The grazing function *g*

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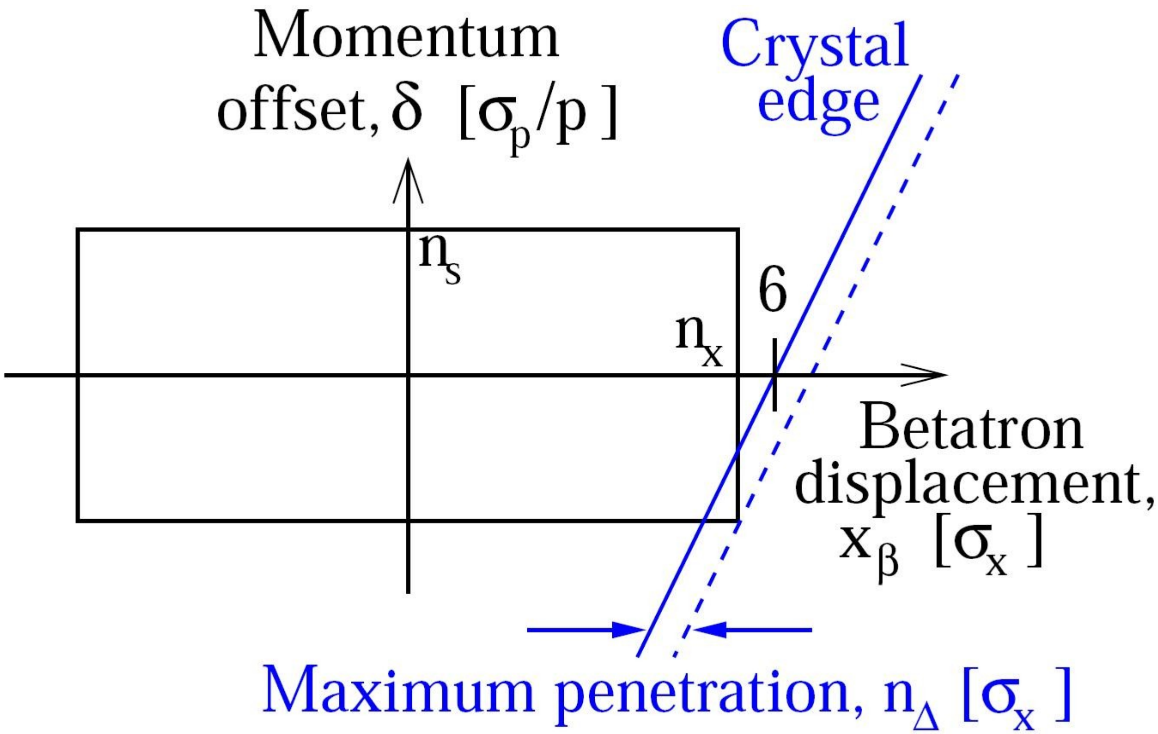
Acknowledgements: R. Assmann, C. Bracco, A. Drees, R. Fliller, J-B. Jeanneret, E. Laface, N. Mokhov, G. Robert-Demolaize, W. Scandale, S. Shiraishi, D. Still, A. Valishev.



A "grazing particle" just touches the crystal during simultaneous betatron & synchrotron oscillation extrema.

$$a_x + |\eta|\, a_s = |x_c|$$

The ONLY page of equations

Thus the grazing angle depends linearly on the synchrotron amplitude a_s according to

$$x'_G = -\frac{\alpha}{\beta} x_c + \mathrm{sgn}(x_c)\,\mathrm{sgn}(\eta)\; g\, a_s \tag{15}$$

where the linear slope of grazing angle with respect to synchrotron amplitude is

$$\boxed{\frac{dx'_G}{da_s} = \mathrm{sgn}(x_c)\,\mathrm{sgn}(\eta)\; g} \tag{16}$$

