Action and Phase jump Analysis: From RHIC to the LHC

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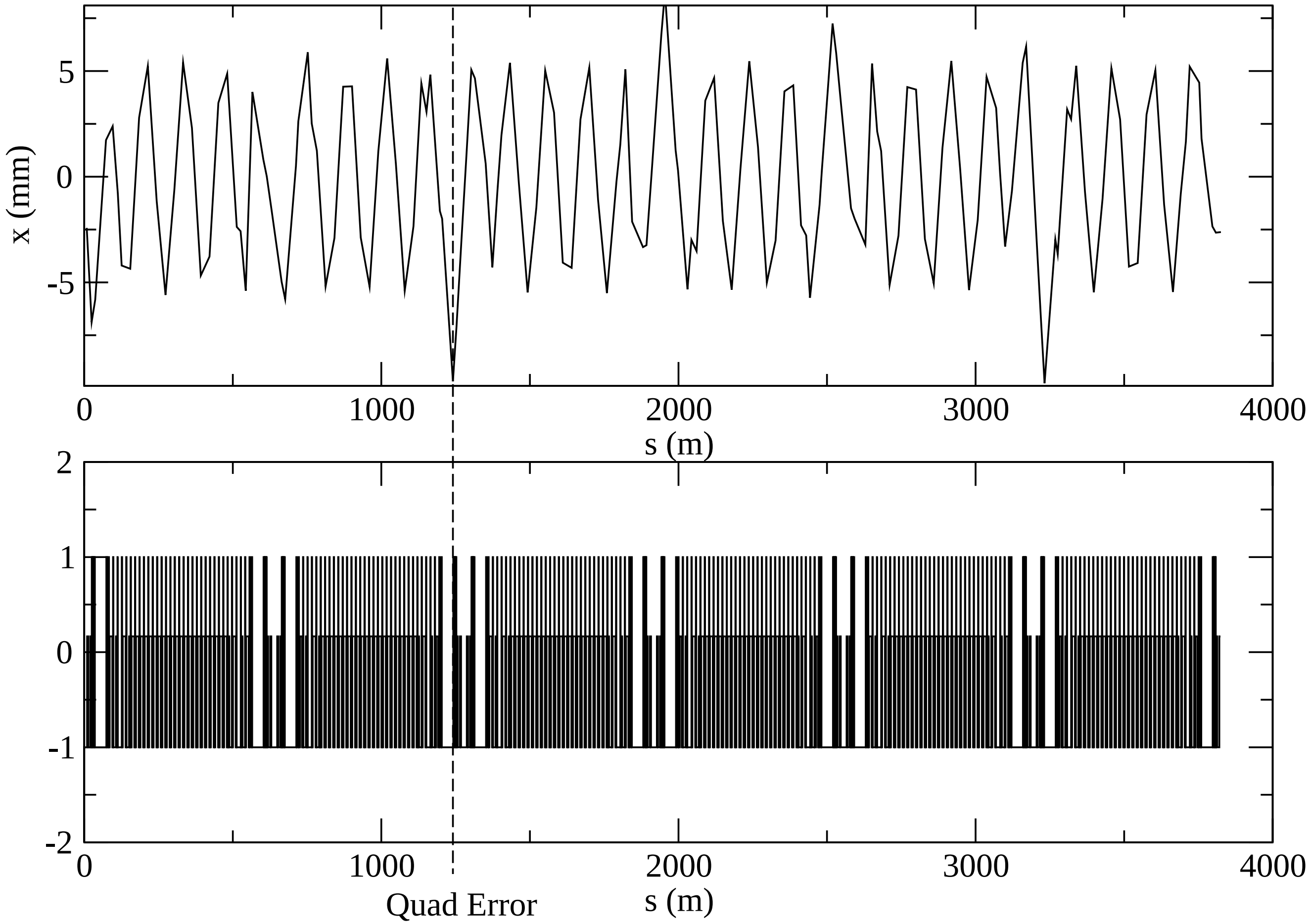
Principle of Action and Phase Jump (1)

Assume a particle is launched in a lattice with a gradient error



Principle (2)

There are two possibilities to describe the particle trajectory:

$$x(s) = \sqrt{2J\beta_N(s)}\sin(\psi_N(s) - \delta)$$

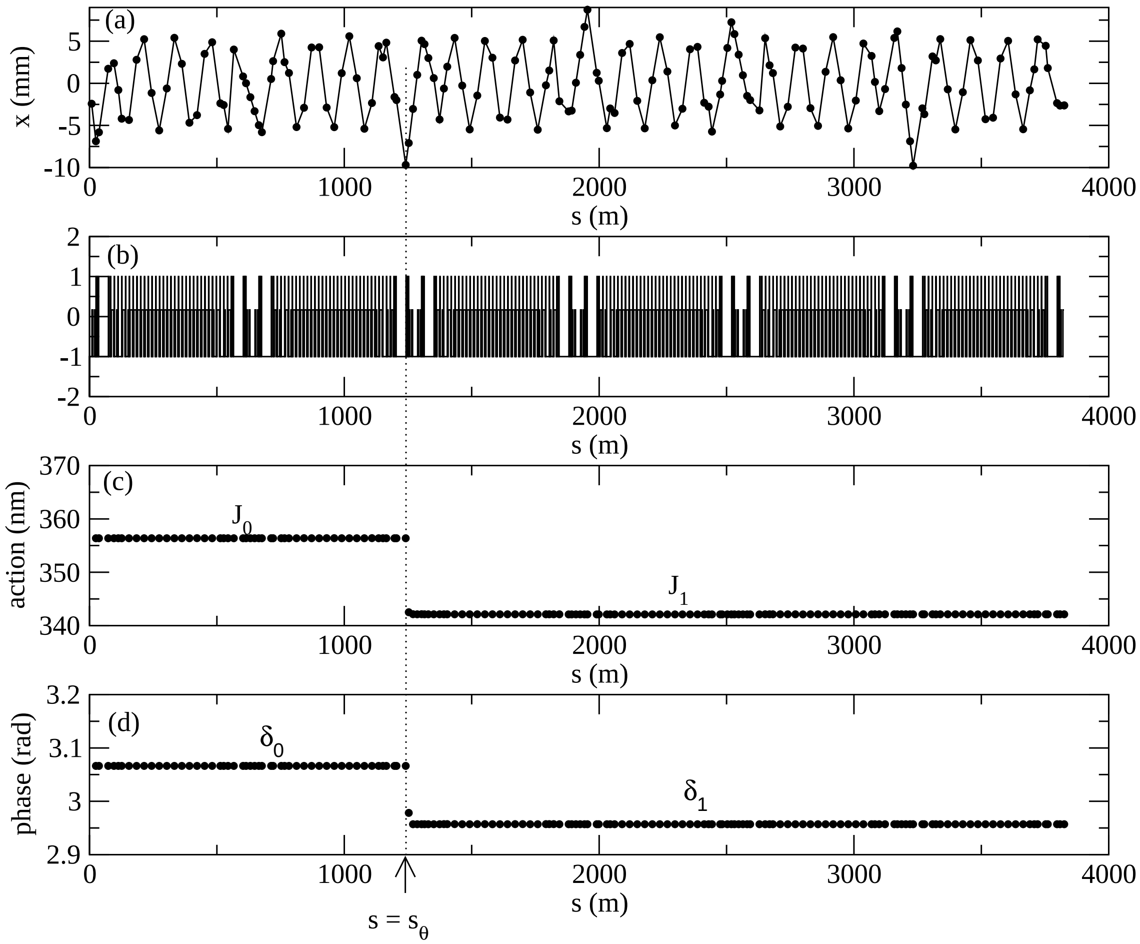
β_N and ϕ_N are the new lattice functions generated by the magnetic error.

Or,

$$\begin{aligned} x(s) &= \sqrt{2J_0\beta_D(s)}\sin(\psi_D(s) - \delta_0) \quad for \quad s < s_1 \\ x(s) &= \sqrt{2J_1\beta_D(s)}\sin(\psi_D(s) - \delta_1) \quad for \quad s > s_1 \end{aligned}$$

β_D and ϕ_D are the designed beta functions. Here, “constants” J and δ change instead of lattice functions.

Principle (3)



(a)
x (mm)
5
0
-5
-10
0
1000
2000
3000
4000
s (m)
(b)
2
1
0
-1
-2
s (m)
(c)
action (nm)
370
360
350
340
J_0
J_1
s (m)
(d)
phase (rad)
3.2
3.1
3
2.9
δ_0
δ_1
s (m)
s = s_θ

Action and Phase from BPM measurements (1)

Plots of J and δ can be build using adjacent BPM measurements:

$$
\begin{aligned}
z_i &= \sqrt{2J_{i+1}\beta_{zi}}\sin(\psi_{zi} - \delta_{i+1}) \\
z_{i+1} &= \sqrt{2J_{i+1}\beta_{zi+1}}\sin(\psi_{zi+1} - \delta_{i+1})
\end{aligned}
$$

Inverting these two Eq., J_{i+1} and δ_{i+1} can be found.

Action and Phase from BPM measurements (2)

$$J_{i+1} = \frac{\left(z_i/\sqrt{\beta_{zi}}\right)^2 + \left(z_{i+1}/\sqrt{\beta_{zi+1}}\right)^2}{2\sin^2(\psi_{zi+1} - \psi_{zi})} - \frac{z_i z_{i+1}\cos(\psi_{zi+1} - \psi_{zi})}{\sqrt{\beta_{zi}\beta_{zi+1}}\sin^2(\psi_{zi+1} - \psi_{zi})}$$

$$\tan\delta_{i+1} = \frac{(z_i/\sqrt{\beta_{zi}})\sin\psi_{zi+1} - (z_{i+1}/\sqrt{\beta_{zi+1}})\sin\psi_{zi}}{(z_i/\sqrt{\beta_{zi}})\cos\psi_{zi+1} - (z_{i+1}/\sqrt{\beta_{zi+1}})\cos\psi_{zi}}$$

This procedure is repeated until all the ring is covered.

