Adding RF-Multipoles to Mad-X

AL, LD, RdM, MG

LCU Meeting, Jan 24 2012

Rationale

- Add RF-Multipolar fields to Mad-X
  - RF-Multipole: RF + Multipole
  - Properties of an RF-CAVITY
  - Properties of a multipole

- Ultimate objective: implement crab cavities with high-order RF multipolar errors

- Staged approach (conservative)

RF-Multipole

is a Multipole with coefficients that change with *z*

$$B_y(x,y,z) + iB_x(x,y,z) = \sum_{n=1}^{N} \left(B_n(r_0,z) + iA_n(r_0,z)\right) \cdot \frac{(x+iy)^{n-1}}{r_0^{n-1}}$$

$$\begin{aligned} r_0 &: \text{ reference radius} \\ x,y,z &: \text{ particle coordinates w.r.t. reference particle} \end{aligned}$$

Where B_n and A_n are the real part of a complex vector (phasor) rotating with angular velocity ω_{RF}

$$\begin{aligned} A_n(r_0,z) &= \mathrm{Re}\left\{A_n(r_0)\, e^{j(k_{\mathrm{RF}} z + \vartheta_n)}\right\} && : \text{ skew multipole component} \\ B_n(r_0,z) &= \mathrm{Re}\left\{B_n(r_0)\, e^{j(k_{\mathrm{RF}} z + \varphi_n)}\right\} && : \text{ normal multipole component} \\ z &= ct && : \text{ longitudinal coordinate w.r.t. reference particle} \\ \boxed{k_{\mathrm{RF}}} &= \frac{\omega_{\mathrm{RF}}}{c} = \frac{2\pi}{\lambda_{\mathrm{RF}}} && : \text{ RF wave number} \end{aligned}$$

$$\begin{aligned} B_n(r_0) &: \text{ modulus of the phasor } \vec{B}_n \qquad & \vartheta_n &: \text{ phase of the phasor } \vec{B}_n \\ A_n(r_0) &: \text{ modulus of the phasor } \vec{A}_n \qquad & \varphi_n &: \text{ phase of the phasor } \vec{A}_n \end{aligned}$$

Two steps for the implementation

1) Add a **new element**: RF-Multipole

2) Modify **EFCOMP** to carry the additional information required by the RF-Multipole:
   - Frequency of the fundamental mode
   - Phase of each multipolar component

Remark: PTC implements a simplified version of the RF-Multipole (1 phase for all orders) that is not usable from Mad-X

(1) New element

```
RFMULTIPOLE,
   L=real, VOLT=real, LAG=real, HARMON=integer,
   FREQ=real, LRAD=real, TILT=real,
   KNL:={…, …, …},  ! Normal coefficients
   KSL:={…, …, …},  ! Skew coefficients
   PNL:={…, …, …},  ! Normal phases [2pi]
   PSL:={…, …, …};  ! Skew phases [2pi]
```

RF-Multipole: RF + Multipole

- Properties of an RF-CAVITY
- Properties of a multipole

(2) Extending EFCOMP

EFCOMP associates field and phase errors to an element

```
EFCOMP, ORDER:=integer, RADIUS:=real,
   DKN:={dkn(0),dkn(1),dkn(2), …},
   DKS:={dks(0),dks(1),dks(2), …},
   DKNR:={dknr(0),dknr(1),dknr(2), …},
   DKSR:={dksr(0),dksr(1),dksr(2), …},
   FREQ:=real,
   DPN:={dpn(0),dpn(1),dpn(2), …},
   DPS:={dps(0),dps(1),dps(2), …};
```

Phase errors are always absolute

Impact on the code (structure)

- EFCOMP:
  - Must be changed to take into account the new input vectors with phases, and the frequency of the fundamental mode
- Add a new element: RF-Multipole
  - Modify several modules: dict, node, …
- Export to SixTrack
  - Modify module: 6track

Once these points achieved, we can start exporting to SixTrack

Impact on the code (physics)

- Add a new element: RF-Multipole
  - – Modify several modules: twiss, tracking, makethin, …
  - ➢ Check closed orbit calculation (4d/6d?)

- Must check out the code already existing for `CRABCAVITY`:
  - – It is implemented in tracking
  - – It is not implemented in twiss
  - – It is exported to SixTrack
  - ➢ What to do: We see two options:
    - Leave it as it is
    - Convert it to a RF-Multipole if needed

Export to SixTrack

- We will adapt our export format to suit any SixTrack implementation of the RF-Multipole

- Preliminary discussions with RT, JB and FS indicate that a staged approach will be chosen (mainly because of limited resources):
  - RFQUAD, RFSEXTUPOLE, RFOCTUPOLE
  - Followed by RFMULTIPOLE at a later moment

Conclusions

Proposed steps:

- Add a new element RF-Multipole
- Modify EFCOMP
- Develop the interface toward SixTrack.
- Twiss and tracking implementations could happen at a later time