The *grazing function* g that enters these equations is an optical quantity defined as

$$g \equiv \left(\frac{\alpha}{\beta}\eta + \eta' \right) \tag{17}$$

The grazing function is thus revealed to be just

$$\boxed{g = \sqrt{\beta}\, \eta'_N}$$

and the rigorous general synchrobetatron condition $g = 0$ is just

$$\boxed{\eta'_N = 0}$$

More realistic condition on g

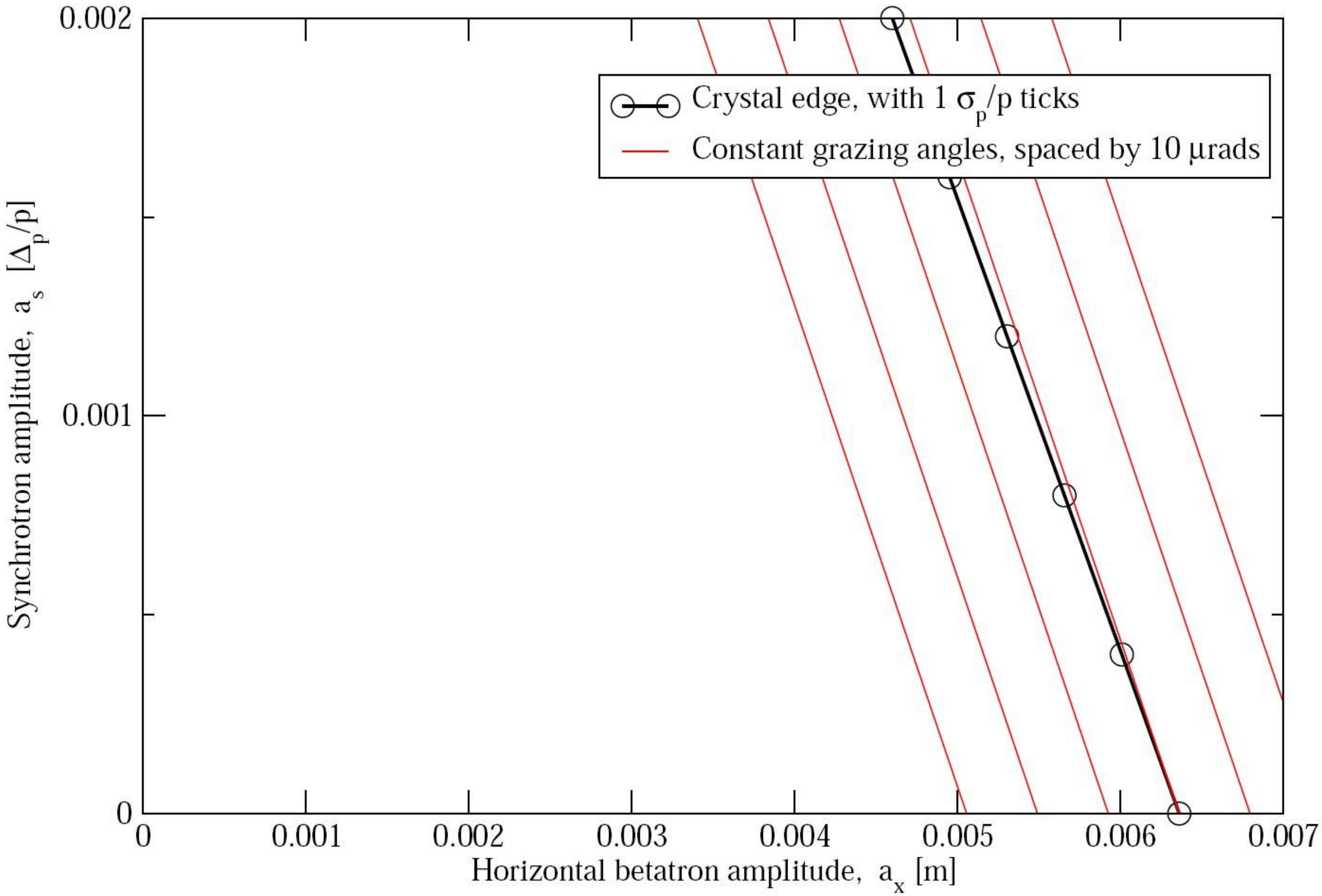
Efficient collimation: the grazing angle must be within the crystal acceptance angle for "all" synchrotron amplitudes.