Error Strength

Not only the location of the error can be easily determined but also the strength of the error:

$$\Delta x' = \theta_z = \sqrt{\frac{2J_0 + 2J_1 - 4 * \sqrt{J_0 J_1}\cos(\delta_1 - \delta_0)}{\beta_z(s_\theta)}}$$

$$\theta_x = -\frac{e\Delta B_y l}{p},$$

$$\theta_y = \frac{e\Delta B_x l}{p}.$$

Multipole Components of Magnetic Errors

The magnetic error is a contribution from different multipole components:

$$\begin{aligned}\theta_x = \quad & B_0 - B_1 x(s_\theta) + A_1 y(s_\theta) + 2A_2 x(s_\theta) y(s_\theta) \\ & + B_2[-x^2(s_\theta) + y^2(s_\theta)] + ...,\end{aligned}$$

$$\begin{aligned}\theta_y = \quad & A_0 + A_1 x(s_\theta) + B_1 y(s_\theta) + 2B_2 x(s_\theta) y(s_\theta) \\ & + A_2[x^2(s_\theta) - y^2(s_\theta)] + ...\end{aligned}$$

where B_n and A_n are values related with the normal and skew multipole components of the error ΔB.

Estimating Errors (One Multipole)

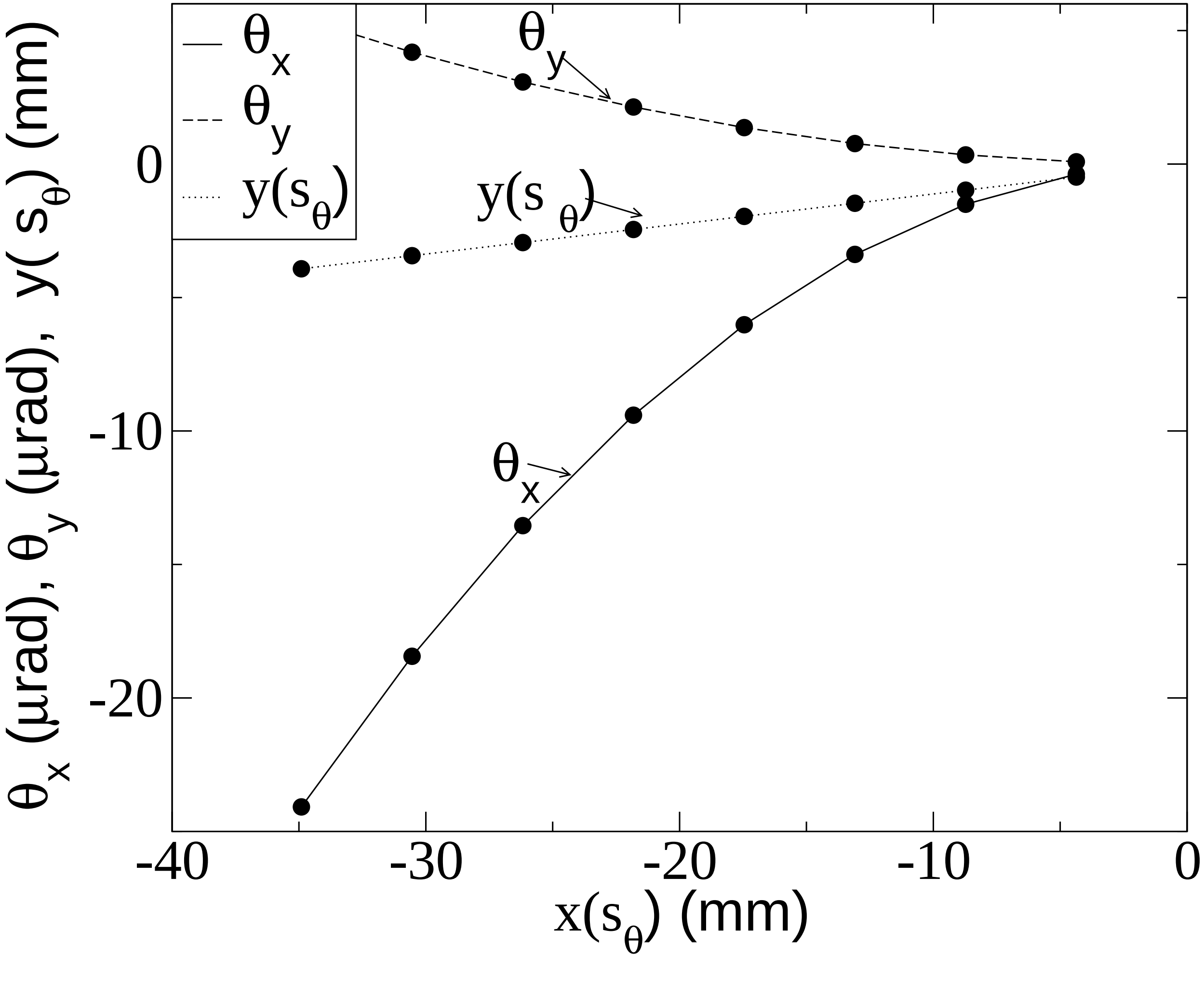
$$A_1 = \frac{\theta_x y(s_\theta) + \theta_y x(s_\theta)}{x^2(s_\theta) + y^2(s_\theta)}$$

$$B_1 = \frac{\theta_y y(s_\theta) - \theta_x x(s_\theta)}{x^2(s_\theta) + y^2(s_\theta)}$$

If only one error multipole component is present, only one particle trajectory is needed.

