Is it possible to link DA and beam losses variation over time?

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Introduction

- All started from LHCCWG (chaired by R. Bailey): What is the tolerance on the LHC beam parameters?
  - Specifically: what happens if dynamic aperture (DA) is not 12 σ as expected? Clearly, this should have an impact on beam losses…
  - In general terms: how to link DA to intensity variation vs. time? General answer does not seem to be known (certainly not to me).
- Comment:
  - In mathematical sense DA does not depend on time
  - Numerical simulations are performed with a specific maximum number of turns (N_{max}): the computed DA does depend on N_{max}
- How does DA depend on N_{max} in numerical simulations? The answer to this is (more or less known)…

DA vs. N_{max}

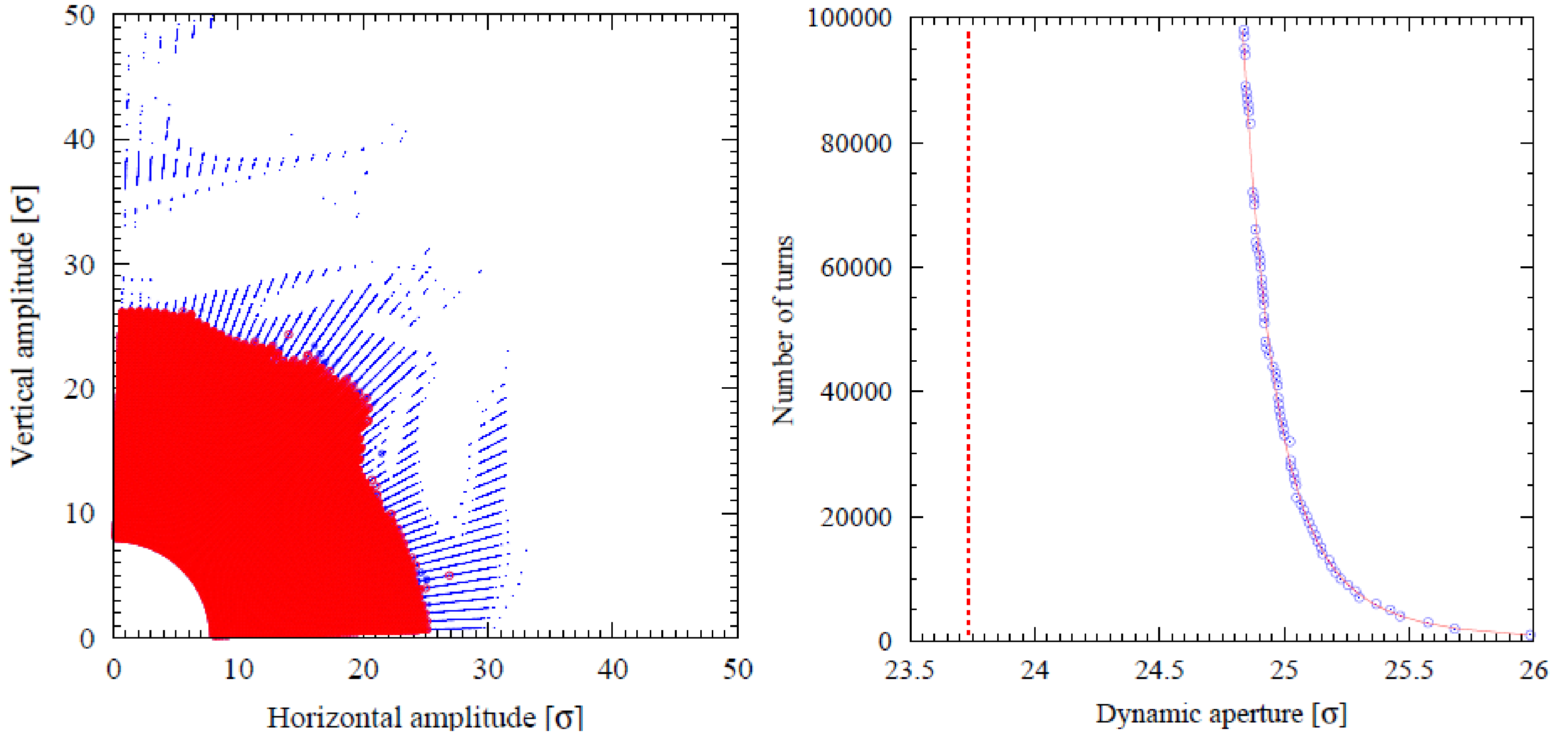
- How to compute DA from numerical simulations?
  - Polar grid of initial conditions.
  - Tracking until they are lost or N_{max} is reached.
  - Compute:

$$D(N) = \frac{2}{\pi} \int_0^{\pi/2} r(\theta; N)\, d\,\theta \equiv < r(\theta; N) >$$

  - NB: more refined approaches can be defined, using different weights for different phase space directions.
  - Then, it can be shown (by fitting numerical data) that

$$D(N) = D_\infty \left(1 + \frac{b}{[\log N]^\kappa} \right)$$

An example of DA for LHC



- Dynamic aperture of a model of the LHC ring (left) in physical space:
  - The red points represent the initial conditions stable up to 10^5 turns
  - The blue points represent unstable conditions and their size is proportional to the number of turns by which their motion is still bounded.
- The time-evolution of the DA is shown on the right.
  - The markers represent the numerical results
  - The continuous line shows the fitted inverse logarithmic law.
  - The dotted line represents D_∞

Phenomenological fit?

- In fact not quite.
- The physical picture is:
  - For r < D_∞
    - The motion is governed by KAM theorem. Fully stable region (only Arnold diffusion for a set of initial conditions of small measure -> irrelevant from the physical point of view).
  - For r > D_∞
    - The motion follows Nekhoroshev theorem, i.e., the stability time N(r) of a particle at radius r is given by
    - This provides a pseudo-diffusion

$$N(r) = N_0 \exp\left(\frac{r_*}{r}\right)^{1/\kappa}$$

Two regimes found

- In 4D simulations:
  - D_∞, b, κ are always positive. This implies a stable region for arbitrary times.
- In 4D simulations with tune ripple or 6D simulations:
  - There could be situations in which no stable region for arbitrary times exists. This corresponds to

$$\begin{cases} D_\infty > 0 & \kappa < 0 & b < 0 \\ D_\infty < 0 & \kappa > 0 & b < 0 \end{cases}$$

Link between DA variation and losses - I

- Assumptions:
  - Non-linear errors from magnetic field imperfections
  - No beam-beam or other collective effects
  - The beam distribution is Gaussian
- Then:

$$\frac{I(N)}{I_1} = 1 - \int_{D(N)}^{+\infty} e^{-\frac{r^2}{2}}\, r\, dr = 1 - e^{-\frac{D^2(N)}{2}}$$

- Assuming also that the system has a stable region. Then

$$\frac{\Delta I}{I_1}(\infty, D_\infty, b, \kappa) = e^{-\frac{D_\infty^2}{2}}$$