$$|g| < \frac{\sigma'_A}{n_{max}\,(\sigma_p/p)}$$

σ'_A [μrad]	$\sim$	2	channeling at 7 TeV
	$\sim$	10	channeling at 0.1 TeV
	$\sim$	100	volume reflection at any energy

	α	β [m]	η [m]	η' [10^{-3}]	g [10^{-3}]	E [TeV]	σ_p/p [10^{-3}]	σ'_G [μ rad]
RHIC	-26.5	1155.0	-0.864	-16.2	3.6	0.10	0.50	1.81
SPS	-2.21	96.1	-0.880	-19.0	1.2	0.12	0.40	0.48
Tevatron	-0.425	67.5	1.925	15.0	2.9	0.98	0.14	0.41
LHC	1.94	137.6	0.559	-8.9	-1.0	0.45	0.31	0.31
						7.0	0.11	0.11

SPS UA9 expectations



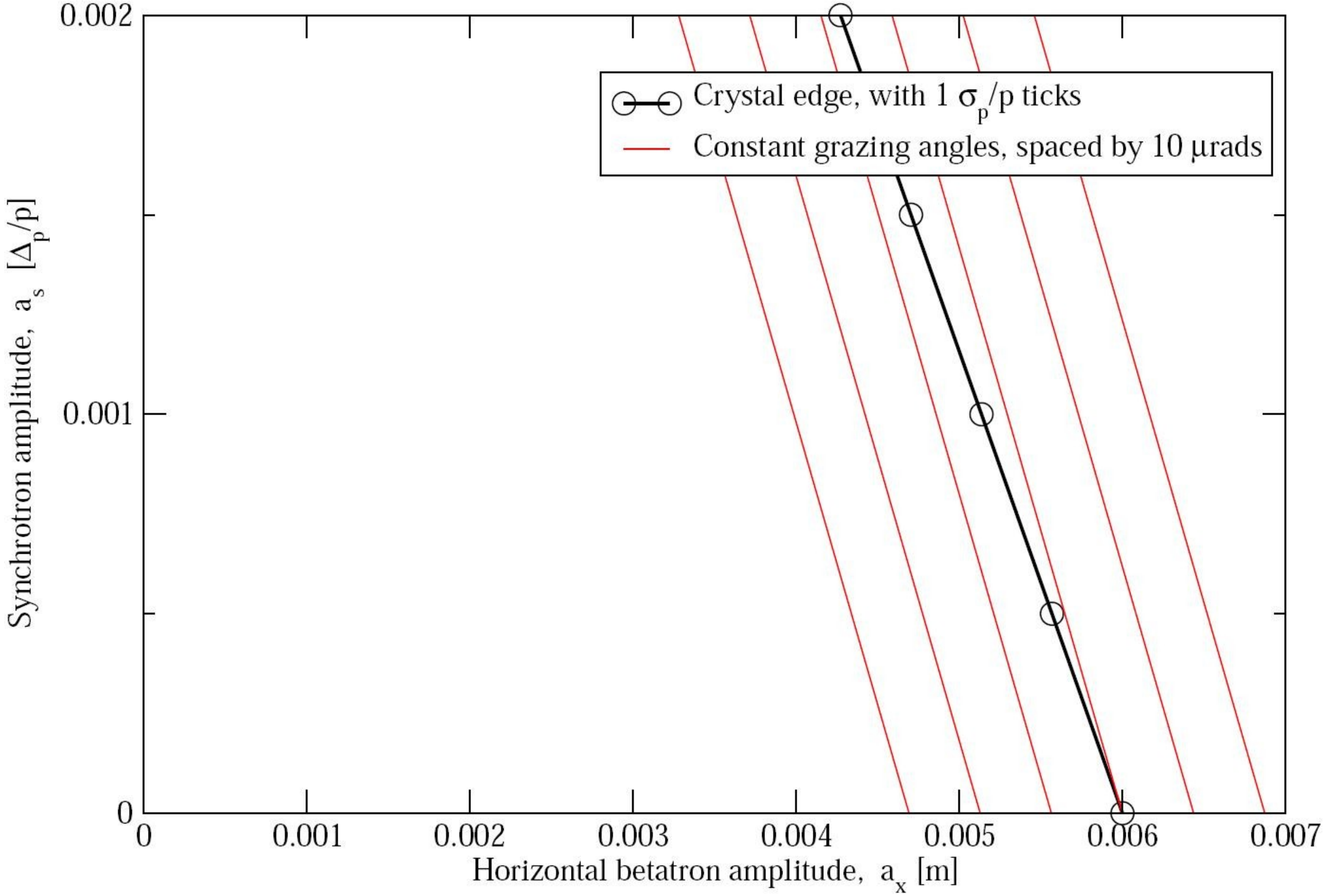
Black line:
Crystal edge

Circles:
Synchrotron amplitudes spaced by $1\ \sigma_p/p$

Red lines:
Constant grazing angles spaced by 10 μrad

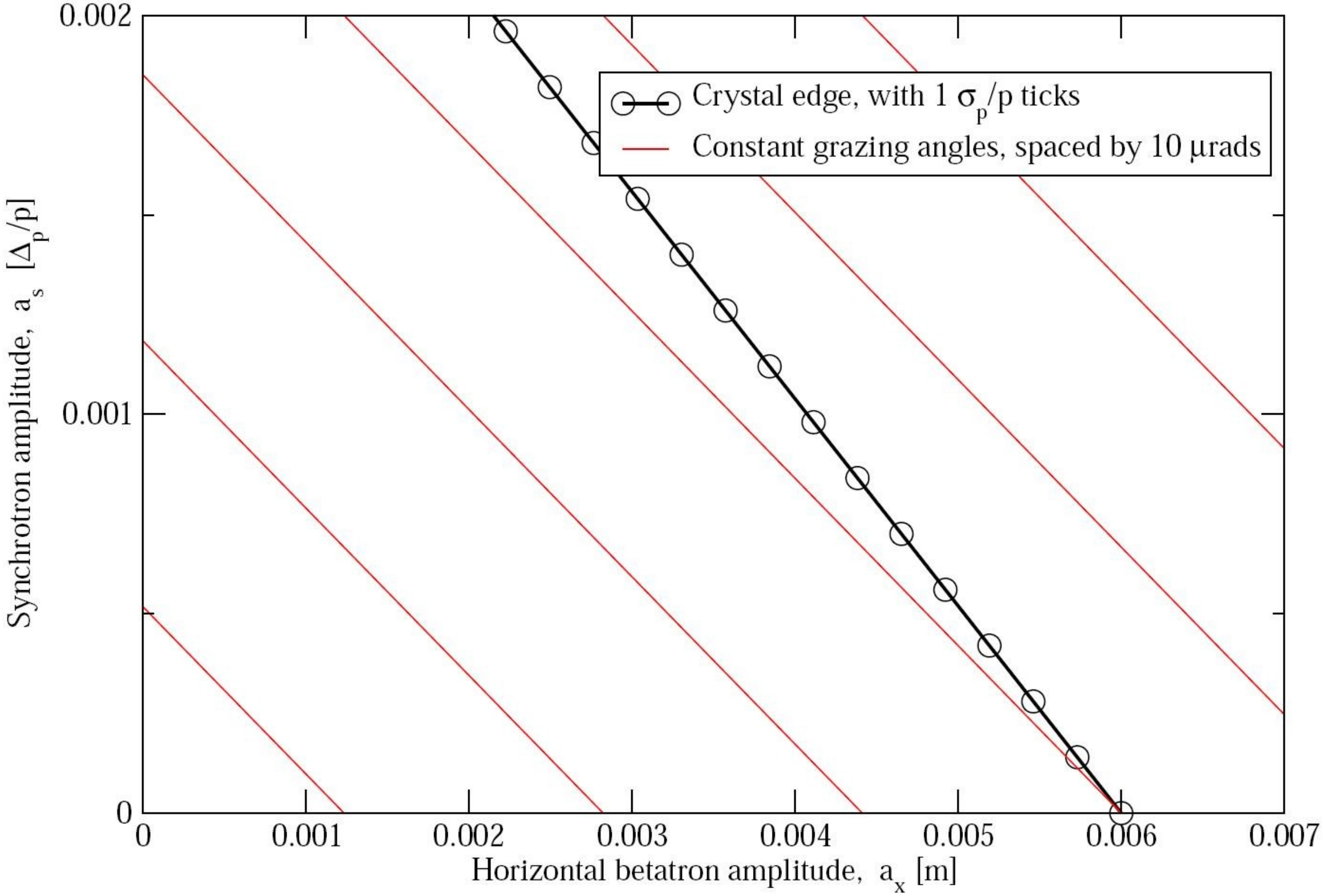
UA9 looks ok: the grazing angle changes by only 0.48 μrad per σ_p/p, despite large negative dispersion at the crystal.