Estimating Errors (Several Multipoles)

Several error multipole components -> several orbits with different amplitudes



Estimating Errors (Several Multipoles)

$$\begin{aligned}\theta_x &= C_{1x}x(s_\theta) + C_{2x}x^2(s_\theta) + cte \\ \theta_y &= C_{1y}x(s_\theta) + C_{2y}x^2(s_\theta) + cte \\ y(s_\theta) &= mx(s_\theta) + b\end{aligned}$$

$$\begin{aligned}A_1 &= \frac{C_{1x}m + C_{1y}}{1+m^2} \\ B_1 &= -\frac{C_{1x} - C_{1y}m}{1+m^2} \\ A_2 &= -\frac{-C_{2y} - 2C_{2x}m + C_{2y}m^2}{1+2m^2+m^4} \\ B_2 &= -\frac{C_{2x} - 2C_{2y}m - C_{2x}m^2}{1+2m^2+m^4}\end{aligned}$$

Results of Simulations

Simulations of particle trajectories with a gradient error, a skew quadrupole error, and a sextupole error were done. The difference between set errors and and errors estimated from action and phase analysis were:

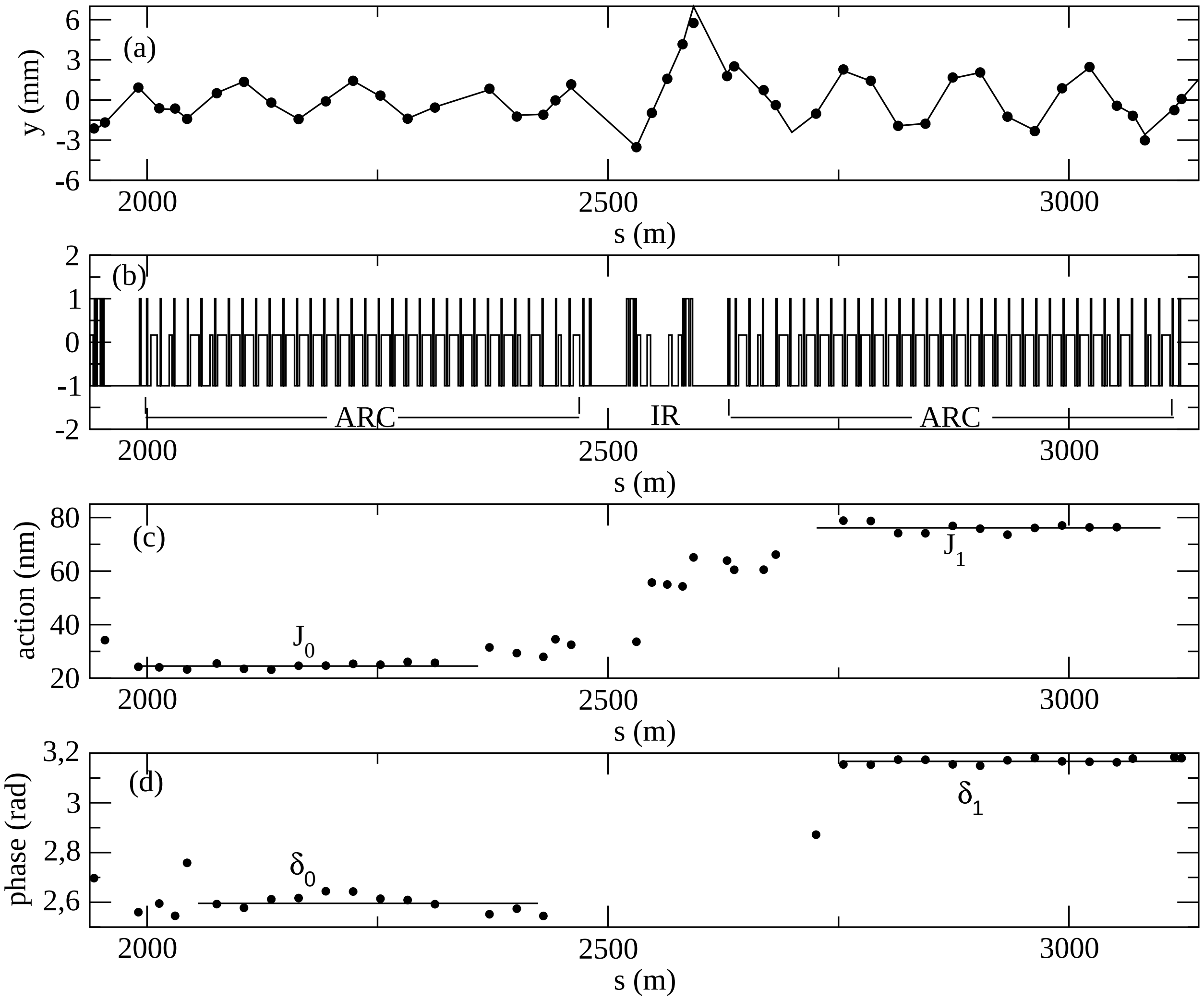
- 0.03% or less for gradient errors.
- 0.01% or less for skew quadrupole errors.
- 3% or less for sextupole errors.

