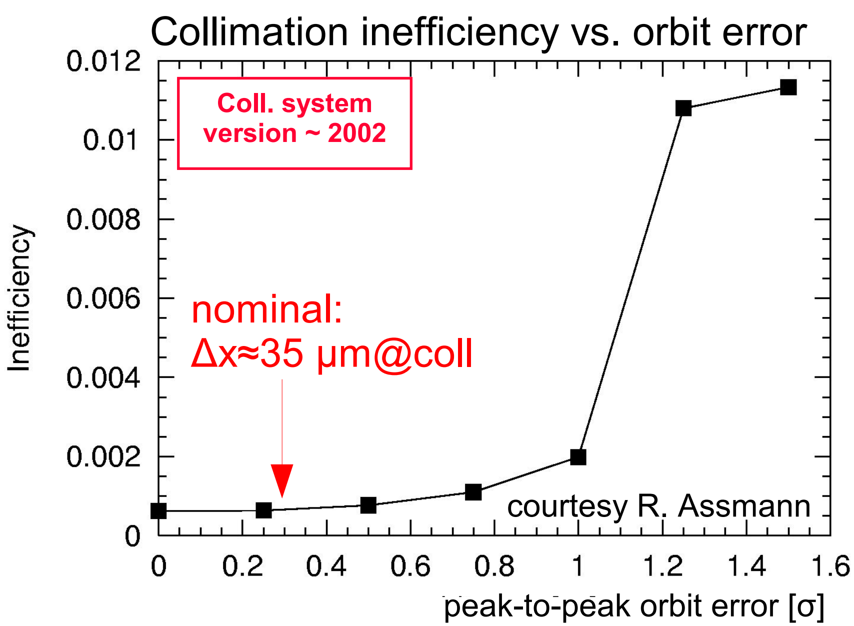
- Maximum allowed safe beam intensity[1,2]:

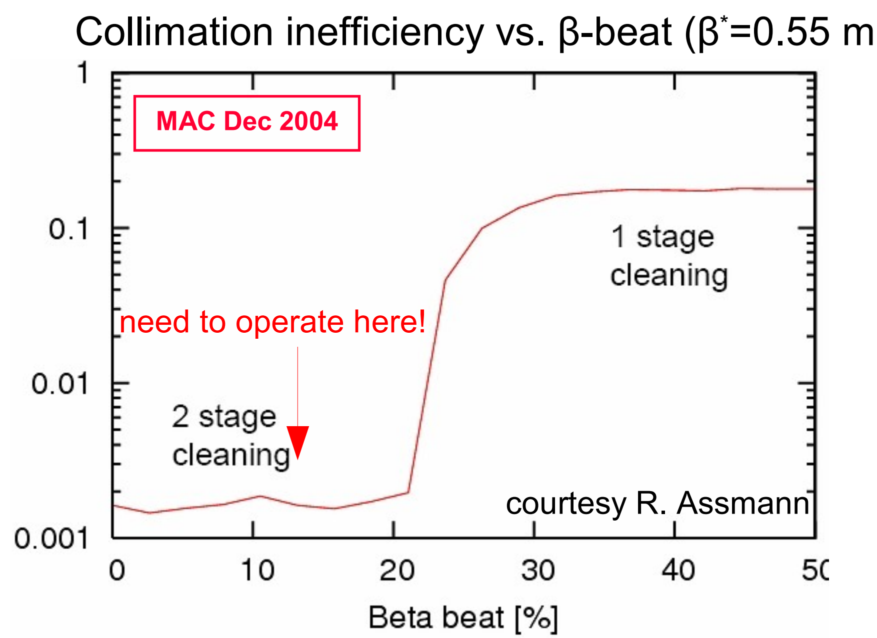
$$N_{max} \leqslant \frac{\tau_{min} \cdot R_q \cdot L_{dil.}}{\eta}$$

  - Min. accept. lifetime: $T_{min} \approx 10$ min.
  - Dilution length: $L_{dil} \approx 50$ m
  - Quench level (@7 TeV) R_q: $R_q \approx 7.6 \cdot 10^6$ prot./m/s
  - Collimation inefficiency: η

Peak-Luminosity:

$$L_{max} \approx \frac{1}{4\pi} \cdot \frac{N_{max} \cdot n_b f_{rev}}{\beta^* \epsilon}$$

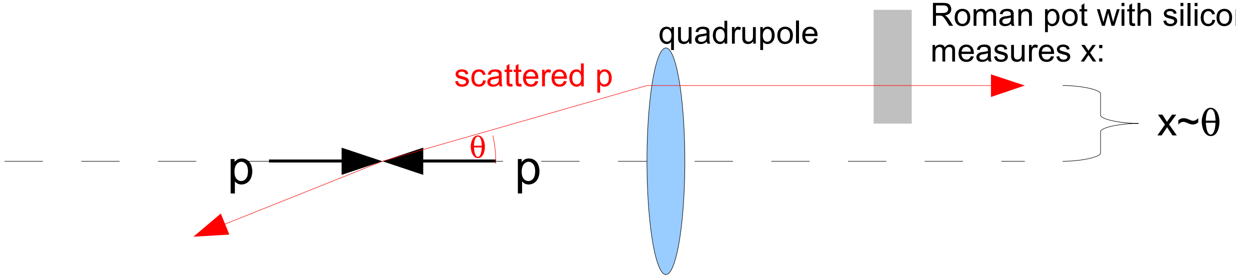




[1] R. Assmann, "Collimation and Cleaning: Could this limit the LHC Performance?", Chamonix XII, 2003
[2] S. Redaelli, "LHC aperture and commissioning of the Collimation System", Chamonix XIV, 2005
[3] R. Steinhagen, "Closed Orbit and Protection", MPWG #53, 2005-12-16

Constraint II: Diffractive Physics
p-p Luminosity using Optical Theorem

- Special parallel to point focusing machine optic ($\beta_0 \approx 1600$ m)



  – Roman Pots move close to the beam halo, measure dN/dt down to:

$$t_{min} = (p\,\theta_{min})^2 \sim \frac{p^2}{\beta_0 \beta_d} \cdot x_{min}^2$$

  – Observables: abs. Luminosity, total p-p cross-section, diffractive physics
    - Requires good knowledge on
      – Beta-functions β_0 at IP and β_d at detector
      – Beam momentum p
      – minimum distance of roman pot x_{min} w.r.t. beam centre

  – Desired: $\Delta L/L \approx 1\% \rightarrow \Delta t/t \approx 1\% \rightarrow 0.5\cdot\Delta\theta/\theta \approx \Delta x/x \approx 5\cdot 10^{-3}$

    → absolute beam position stability at roman pot (x_{min}~ 1mm) < 5 µm!!

    → value of betatron function at IP and RP: → $\Delta\beta/\beta \approx 1\%$!!

Limitations on Squeeze Diagnostic

- Squeeze involves > 45 individual magnetic strength settings (Optics), so far:
  no continuous check on effective optics during/at the end of individual steps

- "Classic" methods may not reach/be compatible with nominal requirements
  - K-modulation induced Q-Changes: $\Delta Q \approx \frac{1}{4\pi} \cdot \beta(s) \cdot \Delta k(s)$
    - Limit: knowledge on quadrupole transfer function
      (hysteresis, D&S, $\beta|^{max} \approx 4.2$km & $\Delta Q^{max} < 10^{-3} \rightarrow \Delta k/k_{nom} < 5 \cdot 10^{-5}$)
  - Kick + turn-by-turn analysis of BPM (phase and/or amplitude), limits:
    - Potential particle loss (beta-functions at triplet) & emittance blow-up
    - Systematic phase errors, amplitude detuning/Landau damping
      - large kicks may probe phase advances (dynamic aperture) which may not be representative for nominal beam operation
      - beam will be collimated at 6 sigma (kick amplitudes < 1.2 mm @7TeV)!
    - … not ideal for continuous monitoring/regular operation (ε blow-up).
  - Closed orbit response analysis (LOCO):
    - resolution/performance compatible with nominal operation
    - Limit: scan requires several minutes per IP (full scan: ~2 OP-shifts)

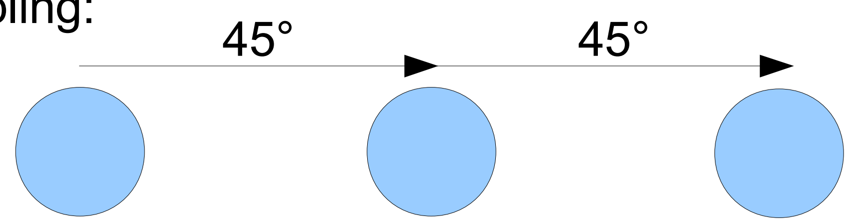
Betatron Function via Phase-Advance Measurement

- Long history at CERN. Original idea dates back to AB-BI report (doctoral thesis) P.Castro, *Luminosity and Betatron Function Measurement at [..] LEP*, CERN SL/96-70 (BI)
- … beating in amplitude related to beating in phase:

$$\frac{\Delta\beta}{\beta}(s) = \frac{1}{2\sin(2\pi Q)}\oint \beta_k \cos(2\cdot|\mu(s)-\mu(a)|-2\pi Q)\Delta k(a)\, da$$

$$\mu(s) := \int_0^s \frac{1}{\beta(a)} da \longrightarrow \frac{\Delta\mu}{\mu}(s) \sim \frac{\Delta\beta}{\beta}(s)$$

- Phase sampling:



Case I: $\Delta\varphi$ →   **0**   ← $-\Delta\varphi$ $\Delta\mu_{12}$ =45° - $\Delta\varphi$, $\Delta\mu_{13}$ =90° - 2 $\Delta\varphi$

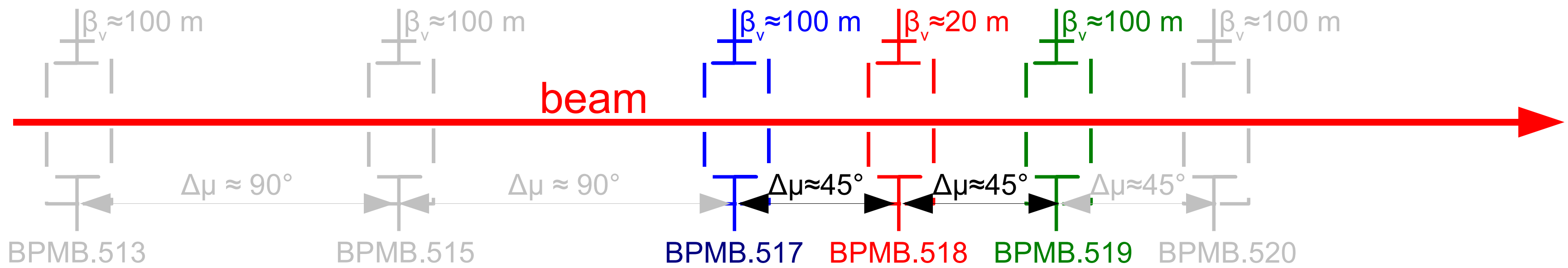
Case II: **0** $\Delta\varphi$ →   **0** $\Delta\mu_{12}$ =45° + $\Delta\varphi$, $\Delta\mu_{13}$ =90°

- Beta-Beat reconstruction (FB/Control would work with phases):