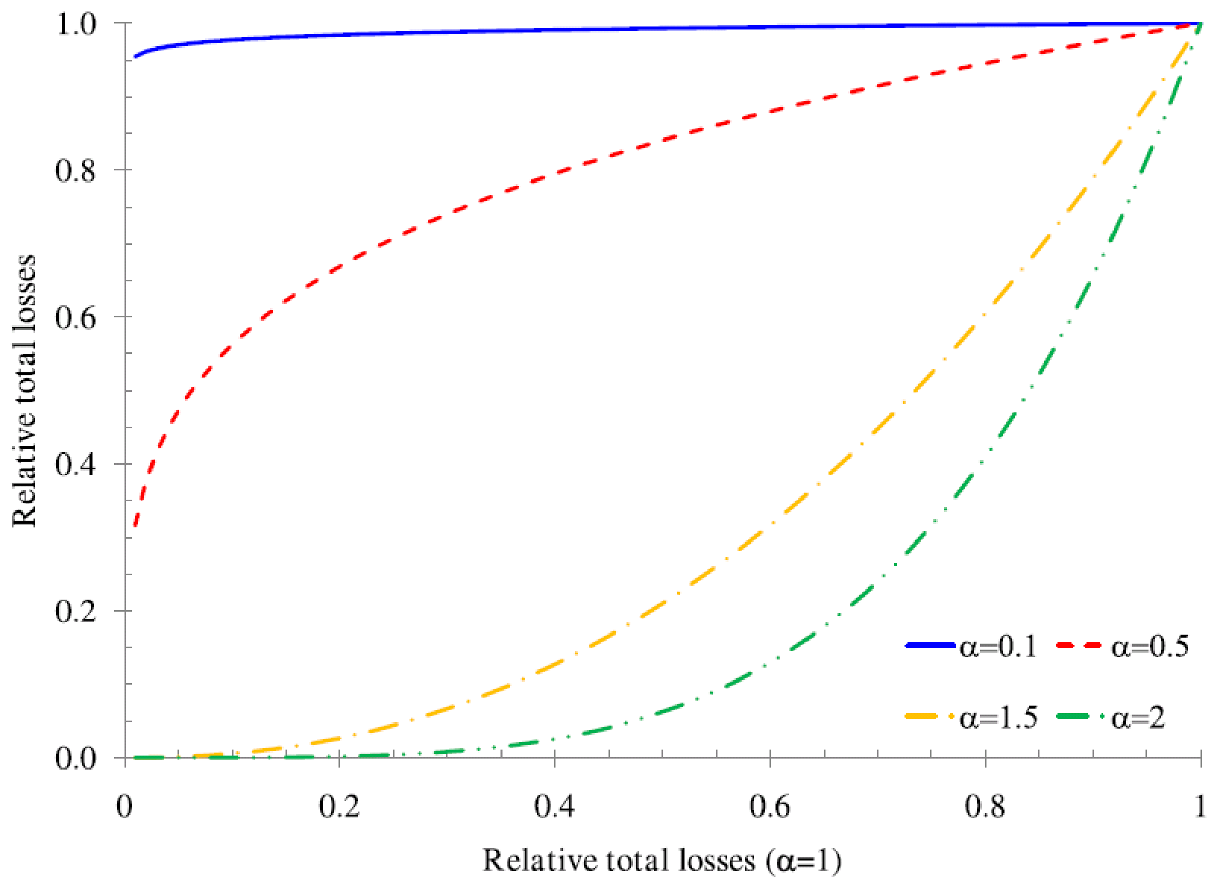
- This is the relation between DA and losses

Link between DA variation and losses - II

- What happens if DA is changed (from design value). Then:

$$\frac{\Delta I}{I_1}(N, \alpha D_\infty, b, \kappa) = \left[\frac{\Delta I}{I_1}(N, D_\infty, b, \kappa)\right]^{\alpha^2}$$

NB: all these considerations can be generalised to different beam distributions (e.g., quasi-parabolic, and Lévy-Student).