Linear Experiments with Beam in RHIC

Experimental Conditions Required

- Large betatron oscillations usually excited by dipole correctors.
- The trajectory should have a maximum at the place where the error is being estimated.
- Systematic errors and dipole errors are eliminated using difference orbits.
- Difference orbits are now made with two turns of the same multiturn orbit.

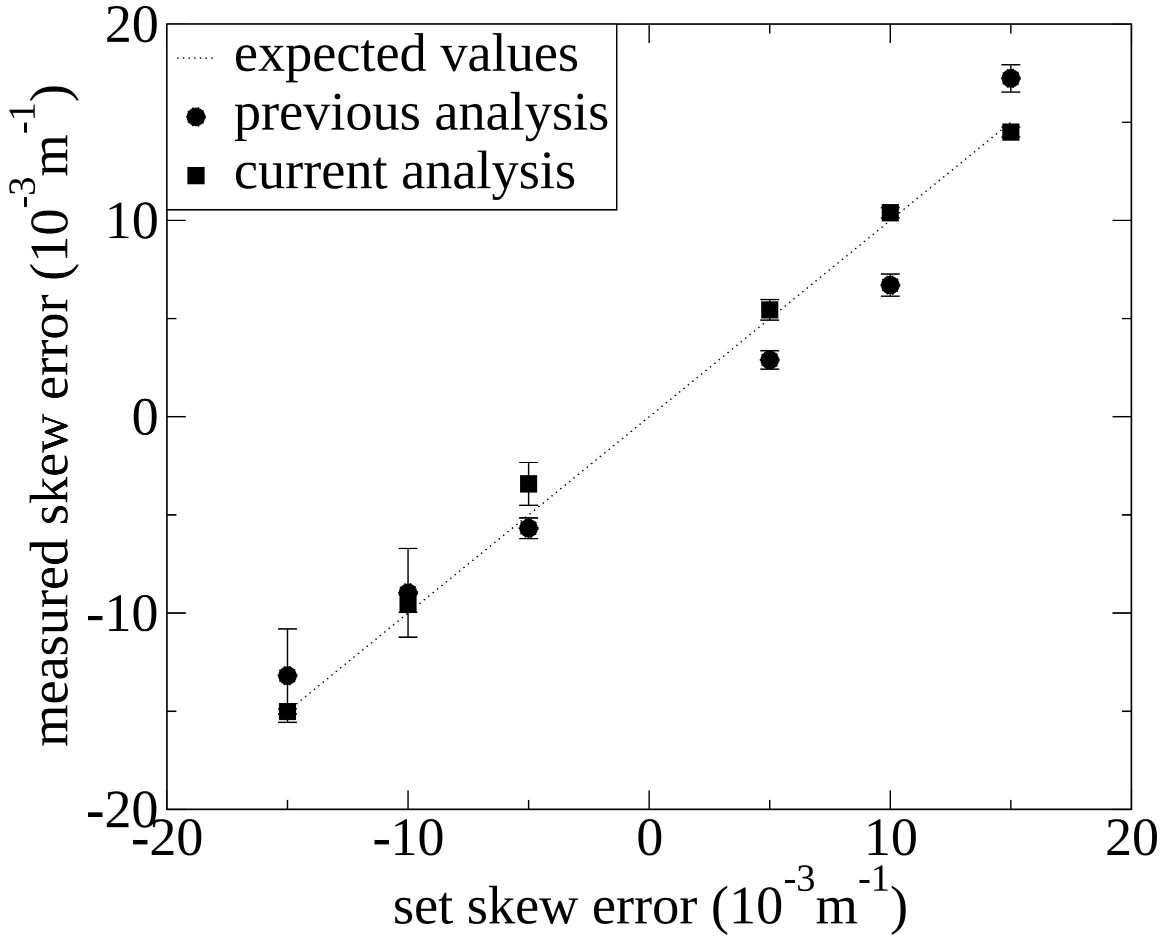
Action and Phase Analysis on a Difference Orbit



Experiments with Known Gradient Errors (1)

- Large beam orbits were excited changing a quadrupole corrector bi8-qs3 at IR8.
- Action and phase analysis of first turn orbits for each setting of bi8-qs3 were done.
- Difference orbits were built using two different methods.

