(SLOW) HEAD-TAIL INSTABILITY IN THE PS ON THE LONG (1.2 s) INJECTION FLAT-BOTTOM

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Work presented at PAC07

⟹ **Comparison between measurements, theory and HEADTAIL simulations**

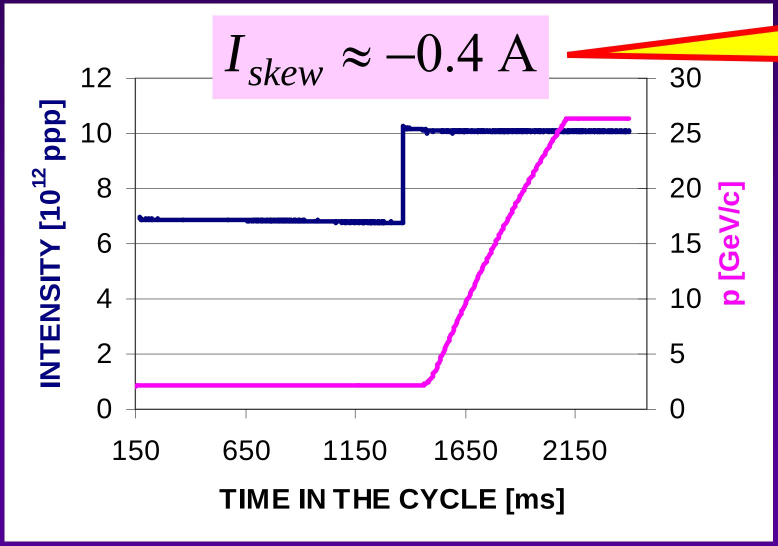
- **Effect of chromaticity**
- **Effect of linear coupling between the transverse planes**

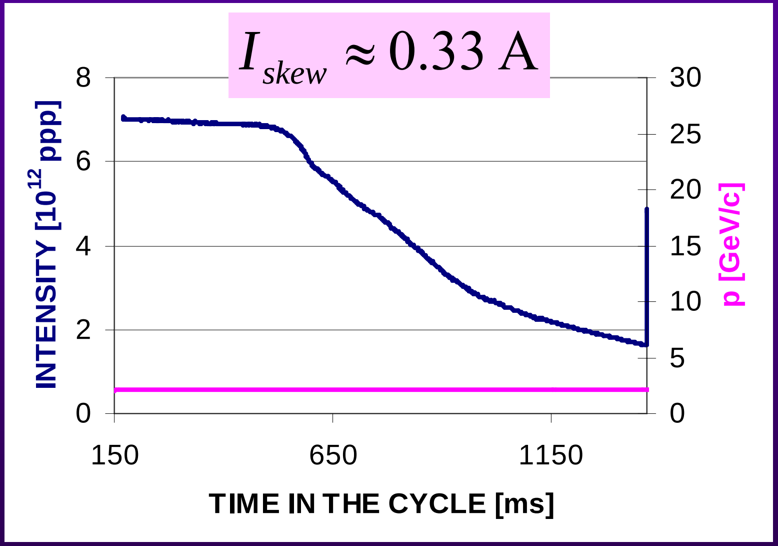
MEASUREMENTS (1/2)

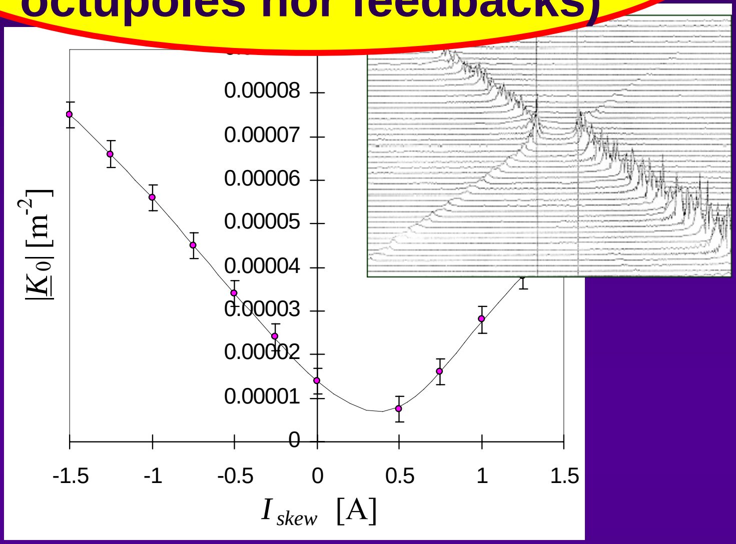
Stabilisation by linear coupling only (i.e. with neither octupoles nor feedbacks)