RHIC – Flillers thesis



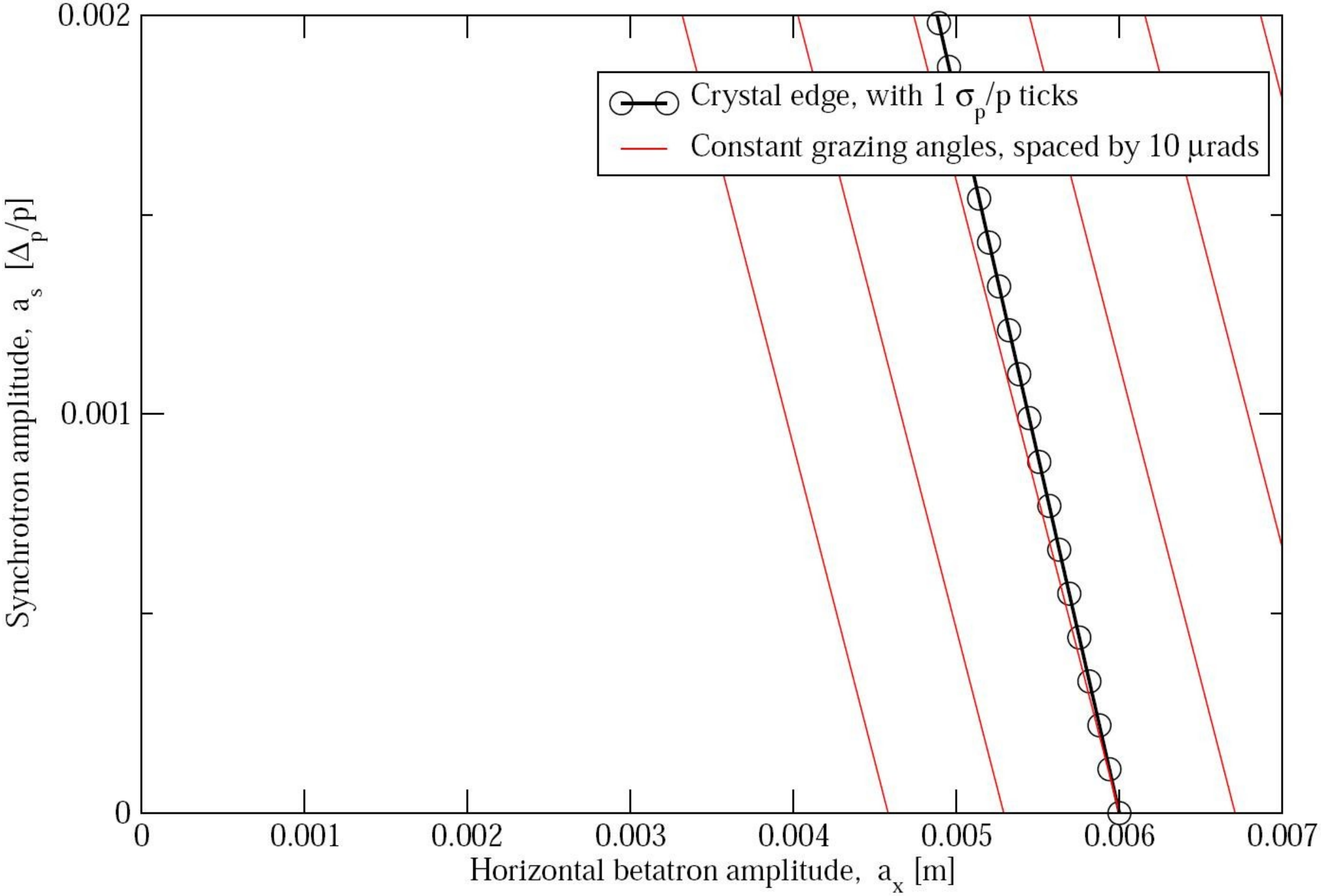
Large σ_p/p: grazing angle changes by 5.4 µrad over 3 σ_p/p. Helps explain disappointing performance?