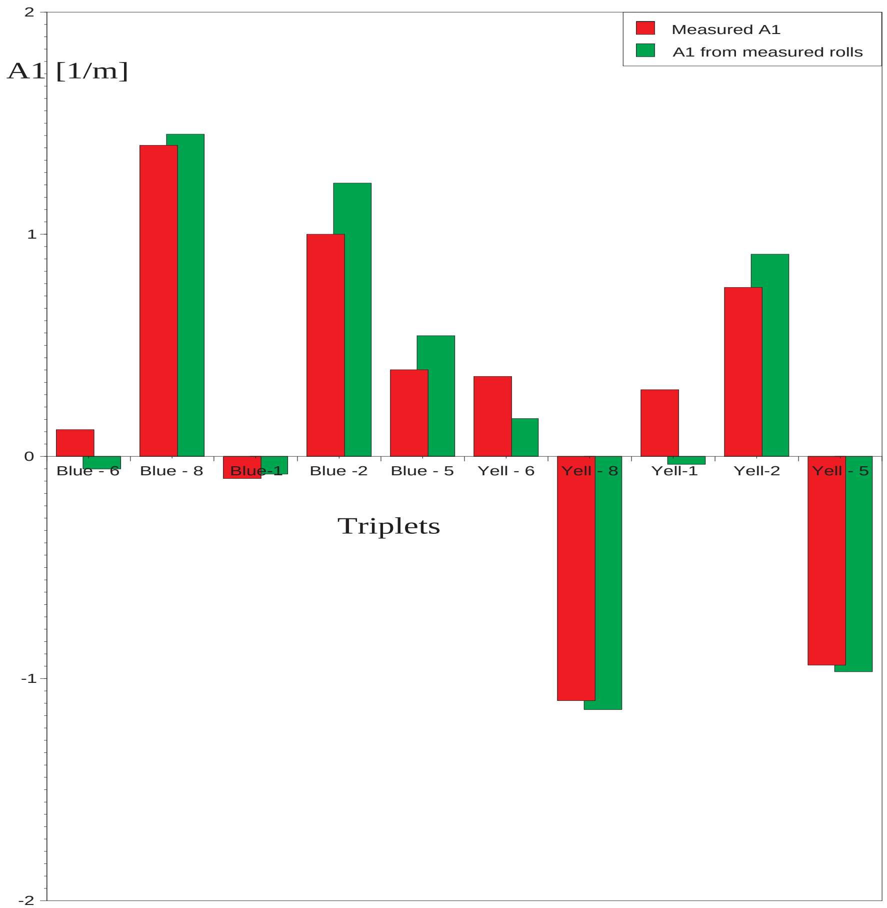
Experiments with Known Gradient Errors (1)



Skew Quad Error Measurements (1)

- Skew quad errors (A_1) were measured for all RHIC IRs using the action and phase analysis.
- Roll angles of the quads were also measured during the 2002 RHIC shutdown period.
- $$A_1^{eq} = \frac{\sum_{i=1}^{3}(-2\frac{\phi_i}{f_i})\sqrt{\beta_x^i\beta_y^i}}{\sqrt{\beta_x^{sc}\beta_y^{sc}}}$$