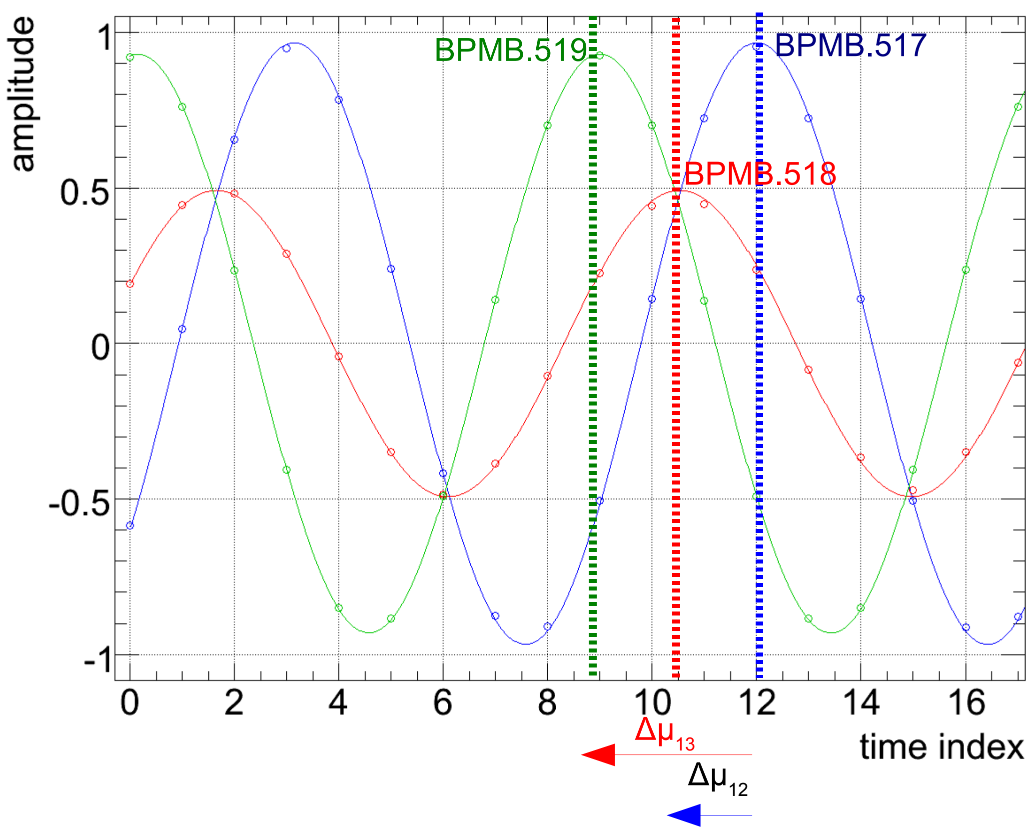
$$\frac{\Delta\beta_1}{\beta_1} = \frac{\cot(\Delta\mu_{12}^{meas.}) - \cot(\Delta\mu_{13}^{meas.})}{\cot(\Delta\mu_{12}^{theo.}) - \cot(\Delta\mu_{13}^{theo.})} \quad \frac{\Delta\beta_2}{\beta_2} = \frac{\cot(\Delta\mu_{12}^{meas.}) - \cot(\Delta\mu_{23}^{meas.})}{\cot(\Delta\mu_{12}^{theo.}) - \cot(\Delta\mu_{23}^{theo.})} \quad \frac{\Delta\beta_3}{\beta_3} = \frac{\cot(\Delta\mu_{23}^{meas.}) - \cot(\Delta\mu_{13}^{meas.})}{\cot(\Delta\mu_{23}^{theo.}) - \cot(\Delta\mu_{13}^{theo.})}$$

N.B. Phase-Beating usually used for correction!

Beta-Beat/LHC BPM Prototype System in the SPS-LSS5



- Measurement (markers), sinusoidal fit (solid line):



$$\frac{\Delta\beta_1}{\beta_1} = \frac{\cot(\Delta\mu_{12}^{meas.}) - \cot(\Delta\mu_{13}^{meas.})}{\cot(\Delta\mu_{12}^{theo.}) - \cot(\Delta\mu_{13}^{theo.})}$$

$$\frac{\Delta\beta_2}{\beta_2} = \frac{\cot(\Delta\mu_{12}^{meas.}) - \cot(\Delta\mu_{23}^{meas.})}{\cot(\Delta\mu_{12}^{theo.}) - \cot(\Delta\mu_{23}^{theo.})}$$

$$\frac{\Delta\beta_3}{\beta_3} = \frac{\cot(\Delta\mu_{23}^{meas.}) - \cot(\Delta\mu_{13}^{meas.})}{\cot(\Delta\mu_{23}^{theo.}) - \cot(\Delta\mu_{13}^{theo.})}$$

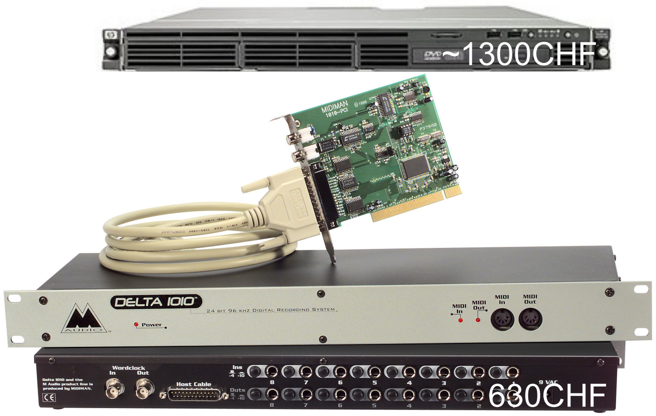
[1]P.Castro, *Luminosity and Betatron Function Measurement at [..] LEP*, CERN SL/96-70 (BI)

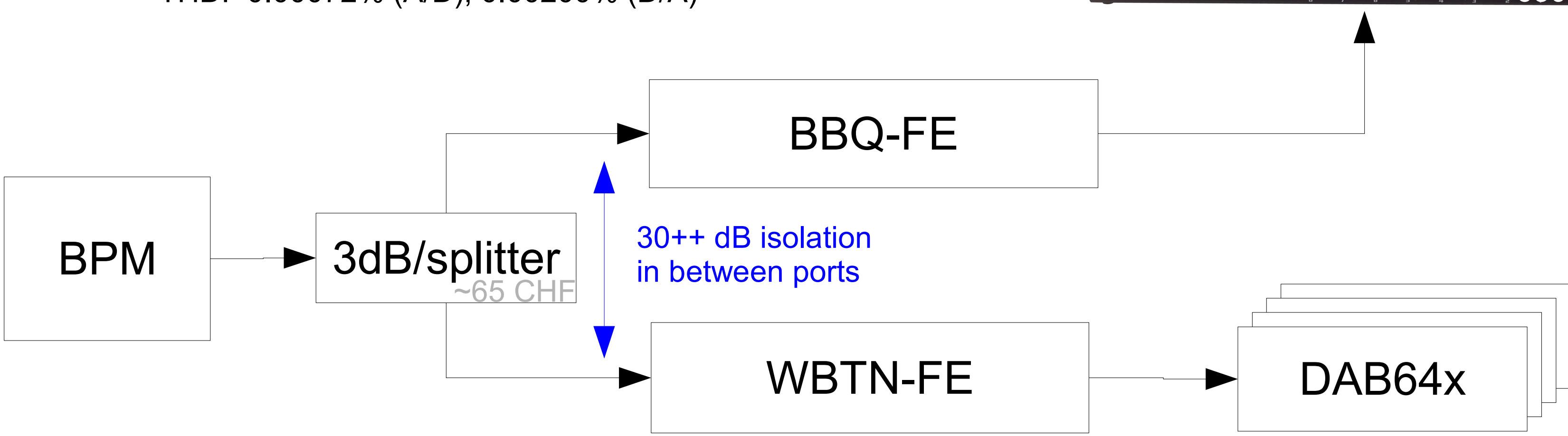
Test Setup

- Yet another exploitation of the BBQ Principle:
  - AC-coupled peak detector:
    - no saturation, self-triggered, no gain changes between pilot and nominal
    - intrinsically down samples spectra: ... 6 GHz → 1kHz ... f_{rev}
    - Base-band operation: very high sensitivity/resolution ADC available
    - Measured resolution estimate: < 10 nm

      → ε blow-up is a non-issue

- Digital acquisition: HP Proliant 16”, 1U + M-AUDIO Delta 1010
  - 8 analogue inputs/outputs, 16”, 1U
  - frequency response: 20Hz-22kHz, +/-0.3dB
  - >100 dB dynamic range/S/N ratio
  - THD: 0.00072% (A/D), 0.00200% (D/A)





Impact of Sharing the same Button Pick-up - Disclaimer

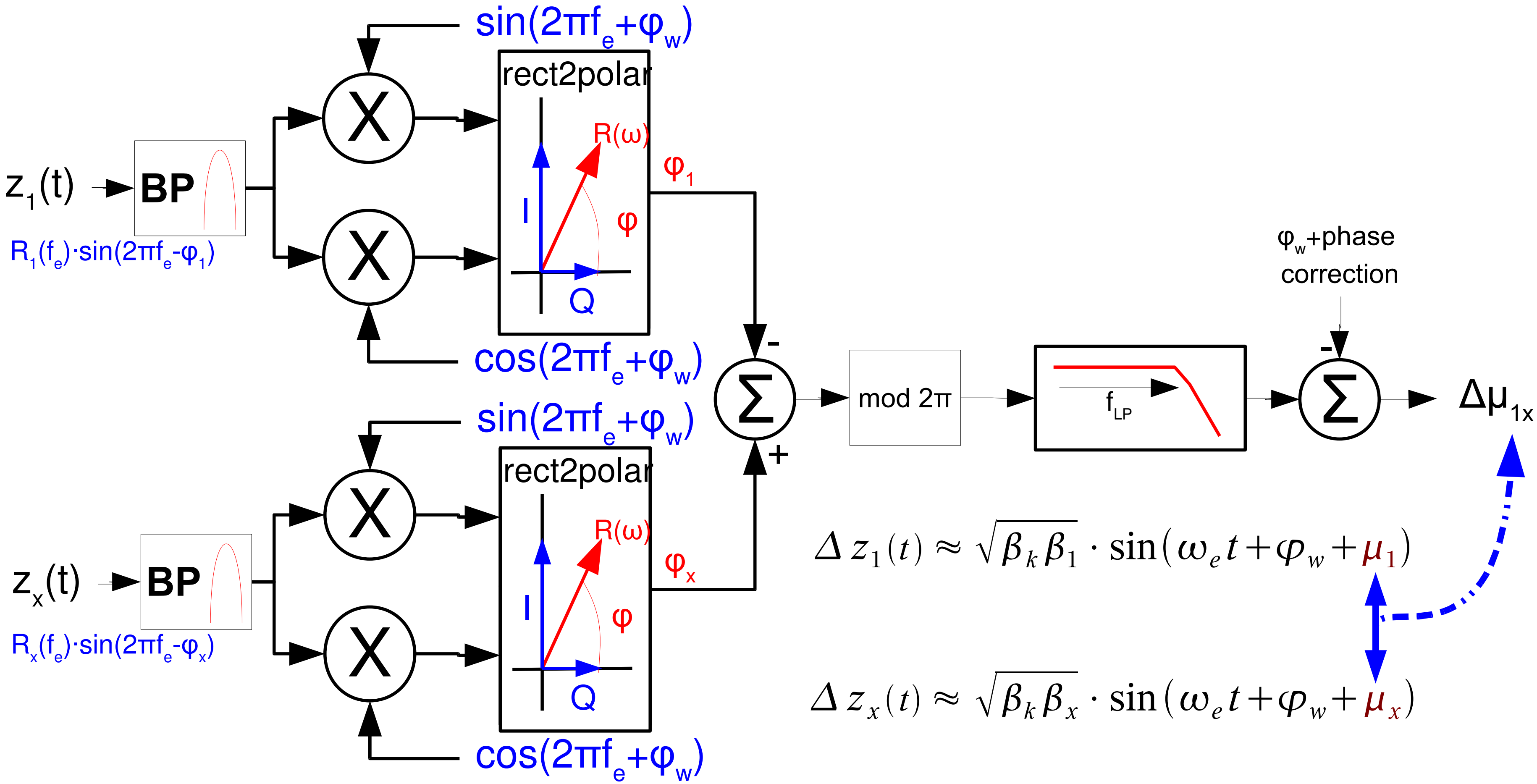
- Tests with beam in the SPS indicate that there is no obvious cross-talk in between the regular LHC WBTN (orbit) and the tested diode-based acquisition electronic used for the continuous beta-beat measurement.