Tevatron T980



Flattened slopes: grazing angle changes by 0.41 μrad per 3 σ_p/p. Measured optics values also available.

LHC



Small σ_p/p, small channeling acceptance: grazing angle changes by only 0.11 (0.31) μrad per σ_p/p at 7 (0.45) TeV.

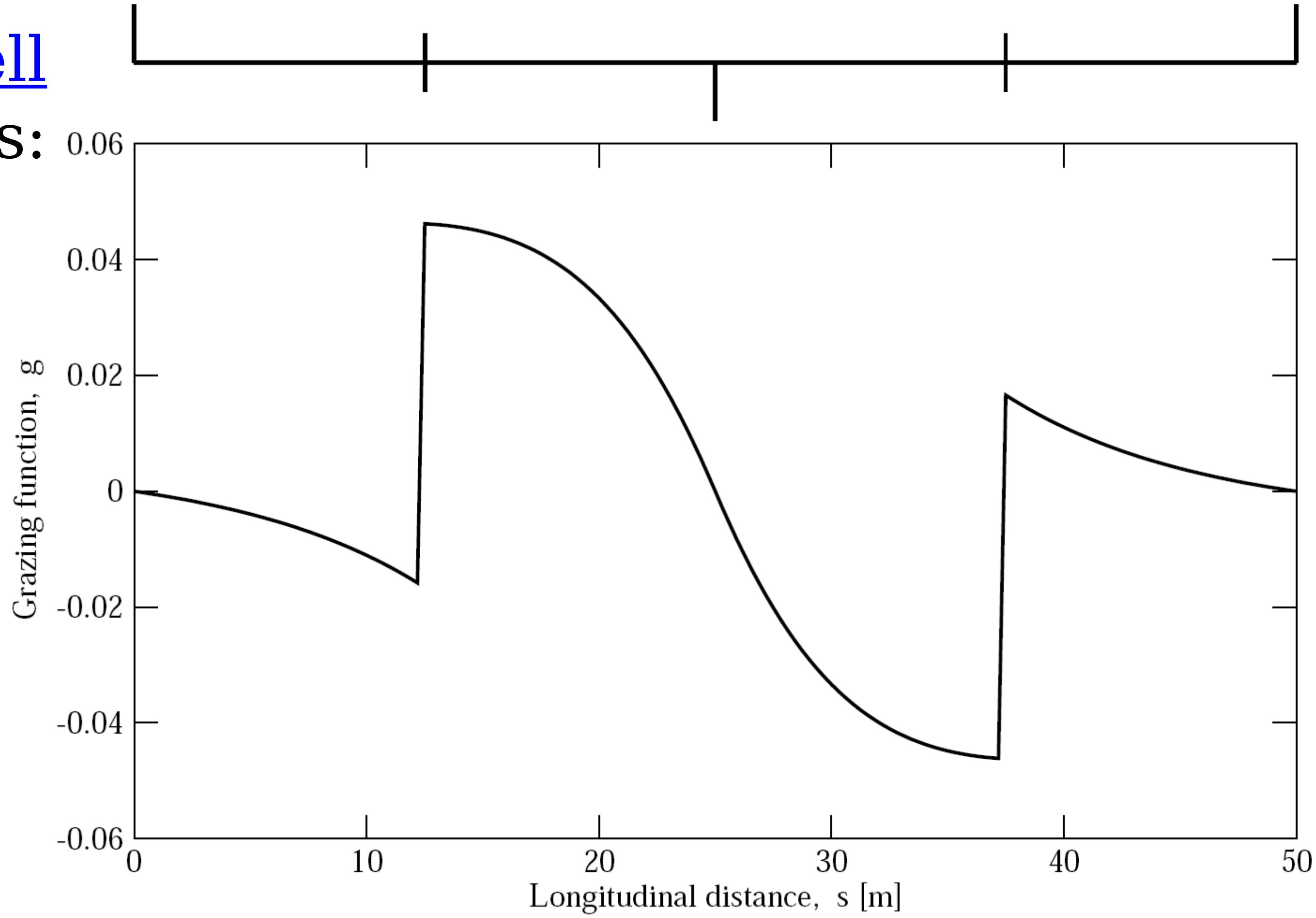
How does g propagate?