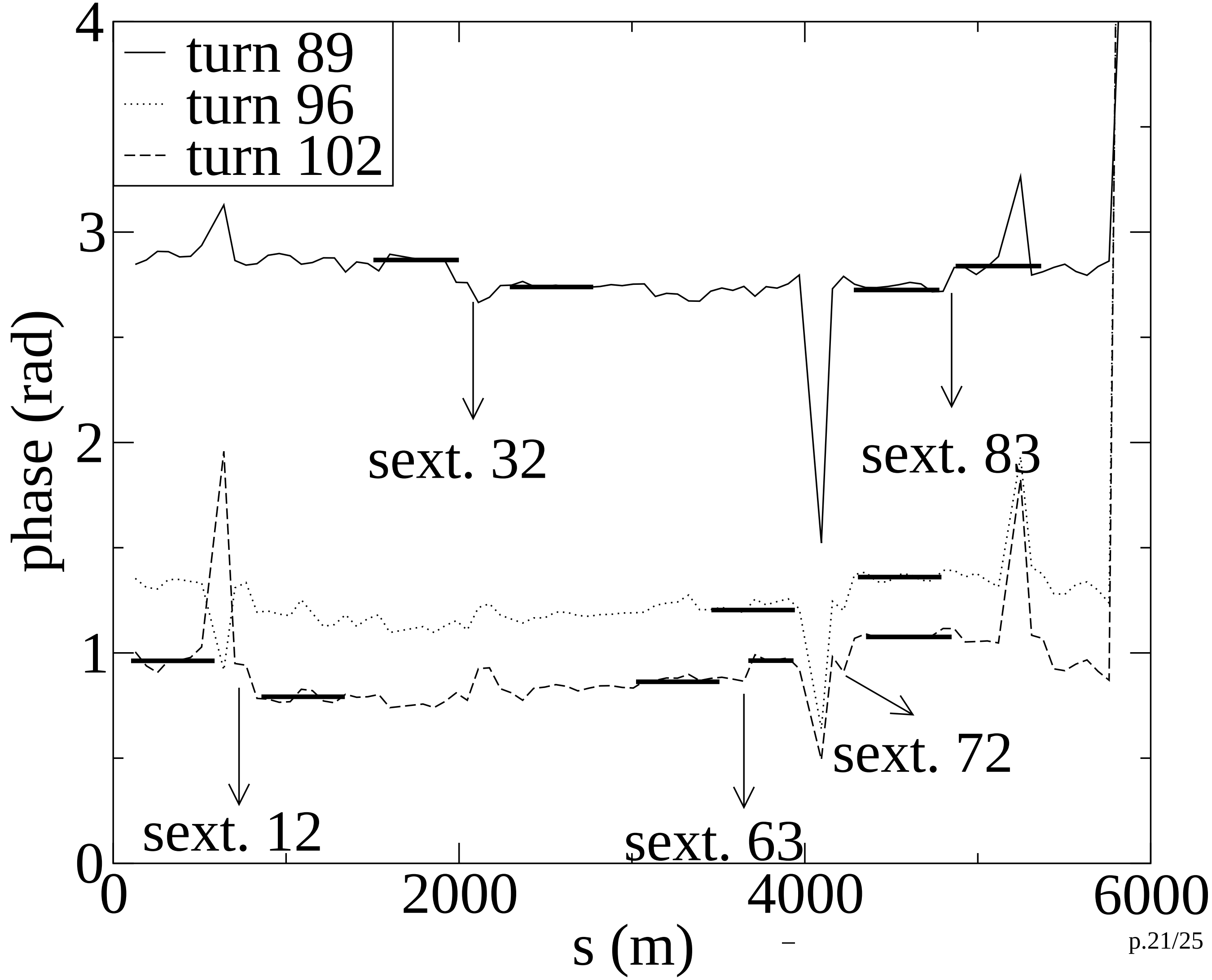
Skew Quad Error Measurements (2)



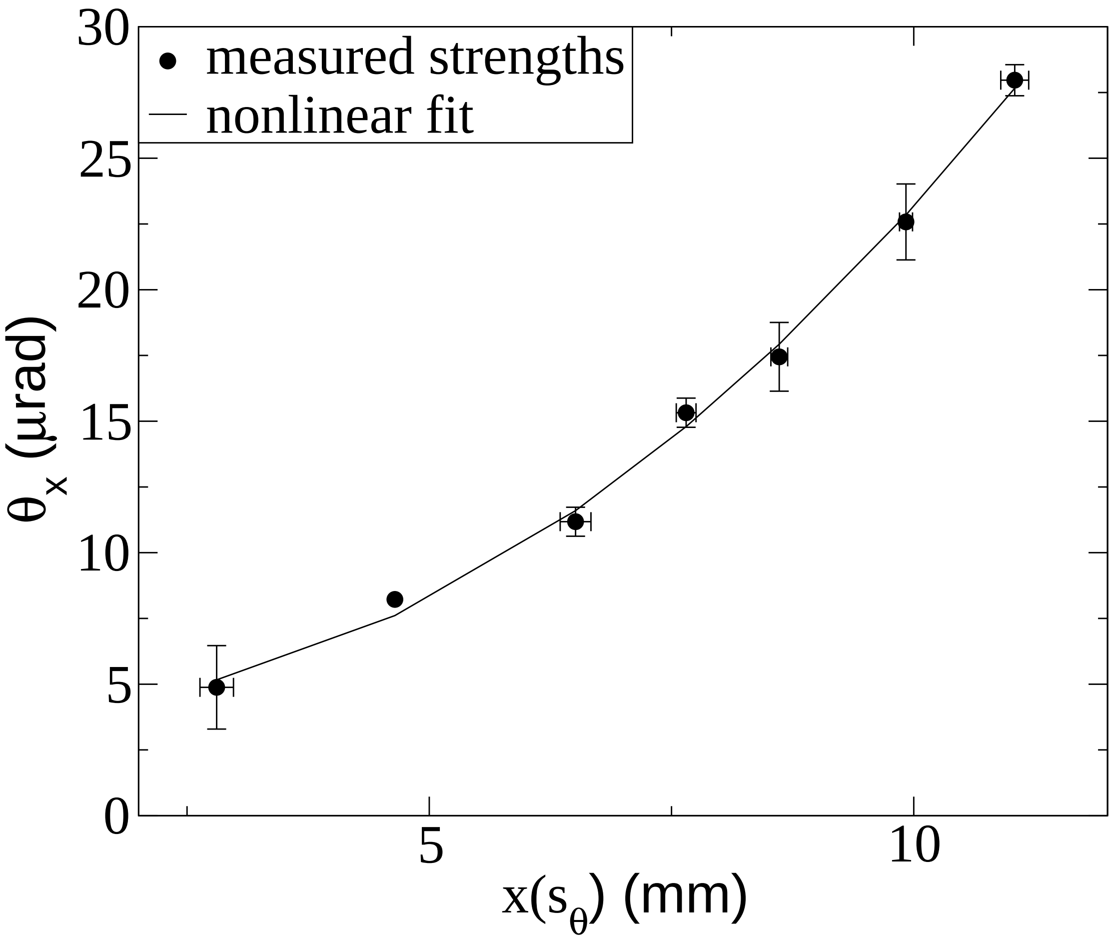
Local skew quadrupole correctors were set according to these values.