- However: sharing intrinsically halves signals seen by the acquisition chain
  - Reduced minimum intensity detectable by button-type BPM ($2 \rightarrow 4\cdot10^9$)
    - Only relevant for IR7, may be less of an issue:
      - redundancies in IR7: multiple BPMs per cell & collimator
      - only affected for below pilot intensities (< $4\cdot10^9$ p/bunch)
    - N.B. Not an issue for strip-line pickups and/or nominal beam:
      - signals are attenuated simplify intensity gain-switching
  - Ongoing tests in the lab...

- If this proves to be really an issue, installation of additional pick-ups in the region of interest may be required.

Sample-by-Sample Phase Demodulation Scheme

- Modified mixing scheme:

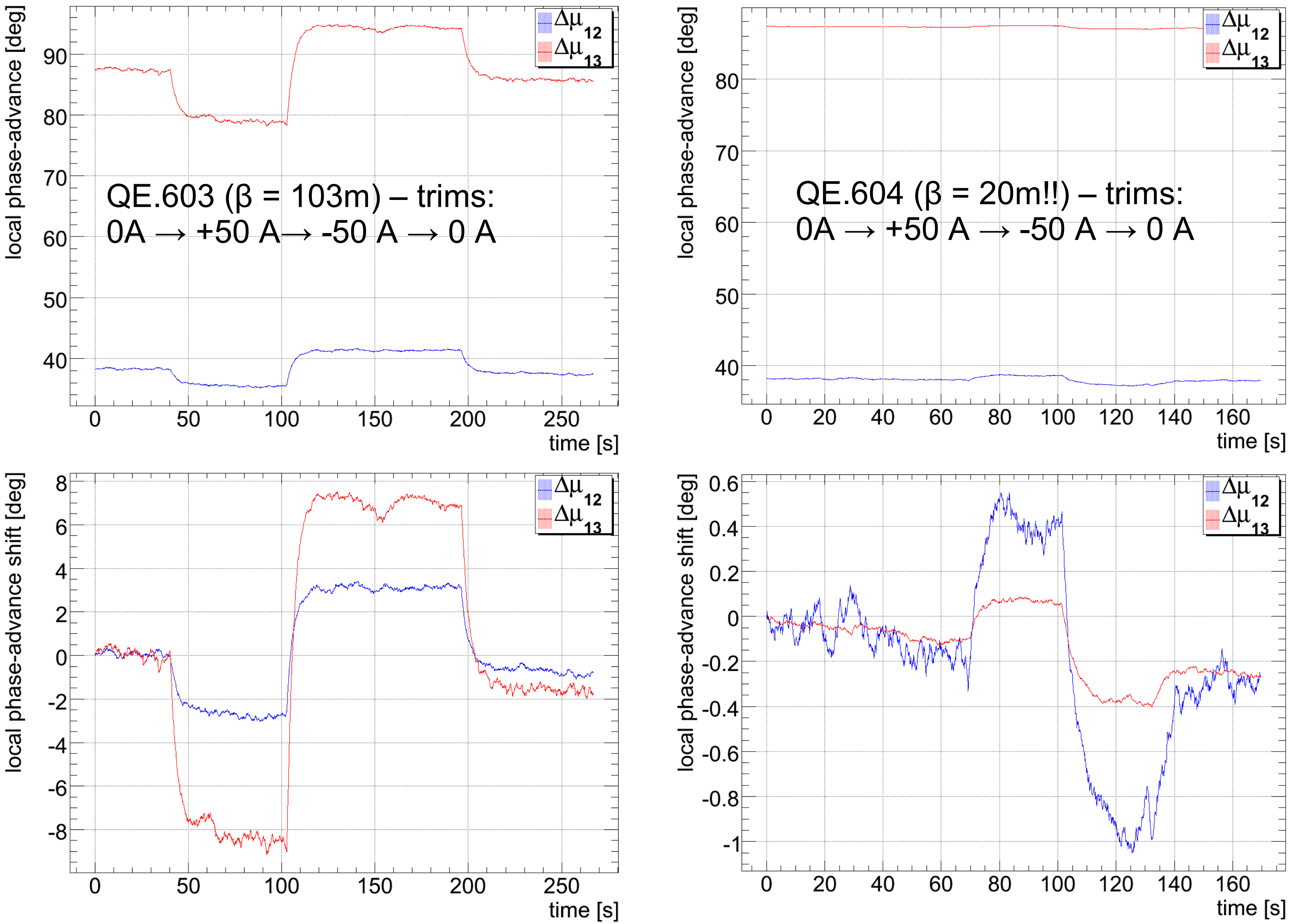


$$\Delta z_1(t) \approx \sqrt{\beta_k \beta_1} \cdot \sin(\omega_e t + \varphi_w + \mu_1)$$

$$\Delta z_x(t) \approx \sqrt{\beta_k \beta_x} \cdot \sin(\omega_e t + \varphi_w + \mu_x)$$

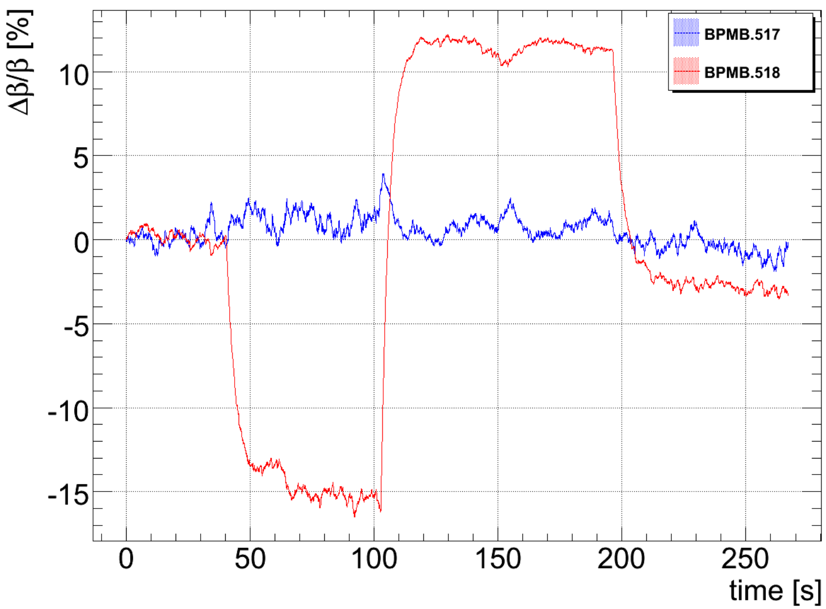
- Alternative to mixing method: IIR Hilbert transformer
  - trade-off: higher bandwidth ↔ lower phase precision

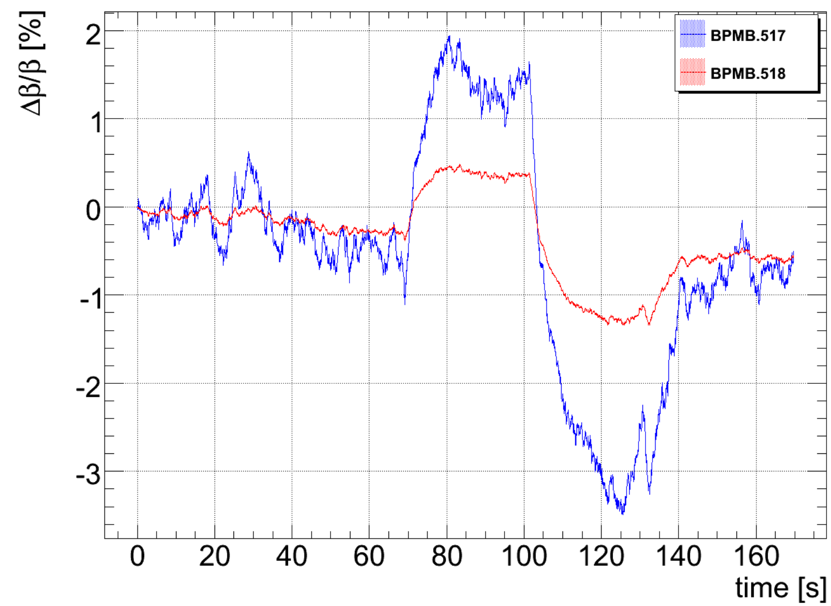
QE.603/QE.604 induced β-Phase-Advance Beating



QE.603/QE.604 induced β-Beating

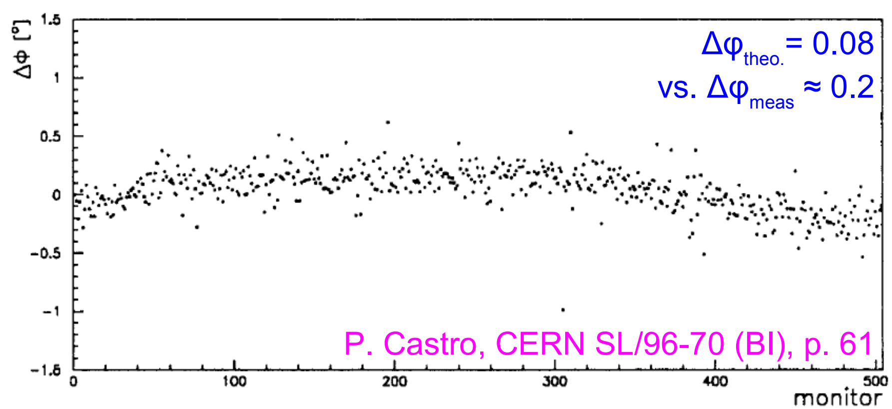
- Corresponding beta-beat: $$\frac{\Delta\beta_1}{\beta_1} = \frac{\cot(\Delta\mu_{12}^{meas.}) - \cot(\Delta\mu_{13}^{meas.})}{\cot(\Delta\mu_{12}^{theo.}) - \cot(\Delta\mu_{13}^{theo.})}$$





- Measured beta-beat is compatible with magnet calibration curves.
- Peak-to-peak β-beat "noise": ~ 0.5 %
  - unlikely due to diagnostic
  - seen already at LEP:
    (though not time resolved)

  → real drift of the optics!