Appendix - Multipole Coefficients

Multipole expansion

$$B_y(x,y) + iB_x(x,y) = \sum_{n=1}^{N} (B_n(r_0) + iA_n(r_0)) \cdot \frac{(x+iy)^{n-1}}{r_0^{n-1}} \quad [\mathrm{T}]$$

$$\begin{aligned} r_0 &: \text{ reference radius [m]} \\ x, y &: \text{ particle coordinates w.r.t. reference particle [m]} \\ B_n(r_0) &: \text{ normal multipole component at } r_0 \text{ [T]} \\ A_n(r_0) &: \text{ skew multipole component at } r_0 \text{ [T]} \end{aligned}$$

Mad-X integrated strength

$$\mathtt{KNL[n]} := l \cdot \frac{1}{B\rho} \frac{\partial^n B_y}{\partial x^n} = l \cdot \frac{1}{B\rho} \frac{B_{n+1}(r_0)}{r_0^n} n!$$

$$\mathtt{KSL[n]} := l \cdot \frac{1}{B\rho} \frac{\partial^n B_x}{\partial x^n} = l \cdot \frac{1}{B\rho} \frac{A_{n+1}(r_0)}{r_0^n} n!$$

$$\begin{aligned} \mathtt{KNL[n]} &: \text{ integrated nominal multipole strength of order } n \text{ } [\mathrm{m}^{-n}] \\ \mathtt{KSL[n]} &: \text{ integrated skew multipole strength of order } n \text{ } [\mathrm{m}^{-n}] \\ l &: \text{ length of the element [m]} \\ B\rho &: \text{ magnetic rigidity [T m]} \end{aligned}$$

Appendix - Field Errors

Absolute errors in Mad-X

$$\texttt{DKN} \equiv \texttt{KNL} \quad [\mathrm{m}^{-n}]$$

$$\texttt{DKS} \equiv \texttt{KSL} \quad [\mathrm{m}^{-n}]$$

Relative error for the multipole strength of order *n* in Mad-X

$$\texttt{KNL}_{[\texttt{n}]} = \texttt{DKNR}_{[\texttt{n}]} \cdot \texttt{KNL}_{[\texttt{ORDER}]} \cdot r_0^{(n-N)}$$

$$\texttt{KSL}_{[\texttt{n}]} = \texttt{DKSR}_{[\texttt{n}]} \cdot \texttt{KSL}_{[\texttt{ORDER}]} \cdot r_0^{(n-N)}$$

$\texttt{DKNLR}_{[\texttt{n}]}$: relative error for normal component of order n [real]

$\texttt{DKSLR}_{[\texttt{n}]}$: relative error for skew component of order n [real]

Where the relative errors are a scaling factor for b_n (and a_n), relative to $B_N(r_0)$

$$B_n(r) = B_N(r_0)\, b_n(r)$$

$$b_n(r) = b_n(r_0)\left(\frac{r}{r_0}\right)^{n-N}$$

r_0 : reference $\texttt{RADIUS}$ [m]

N : $\texttt{ORDER}$ [$\texttt{integer}$]

r : arbitrary radius [m]

$B_N(r_0)$: main field at radius r_0 [T]

$B_n(r)$: normal multipole component at radius r [T]

$b_n(r)$: normal relative multipole coefficient related to the main field $B_N(r_0)$ [real]

Appendix - Multipole Expansion

Given

$$\tilde{\mathbf{E}} = \mathbf{E}(x, y, z)\, e^{-j(\omega t+\phi)},$$
$$\tilde{\mathbf{H}} = \mathbf{H}(x, y, z)\, e^{-j(\omega t+\phi)}$$

- The magnetic strength [A/m] is converted into magnetic induction, B [T]

$$\mathbf{B} = \mu_0 \mathbf{H}$$

- The electric field and the magnetic strength are added, using the Lorentz force

$$\mathbf{E}_{\text{equiv.}} = \mathbf{E} + \mathbf{v} \wedge \mathbf{B}$$

that is,

$$\mathbf{E}_{\text{equiv.}} = \mathbf{E} + \det \begin{pmatrix} \hat{\mathbf{e}}_x & \hat{\mathbf{e}}_y & \hat{\mathbf{e}}_z \\ 0 & 0 & c \\ B_x & B_y & B_z \end{pmatrix} = \begin{pmatrix} E_x - cB_y \\ E_y + cB_x \\ E_z \end{pmatrix},$$

where $\mathbf{v} = (0, 0, c)$. Note that $B_z = 0$ if the excited mode is TM.

Then the resulting field E can be expanded in either ways:

$$E_y + iE_x = \sum_{n=1}^{N} C_n\,(x+iy)^{n-1}$$

$$E_z = \sum_{n=0}^{N} D_n\,(x+iy)^{n}$$