A thin dipole: $$\Delta g = \left(\Delta \eta' - \frac{\eta \, \Delta b'}{b} \right) = \Delta \theta$$

A thin quad: $$\Delta g = \eta \left(\frac{\Delta \eta'}{\eta} - \frac{\Delta b'}{b} \right) = 0$$

A matched FODO cell

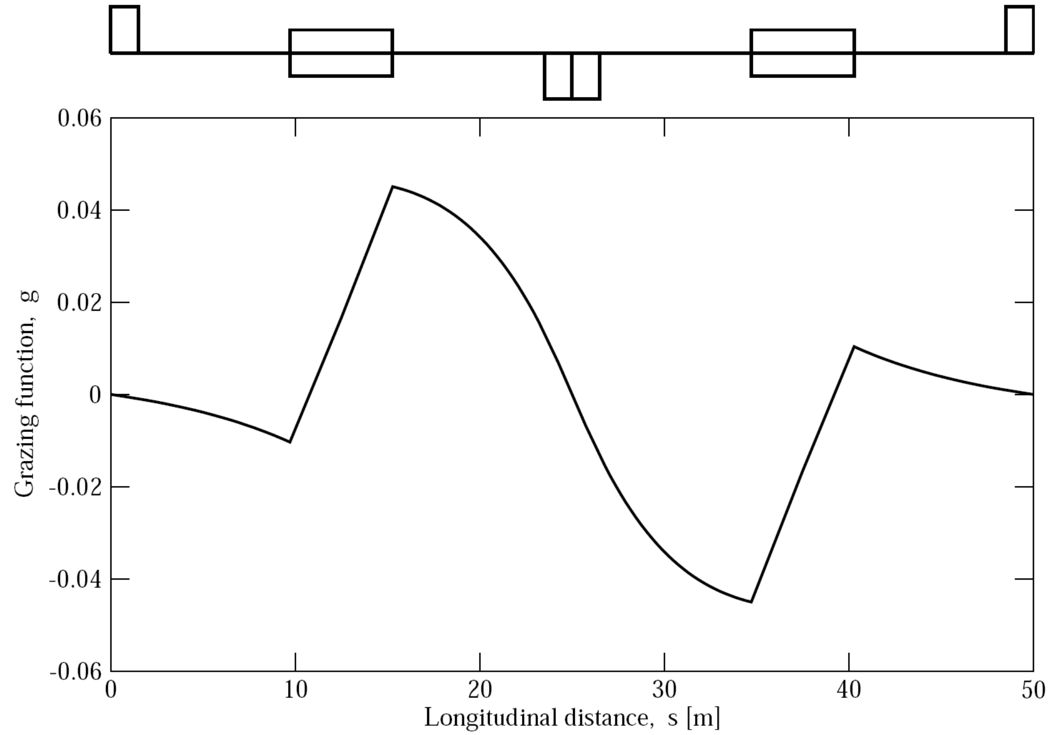
thin dipoles & quads:



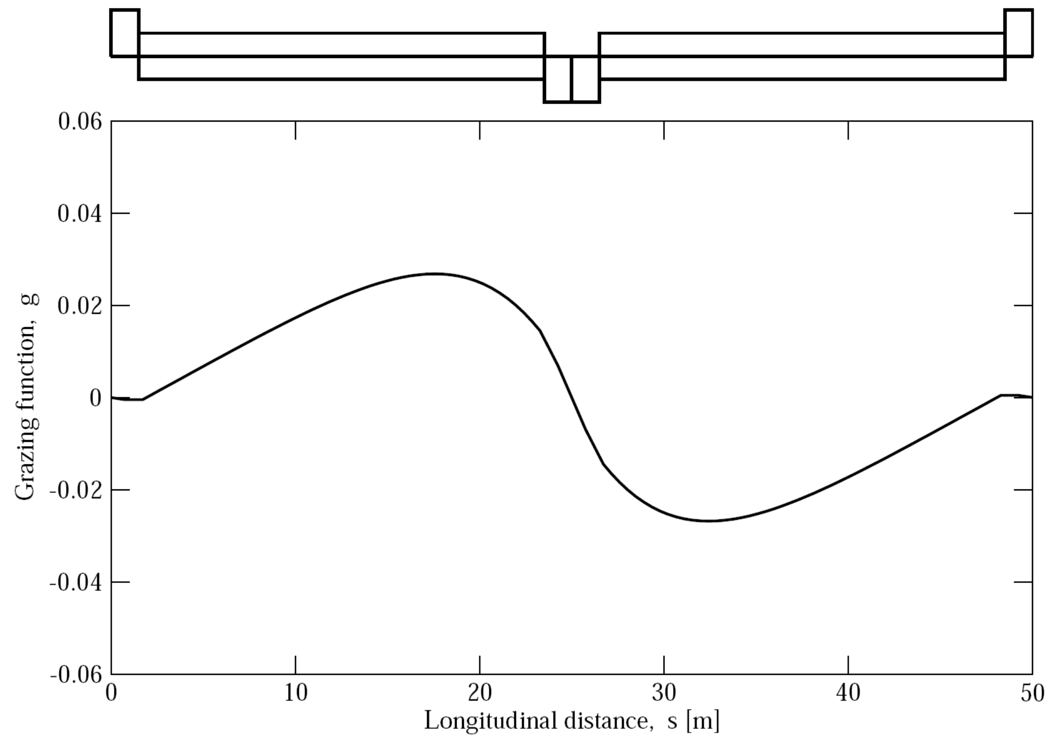
Propagation – 2

LARP APL

A matched FODO cell
short dipoles
thick quads:



A matched FODO cell
long dipoles
thick quads:



g is small when the normalized dispersion is almost constant

$$g = \sqrt{\beta}\, \eta'_N$$

Simple prediction vs numerical testing

Predict $$|g|_{max} \approx \theta \frac{3}{4\sqrt{2}} \frac{\sqrt{1+C^2}}{S^2 C} \left((2-S)\sqrt{1+S} - (2+S)\sqrt{1-S} \right)$$

where $S \equiv \sin\phi/2, \qquad C \equiv \cos\phi/2$

for a matched FODO cell, phase advance per cell ϕ, half cell length *L*, half cell bend angle θ

In reasonable agreement with numerical testing:

$$g_{max} \approx 0.427 \; L^0 \, \theta^1 \tag{51}$$

when the phase advance per full-cell is 90 degrees. There is no dependence on the cell length! A fair rule of thumb is that

$$g_{max} \approx \theta/2 \tag{52}$$

CAUTION: these best case results assume matched optics.

Conclusions

1) The grazing function g parameterizes the rate of change of total angle with synchrotron amplitude.

2) A pure optics function, it is related to the slope of the normalized dispersion. Ideal crystal value $g = 0$

3) g should be small enough that "all" synchrotron amplitudes are within the crystal acceptance angle.

4) This appears to be reasonable to achieve in practice, especially in VR mode, and at lower energies.

5) Place crystal away from betatron & dispersion waves, since they may increase g by an order of magnitude.

6) Planning for future crystals should include a grazing function analysis, both in design and in error analysis.