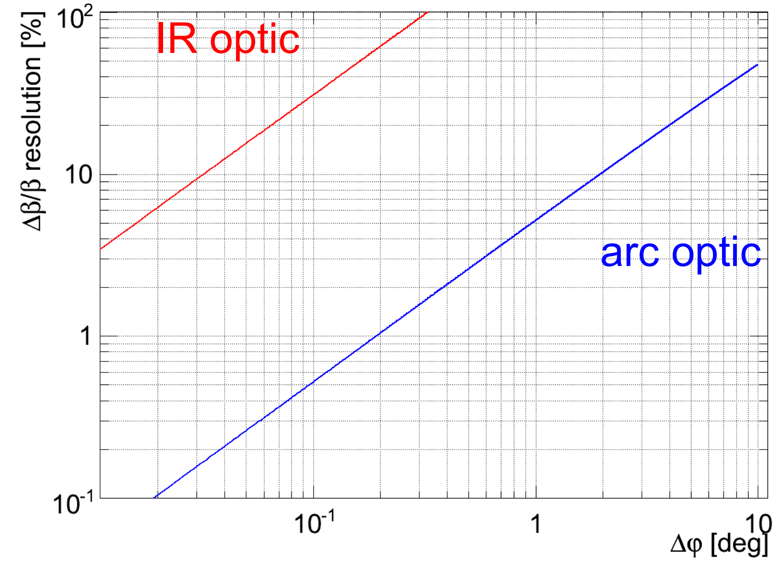
- Residual resolution/systematic error

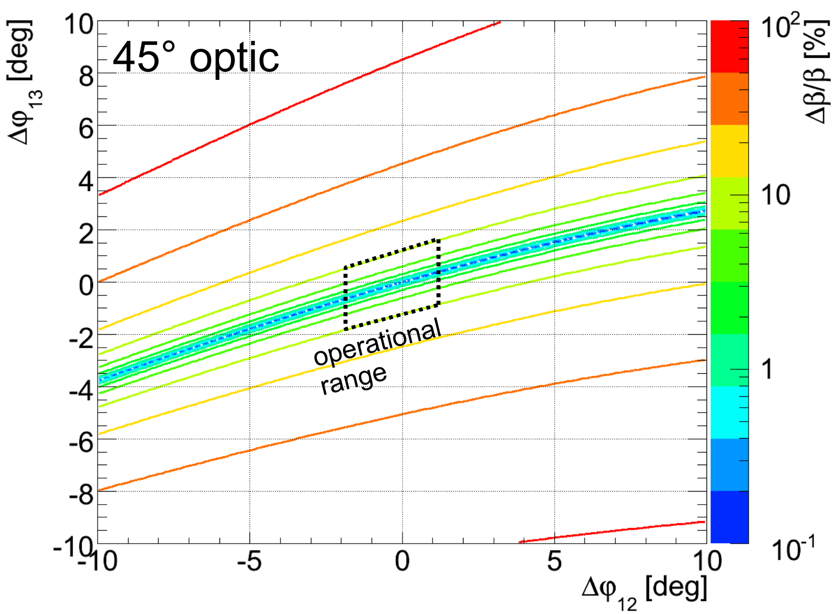
$$\frac{\Delta\beta_1}{\beta_1} = \frac{\cot(\Delta\mu_{12}^{meas.}) - \cot(\Delta\mu_{13}^{meas.})}{\cot(\Delta\mu_{12}^{theo.}) - \cot(\Delta\mu_{13}^{theo.})}$$

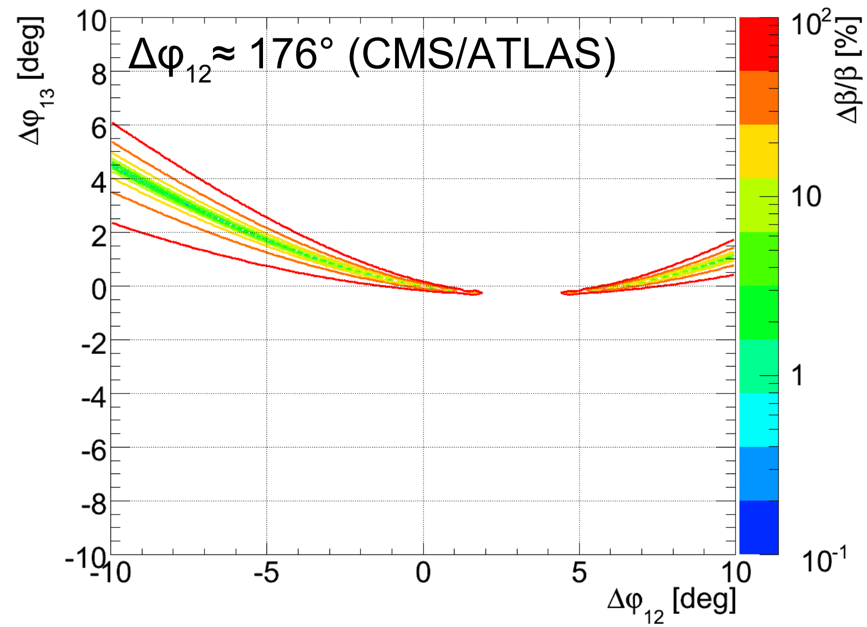
$$\Delta\mu_{1i}^{meas.} := \Delta\mu_{1i}^{theo.} + \Delta\varphi_{1i}$$

  – ARC optics: requires error below ~1°
  – IP optics: requires error below ~0.02°

N.B. Plots have logarithmic z-scale!



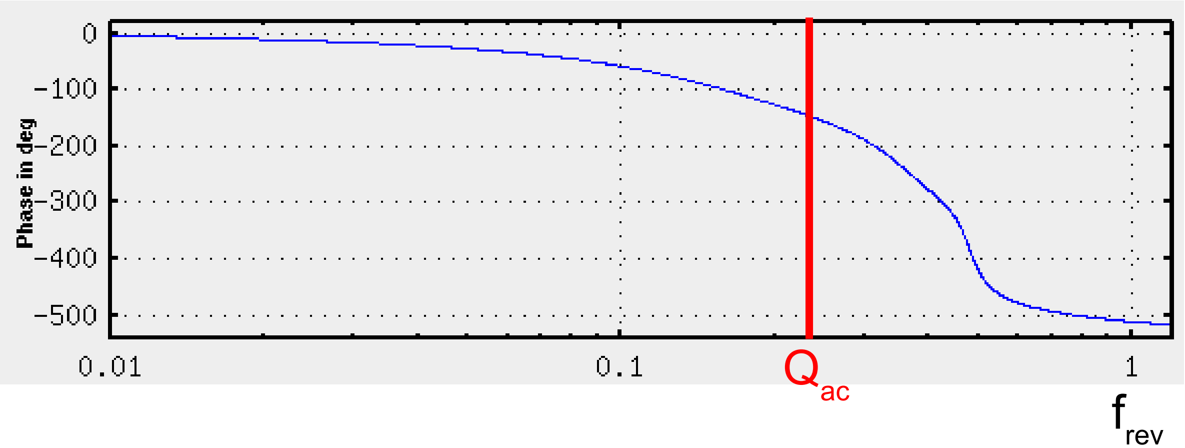




Beta-Beat Measurement Error Sources I/II
Systematic Phase Errors

- Sources – usually depend on observation/excitation frequency
  - Systematic delays: $\Delta\varphi[deg] = 360° \cdot \Delta\tau f$
    - Pick-up to acquisition system cable length (e.g. 100 m@ Q_{AC}=0.25 f_{rev})
      - SPS: Δφ ≈ 2° LHC Δφ ≈ 0.5°: $\Delta\beta/\beta_{sys.}$ ≈ 3-10% (45° lattice)

        → cable delay compensation mandatory for direct β*-Measurements
  - Low-frequency pre-processing and analogue front-end asymmetry (mostly filters, N.B. Current has been not optimised for those issues)
    - Delta 1010 – analogue pre-filter: Δφ ≈ 7° (measured)
    - BBQ front-end: Δφ ≈ 10° (measured, here: only Chebychev stage shown)

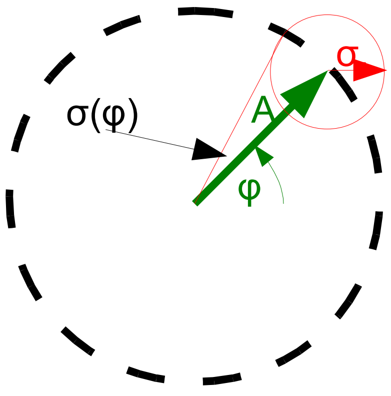


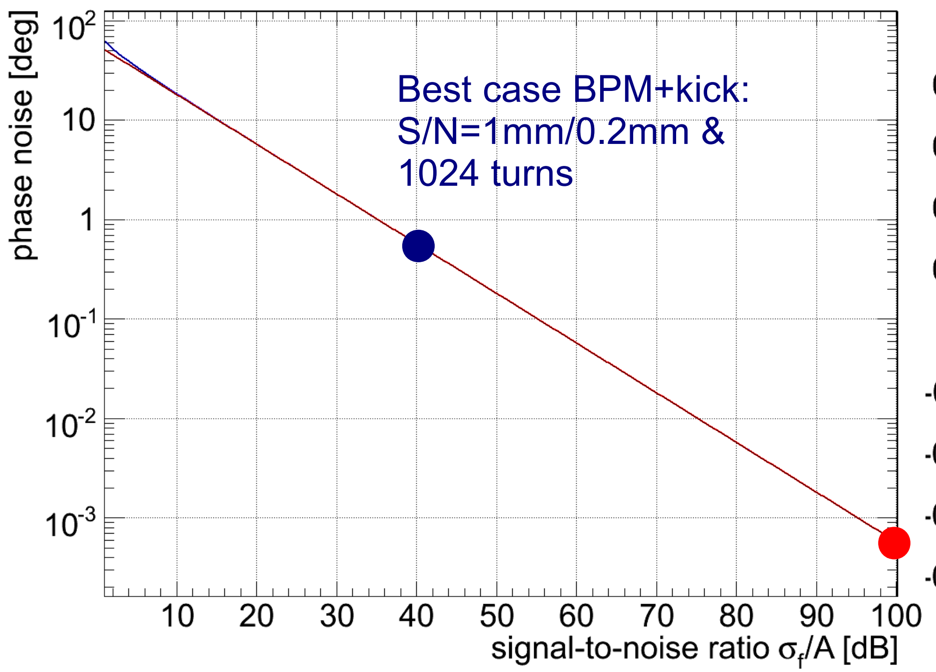
    - Systematic drift: < 0.1° → will be further reduced

- Statistical noise adds vectorial to the carrier signal:
  - excitation amplitude (carrier signal): A
  - noise in time (frequency) domain: σ_t (σ_f)
  - Equivalent number of turns: N