Nonlinear Experiments with Beam in the SPS (1)

Sextupoles were intentionally turn on at specific locations



Nonlinear Experiments with Beam in the SPS (2)

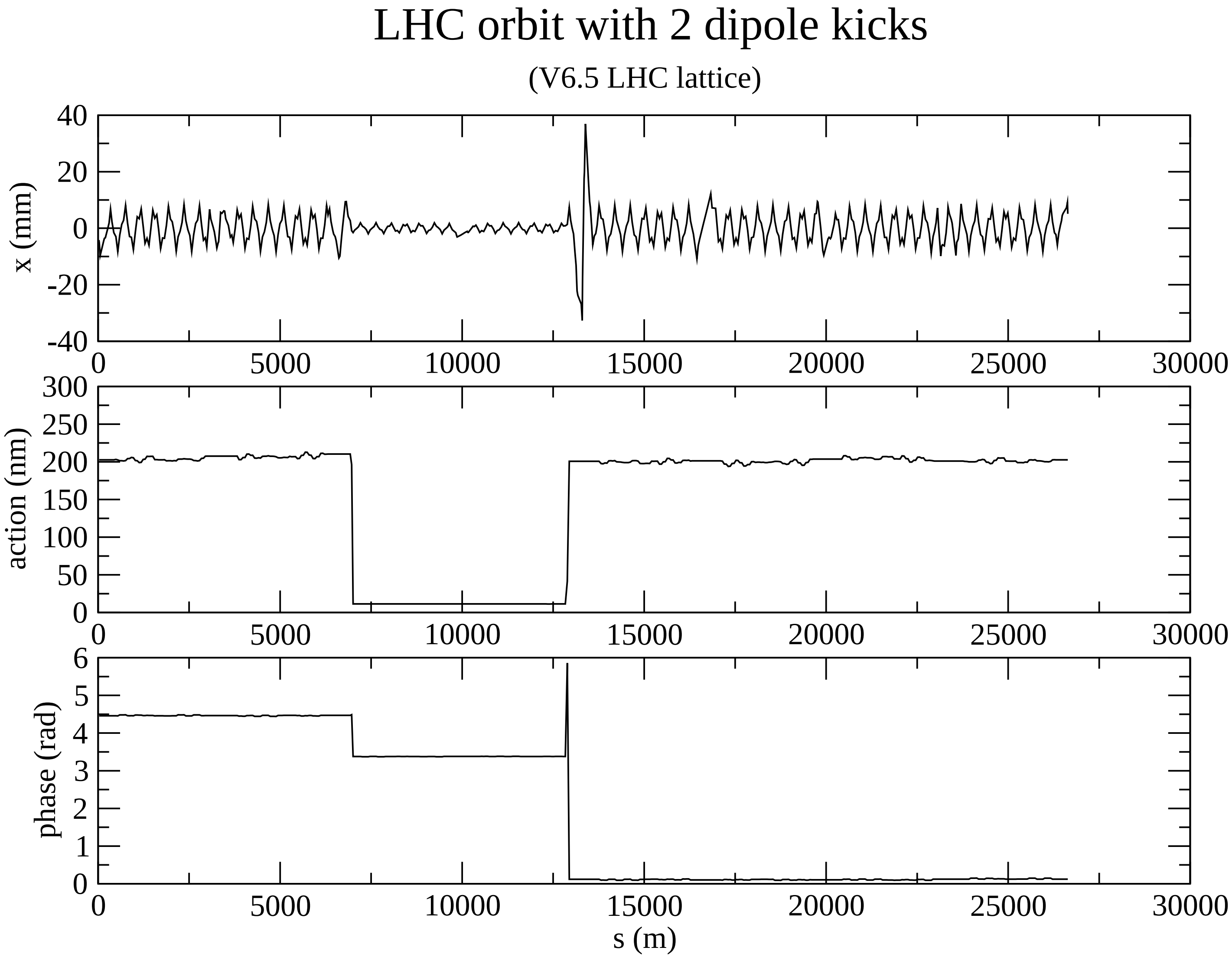


$K2L = (0.438 \pm 0.032)m^{-2}$ from the fit of the experimental points which is in good agreement with the set value.

Action and Phase Analysis for the LHC

- Simulation of LHC orbits with linear and nonlinear errors.
- Action and phase analysis of simulated orbits.
- Estimation of magnetic errors from the simulated orbits.
- Action and phase analysis of experimental orbits.
- Comparisons with other methods ?.

Sample of LHC simulations so far



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