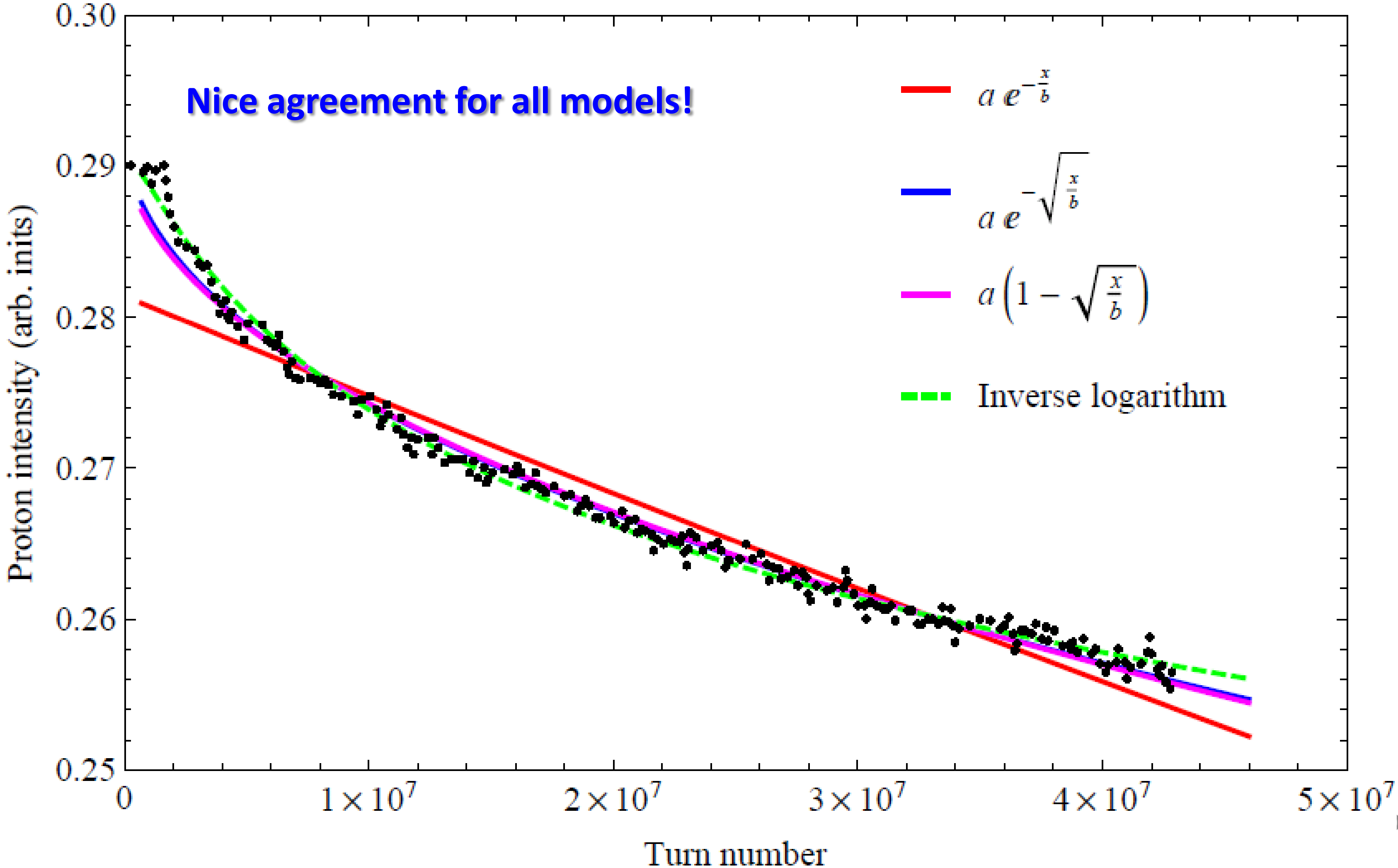


Experimental verification

- Not easy (data available do not satisfy the assumptions). But:
  - One data set found from Tevatron at injection: T. Sen, P. Lebrun, R. Moore, V. Shiltsev, M. Syphers, X. L. Zhang, W. Fischer, F. Schmidt, F. Zimmermann, "Beam Losses at Injection Energy and During Acceleration in the Tevatron", proceedings of 2003 Particle Accelerator Conference, edited by J. Chew, P. Lucas and S. Webber, (IEEE Service Center, Piscataway - NY, 2003), p. 1754.
  - One data set found from SPS data for coast at 55 GeV (long range beam-beam tests – thanks to Frank and Elias) .