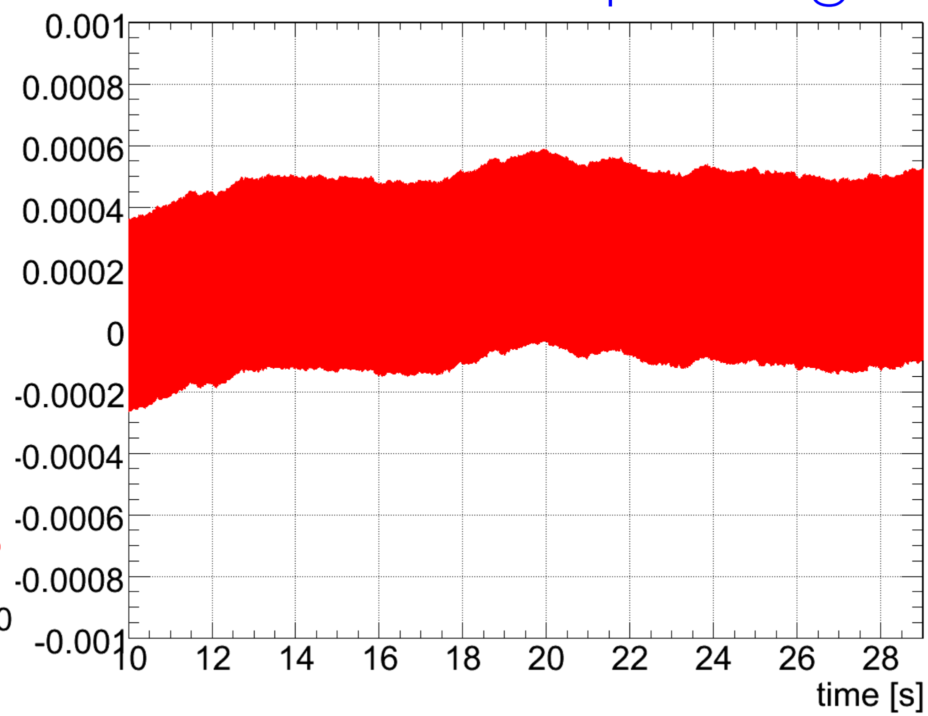
$$\sigma(\varphi) = \arcsin\left(\frac{\sigma_f}{A}\right) = \arcsin\left(\sqrt{\frac{2}{N}}\frac{\sigma_t}{A}\right)$$

for small noise to signal ratios $\approx \sqrt{\frac{2}{N}}\frac{\sigma_t}{A}$



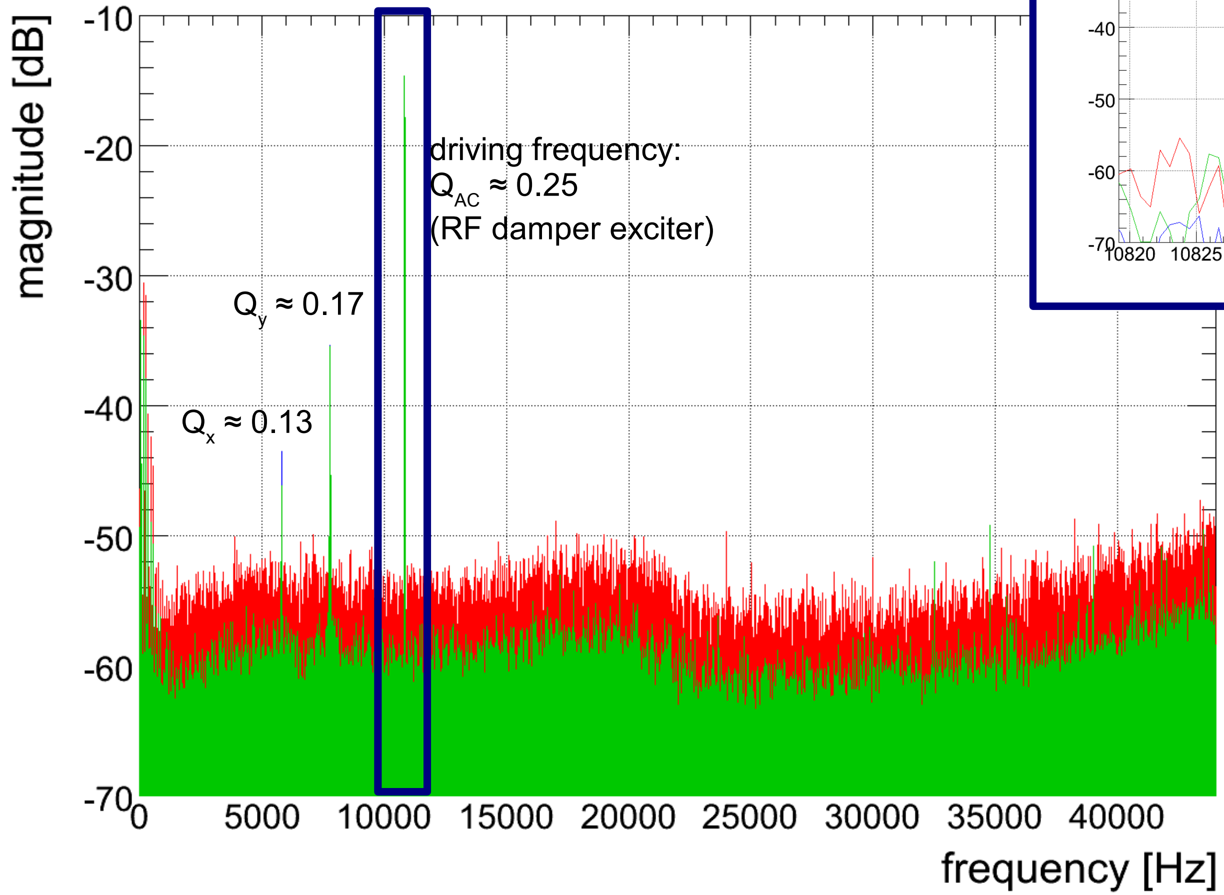


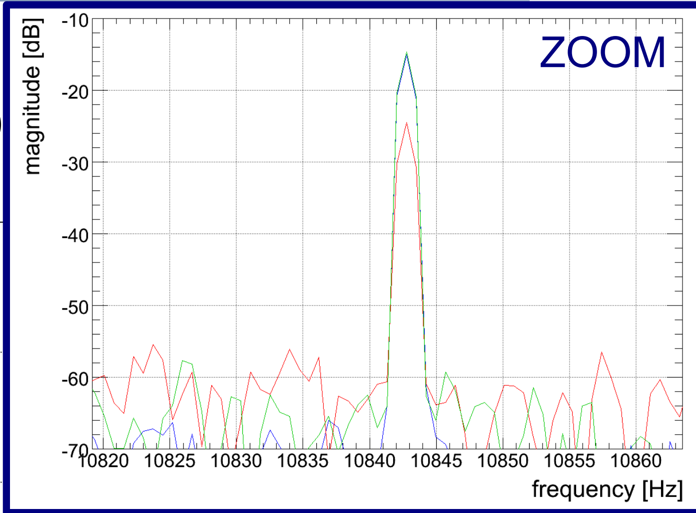
Delta 1010 intrinsic phase noise@1Hz



Typical SPS Beam Spectrum single bunch, ~ 7 ·10^{10} protons@270GeV (coasting)

- Based on 128k turns (~ 1.3 seconds)
  - noise floor (time domain): ~100 nm (BBQ1: ~5 nm)
  - driven 'AC-dipole' signal: ~20-30 um

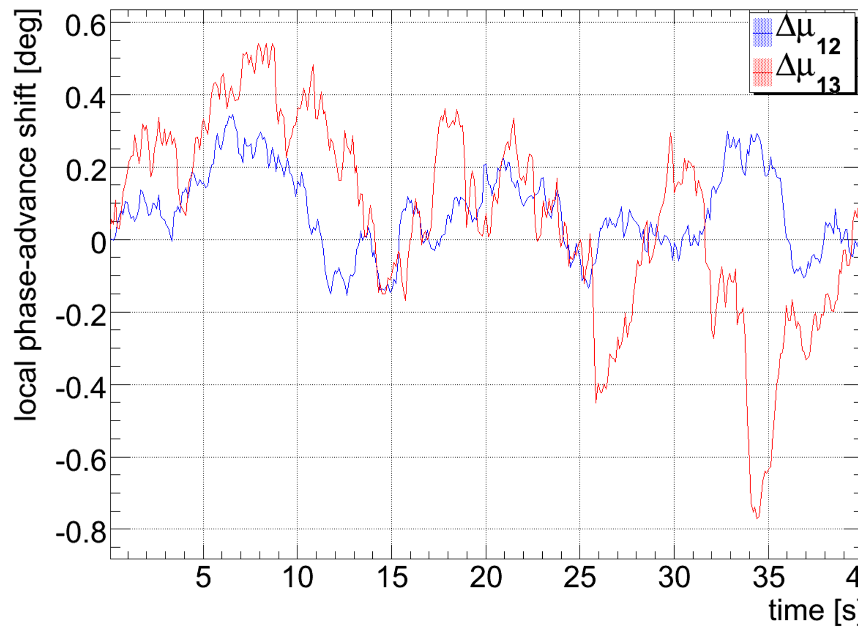


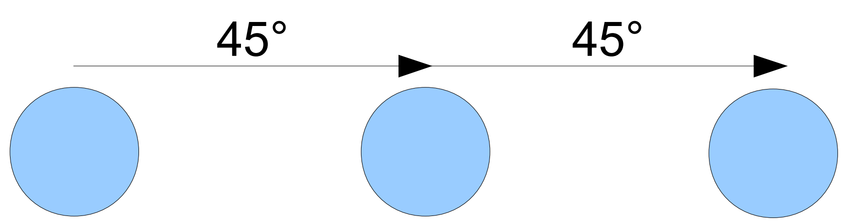


  - LHC BPMs give ~30 dB less signal than BBQ1 installation (buttons vs. 30 cm strip-line)
  - Residual tune signals ~ 0.5/2 um (calibrated w.r.t. Signal seen on SPS BPMs)
  - off-resonance excitation → no emittance blow-up

Residual Beta-Beat Drifts - Revisited

- Residual phase motion (blue: BPM1->2, red: 0.5*BPM1->3)
  - Acquisition/electronic induced noise would be “equal”/randomly distributed over all channels
  - GM induced sextupole shifts
    - $\Delta x \approx 100$ um r.m.s. $\rightarrow \Delta\beta/\beta \approx 0.1\text{-}0.2\%$
    - bit too large to be the only perturbation source...





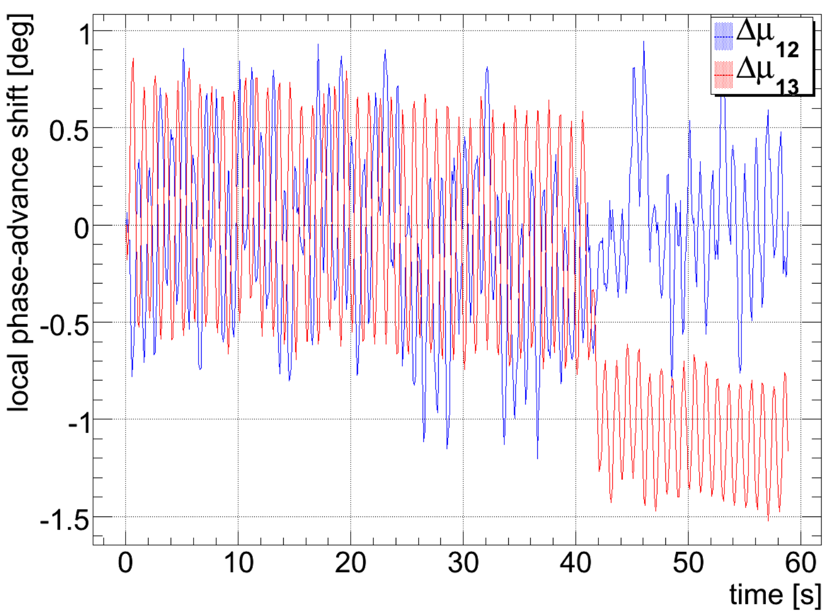
Case I: $\Delta\varphi$ → 0 ← $-\Delta\varphi$ $\Delta\mu_{12}$ =45° - $\Delta\varphi$, $\Delta\mu_{12}$ =90° - 2 $\Delta\varphi$

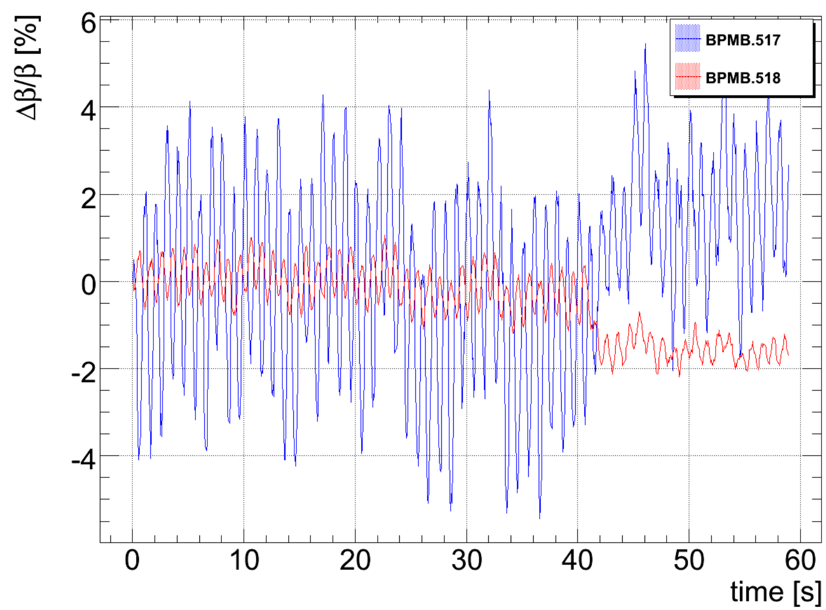
Case II: 0 $\Delta\varphi$ → 0 $\Delta\mu_{12}$ =45° + $\Delta\varphi$, $\Delta\mu_{12}$ =90°

Further Exploitation Possibilities

- System can be further exploited for fast and transparent measurements of physics affecting $\Delta\beta/\beta$ that earlier required significant amount of beam time

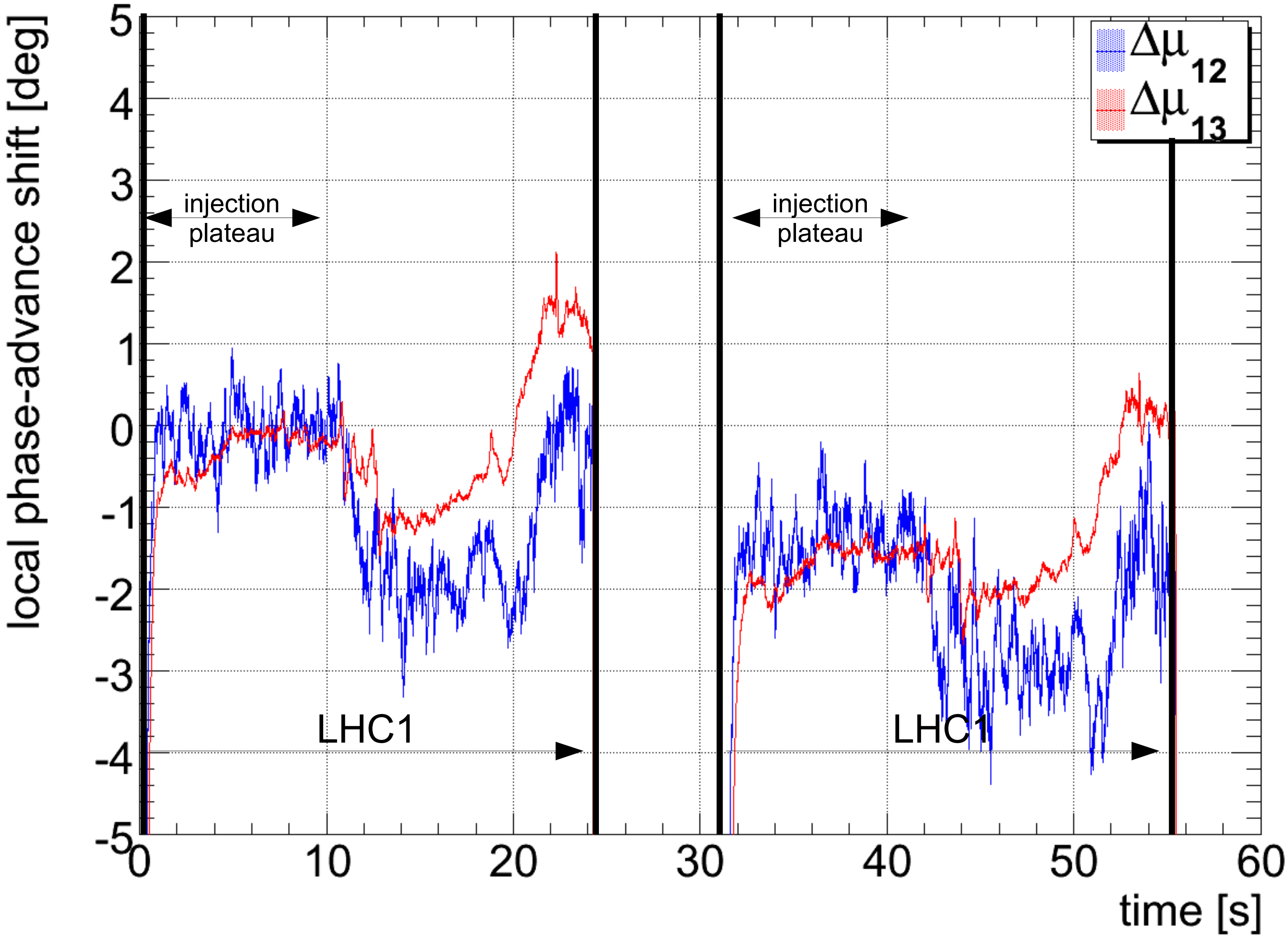
- Example: vertical off-momentum β-Beat:
  - Continuous radial modulation: $\Delta p/p \approx 1\cdot10^{-3}$ @ 1 Hz
  - One full measurement data set every second!
  - N.B. Step in phase → off-centre horizontal orbit in lattice sextupoles





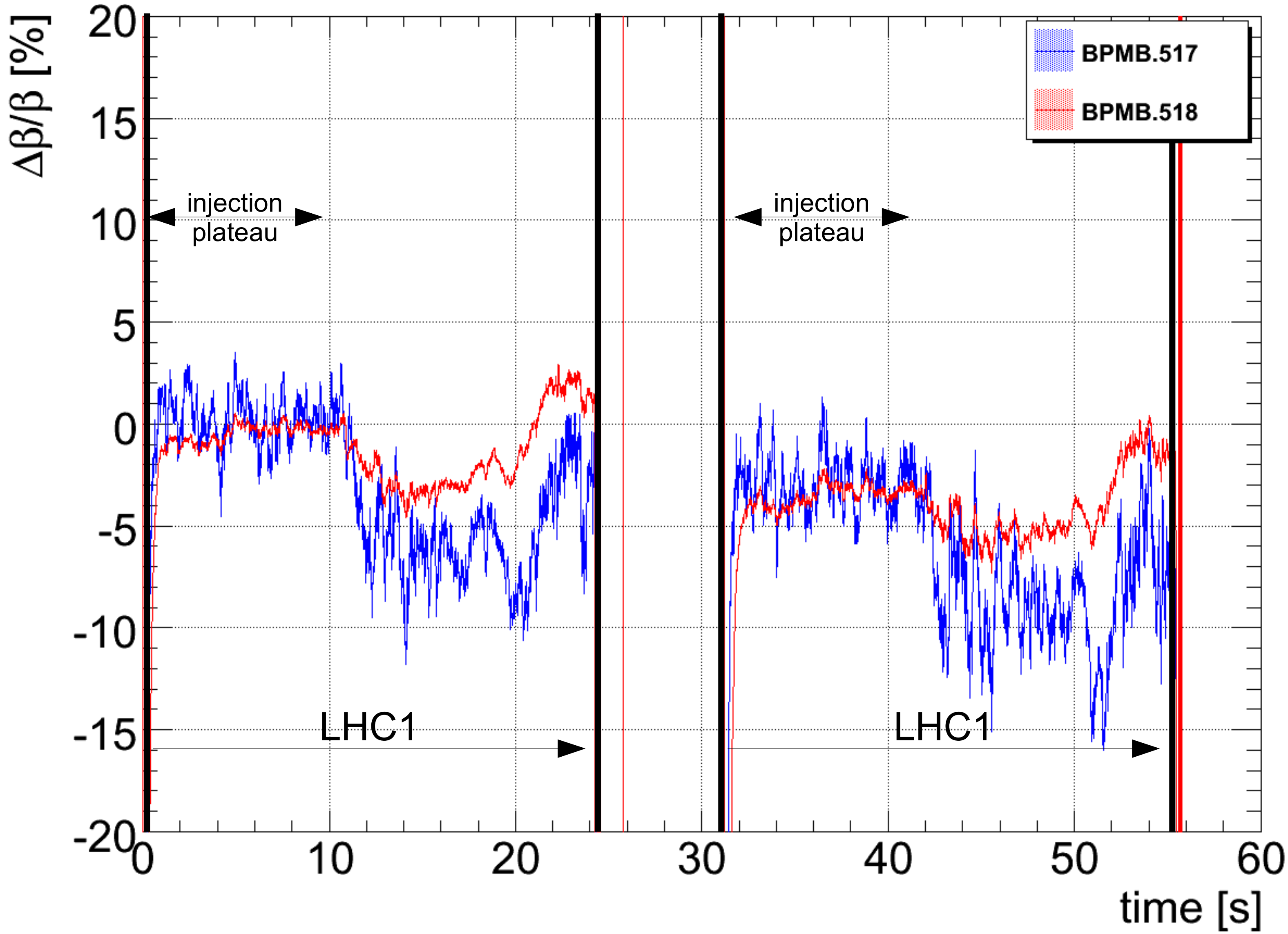
Example: SPS LHC1 Cycle-to-Cycle Stability II/II Phase-Beating

- In between two coasts...