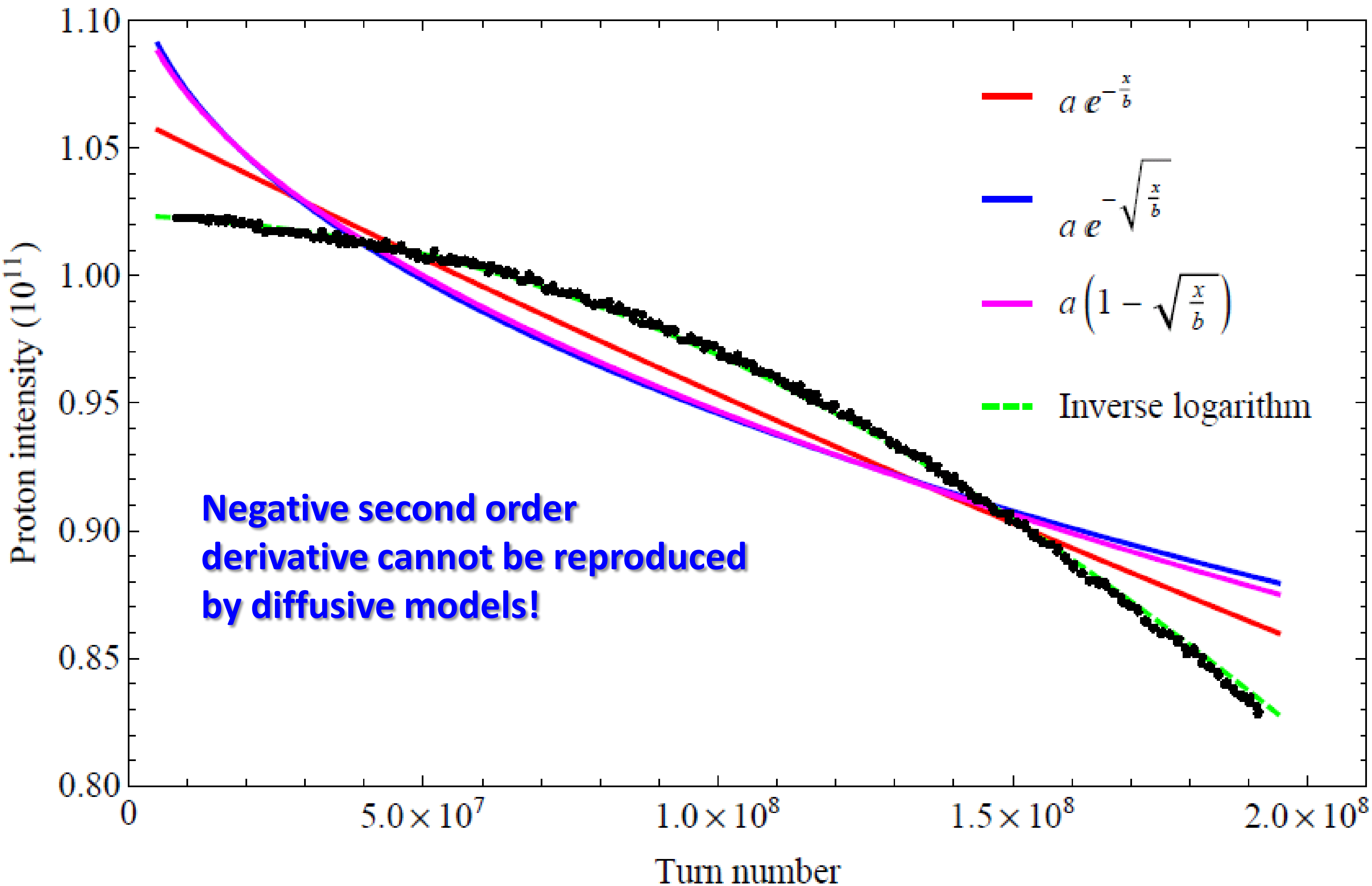
Tevatron data

- Proton bunch at injection.
  - Estimates from purely diffusive model included.



SPS data

- Proton bunch at 55 GeV in coast.
  - Estimates from purely diffusive model included.



Conclusions

- Extension of proposed approach to include other effects in progress.
- The method could be tested at the LHC.
- It could be used to perform DA measurements.