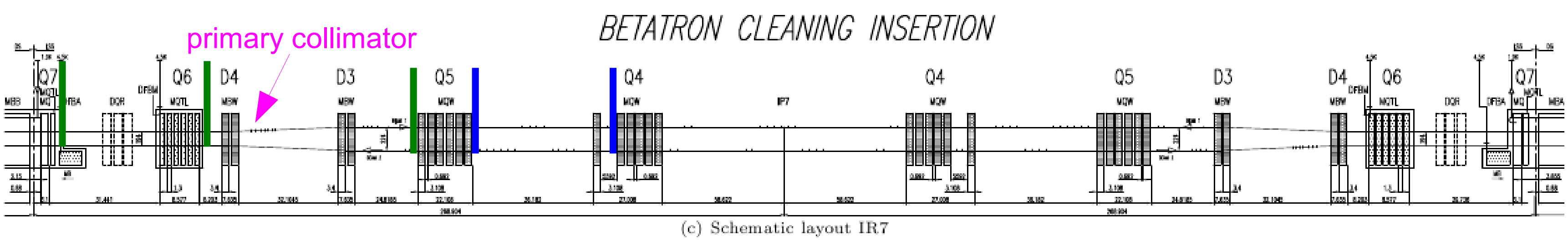
Example: SPS LHC1 Cycle-to-Cycle Stability II/II - Beta-Beat

- In between two coasts...



Next Steps I/II: Improvement to LHC Test Installations:

- Beside the intrinsic loss of signal due to the 3dB-signal splitter, initial tests show that sharing and cross-talk effects in been the regular WBTN and Beta-Beat system appears to be minimal.
  - tested with 75 ns/50 ns/25 ns/single nominal & pilot bunch configurations
  - 3dB-Splitter are being further tested in the lab using a bunch simulator setup
  - Affects mainly performance with ultra-low intensity bunches ($<2\cdot10^9$ p/bunch)
- Further Setup Improvement:
  - systematic drifts of analogue front-end stages 0.1°/0.5 hour (β* meas)
  - Scalability and possible system integration (in view of LHC application)
  - Install 3 (+2) β-beat acquisition chains, both planes, either B1 or B2, in parallel to the regular BPM system, e.g. in LHC beta-cleaning insertion:



(c) Schematic layout IR7

    - the minimum installation without pulling additional cables

Next Steps II/II: Beta-beat Control

The LHC Prototype system's usefulness is two-fold:

- Provided β does not change: study real beta-beat as a function of time and use measured values to possibly relax collimation requirements
- Provided β does change: same as above but – in addition – use real beta-beat values as an input to a real-time feedback loop (e.g. primary/secondary collimators, IPs)
  - Correction scheme similar to LHC Orbit FB system using the dispersion suppressor's and other individually powered quadrupoles (N.B. we are not as "free" in correcting μ as for correcting the orbit)

$$\frac{\vec{\Delta}\mu}{\mu} = \underline{R}_{\mu} \cdot \vec{\delta}_{DS} \quad \overset{SVD}{\longrightarrow} \quad \vec{\delta}_{DS} = \underline{\tilde{R}}_{\mu}^{-1} \cdot \frac{\vec{\Delta}\mu}{\mu}$$

  - Additional regions of interest: experimental insertions, Inj./Extr. IRs, ...
- The possible merit...
  → remove/reduce protection/cleaning limitations on β* & stored intensity
  → +20% effective Luminosity

"Isn't this worth being further investigated?"

Conclusions

- A real-time β-beat measurement system has been successfully tested at the SPS based on the continuous measurement of the cell-to-cell phase advance.
  - Achieved resolution: $\Delta\beta/\beta < 1\%$ @ 1 Hz measurement bandwidth
  - Required S/N ratio: ~ 20 μm/100 nm → ε blow-up is a non-issue
    - compatible with nominal LHC operation
  - Shared pickup scheme (to 1st order) compatible with regular WBTN oper.
    - price of sharing: loose ~ 3dB signal, OK for strip-line, buttons → may affect smallest detectable bunch intensity for some IR7 BPMs

- Measured residual 1% drift of SPS lattice and off-momentum beta-beat

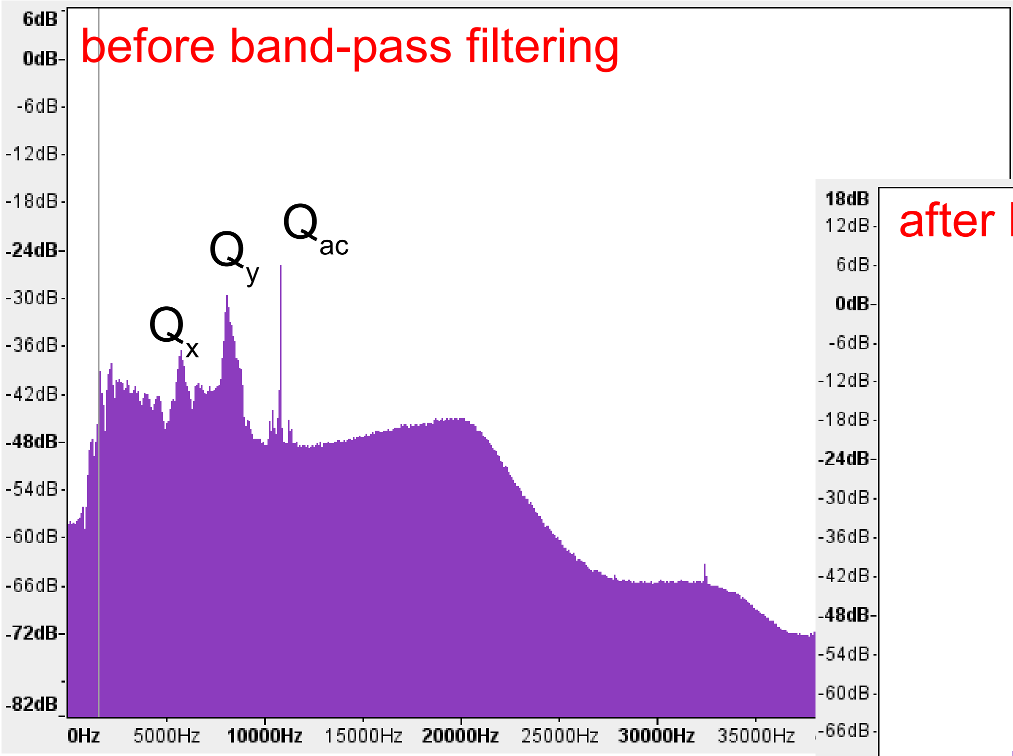
- Present limitations of the system:
  - For the time being: only local measurements
  - 45° optics (LHC arcs): < 0.01° ↔ $\Delta\beta/\beta \ll 1\%$ @ 1Hz bandwidth
    - residual beta-function stability and S/N ratio
  - Exp. Insertion optics: ~ 0.1° ↔ $\Delta\beta/\beta \approx 30\%$ ($\Delta\mu_{12} \approx 178°$)
    - systematic phase and drifts, can be improved, target: $\Delta\beta/\beta_{IP} \approx 3\%$

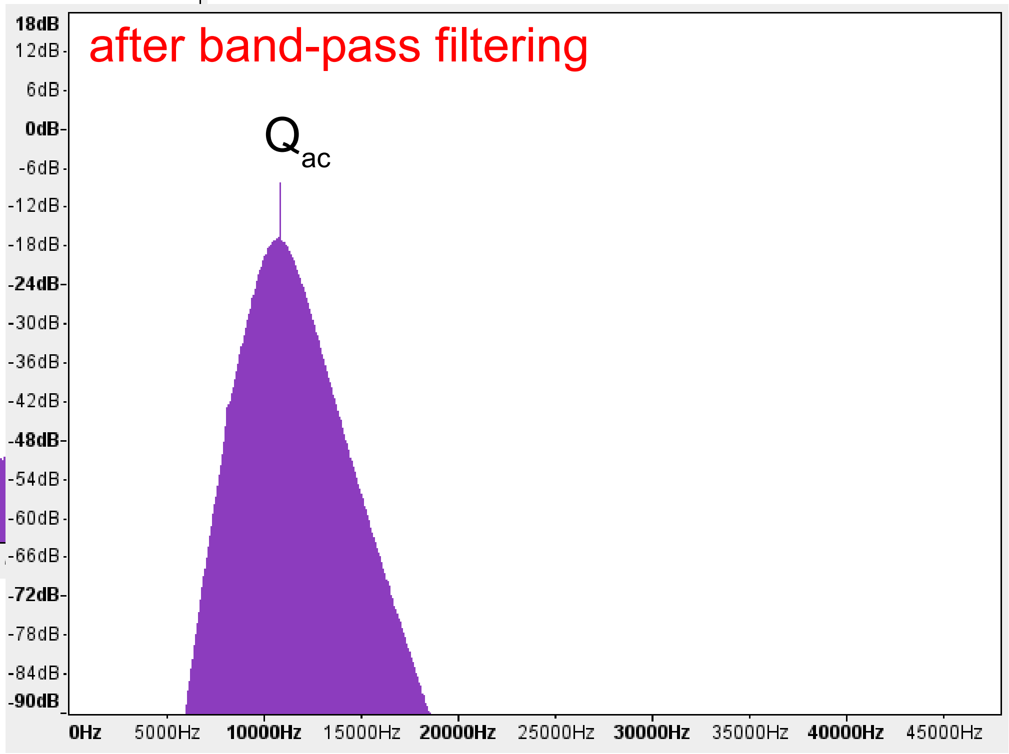
- Provided LHCPerfC approval: We would like to install a similar setup in LHC-IR7 for 2009 to investigate residual beta-function drifts and possible exploitation of this measurement techniques within a real-time β-beat FB loop.

additional supporting slides

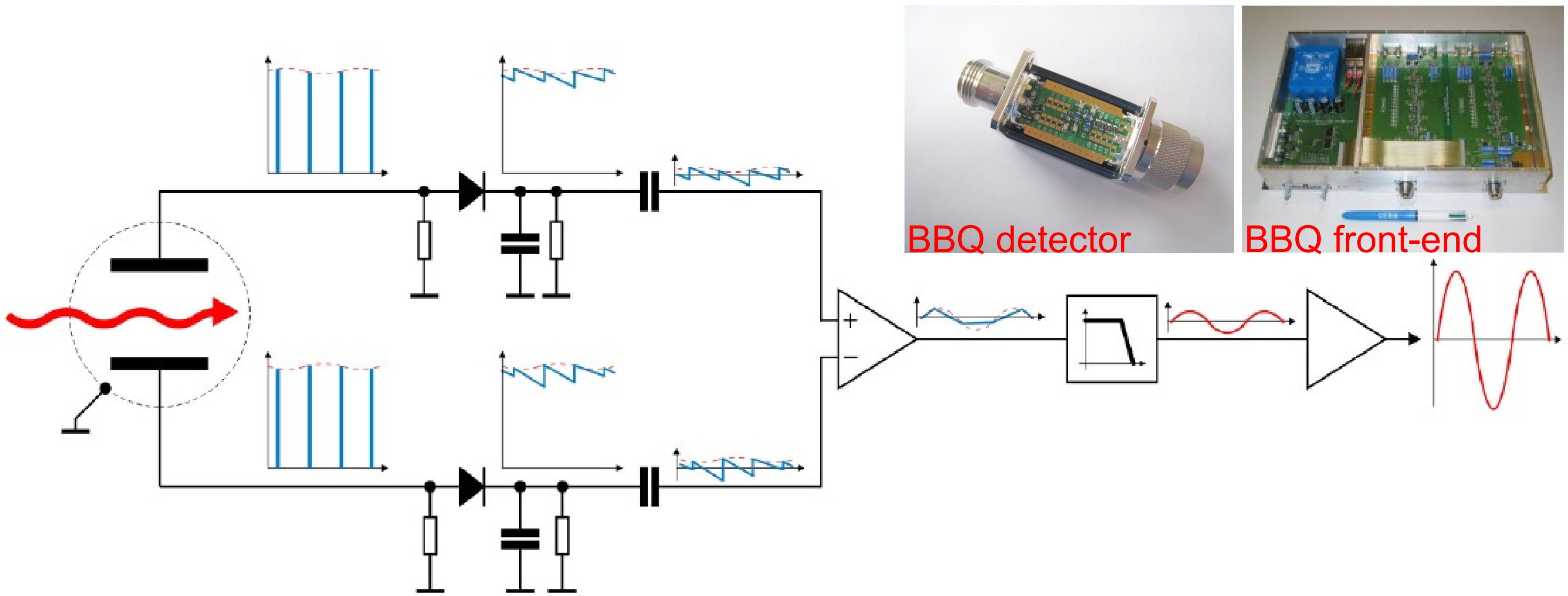
Example: SPS LHC1 cycle-to-cycle stability necessary correction

- Fourier Spectrum before and after band-pass filter (carrier at 10.8 kHz)





LHC Q Base-Line Q Instrumentation Back-bone: Base-Band-Q Principle on a Slide



- Basic principle: AC-coupled peak detector
  - no saturation, self-triggered, no gain changes between pilot and nominal
  - intrinsically down samples spectra: ... 6 GHz → 1kHz ... f_{rev}
    - Base-band operation: very high sensitivity/resolution ADC available
    - Measured resolution estimate: < 10 nm → ε blow-up is a non-issue
- Since 2007: added low-pass filter in front of diodes to suppress intrinsic longitudinal (carrier) signal propagation into transverse oscillations

- Beta-Beat Sources
  - Quadrupole gradient and Momentum errors
  - Feed-down due to off-centre horizontal orbit in lattice sextupoles
  -
- Requirements: Brüning, Fartoukh, *LHC Project Report 501*

Peak hor. β-beating [%]	Mechanical aperture	14 / 15 (inj. / col.)
Peak vert. β-beating [%]	Mechanical aperture	16 / 19 (inj. / col.)
R.M.S. hor. β-beating [%]	Mechanical aperture	4.8 / 5.2 (inj. / col.)
R.M.S. vert.. β-beating [%]	Mechanical aperture	5.5 / 6.6 (inj. / col.)

Space Domain: - No "black feedback magic"

- Effects on orbit, Energy, Tune, Q' and C^- can essentially cast into matrices:

$$\Delta\vec{x}(t)=\underline{R}\cdot\vec{\delta}(t) \quad with \quad R_{ij}=\frac{\sqrt{\beta_i\beta_j}}{2\sin(\pi Q)}\cdot\cos(\Delta\mu_{ij}-\pi Q)+\frac{D_i D_j}{C(\alpha_c-1/\gamma^2)}$$

matrix multiplication

  - LHC matrices' dimensions:

$$\underline{R}_{orbit}\in\mathbb{R}^{1070\times 530} \quad \underline{R}_{Q}\in\mathbb{R}^{2\times 16} \quad \underline{R}_{Q'}\in\mathbb{R}^{2\times 32} \quad \underline{R}_{C^-}\in\mathbb{R}^{2\times 10/12}$$

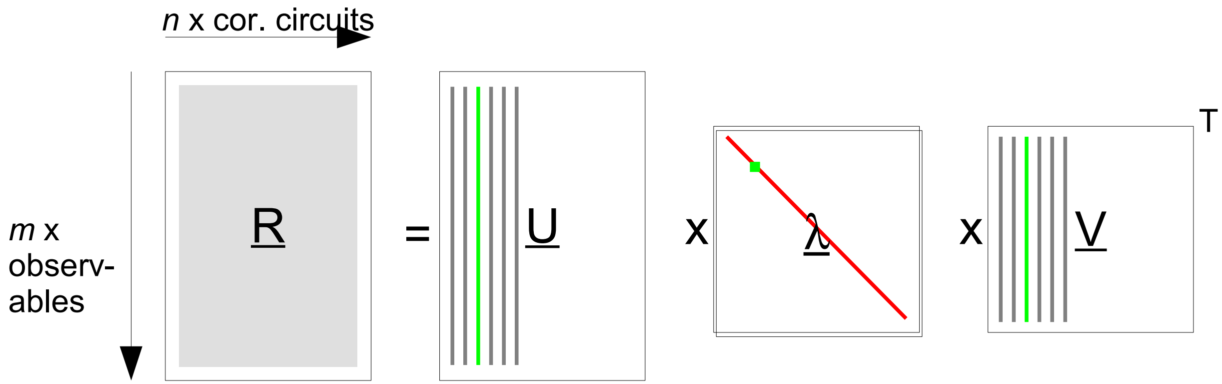
  - control consists essentially in inverting these matrices:

$$\left\|\vec{x}_{ref}-\vec{x}_{actual}\right\|_2=\left\|\underline{R}\cdot\vec{\delta}_{ss}\right\|_2<\epsilon \;\rightarrow\; \vec{\delta}_{ss}=\tilde{R}^{-1}\Delta\vec{x}$$

- Some potential complications:
  - Singularities = over/under-constraint matrices, noise, element failures, spurious BPM offsets, calibrations, ...
  - Time dependence of total control loop → "The world goes SVD...."

Space-Domain: Singular Value Decomposition (SVD) on a slide

Linear algebra theorem*:



eigen-vector relation:

$$\lambda_i \vec{u}_i = \underline{R} \cdot \vec{v}_i$$

$$\lambda_i \vec{v}_i = \underline{R}^T \cdot \vec{u}_i$$

- though decomposition is numerically more complex final correction is a simple vector-matrix multiplication:

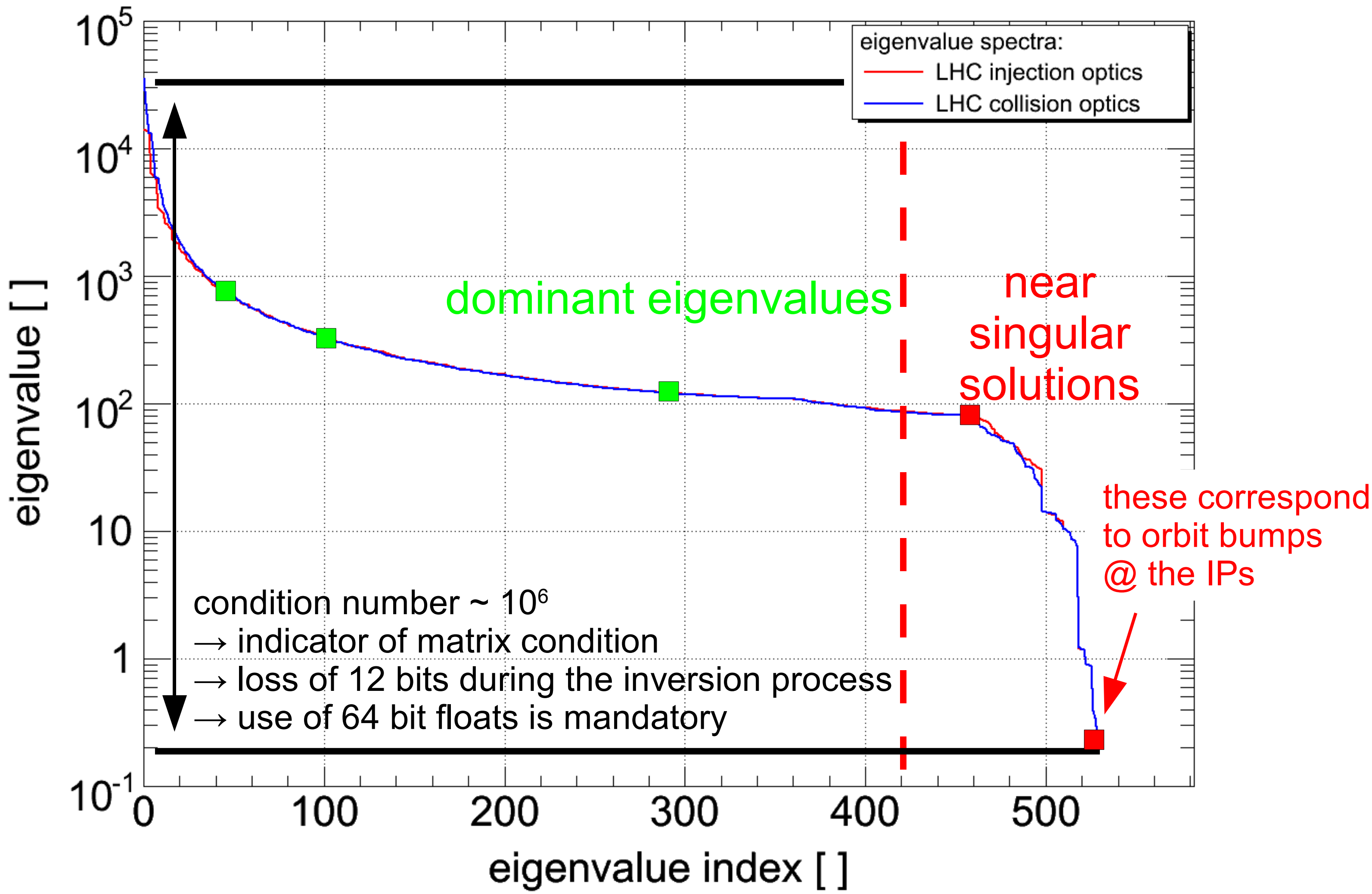
$$\vec{\delta}_{ss} = \tilde{R}^{-1} \cdot \Delta \vec{x} \quad with \quad \tilde{R}^{-1} = \underline{V} \cdot \underline{\lambda}^{-1} \cdot \underline{U}^T \quad \Leftrightarrow \quad \vec{\delta}_{ss} = \sum_{i=0}^{n} \frac{a_i}{\lambda_i} \vec{v}_i \quad with \quad a_i = \vec{u}_i^T \Delta \vec{x}$$

- numerical robust, minimises parameter deviations Δx and circuit strengths δ
- Easy removal of singularities, (nearly) singular eigen-solutions have $\lambda_i \sim 0$
  - to remove those solution: if $\lambda_i \approx 0 \rightarrow$ '$1/\lambda_i$:= 0'
  - discarded eigenvalues corresponds to solution pattern unaffected by the FB

*G. Golub and C. Reinsch, “*Handbook for automatic computation II, Linear Algebra*”, Springer, NY, 1971

Space-Domain:
SVD example: LHC eigenvalue spectrum

Eigenvalue spectra for vertical LHC response matrix using all BPMs and CODs:



- Optics imperfections may deteriorate the convergence speed but do not affect absolute convergence (response functions are 'monotonic'):
- Example: 2-dim orbit error surface projection