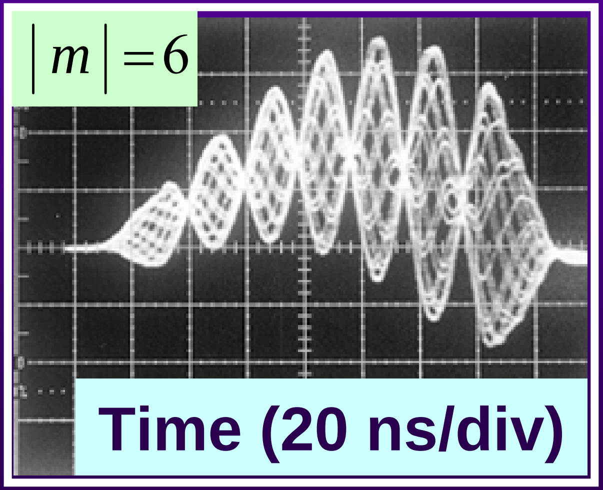
$Q_x = 6.22$

$Q_y = 6.25$









MEASUREMENTS (2/2)

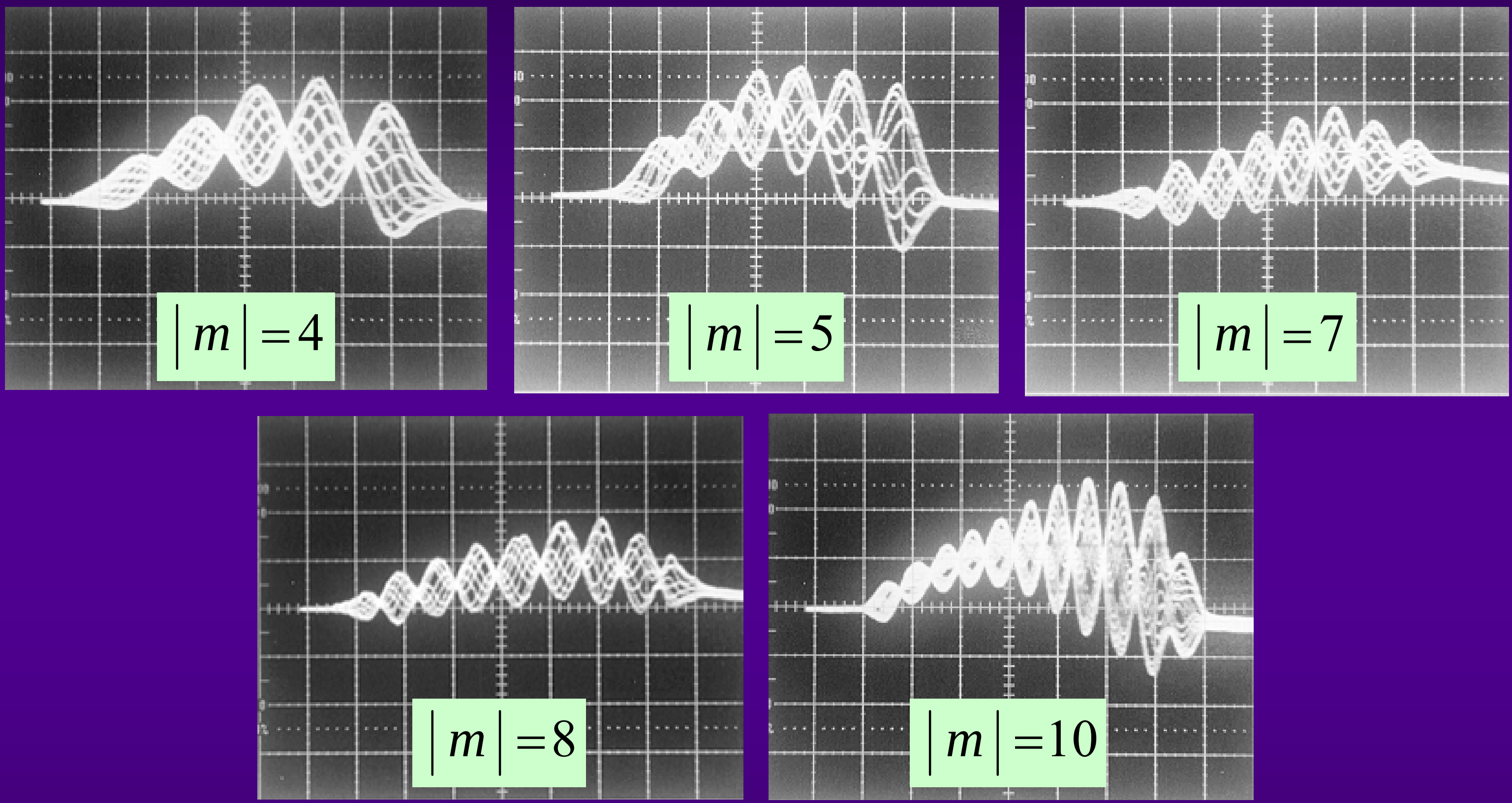


Figure 4: Measured ΔR signals from a radial beam-position monitor during 20 consecutive turns, in the PS with minimum coupling [5]: (a) $\xi_x \approx -0.5$, (b) $\xi_x \approx -0.7$, (c) $\xi_x \approx -1.1$, (d) $\xi_x \approx -1.2$, (e) $\xi_x \approx -1.3$. Time scale: 20 ns/div.

HEADTAIL SIMULATIONS (1/7)

Table 1: Basic beam and PS parameters relevant for this simulation study.

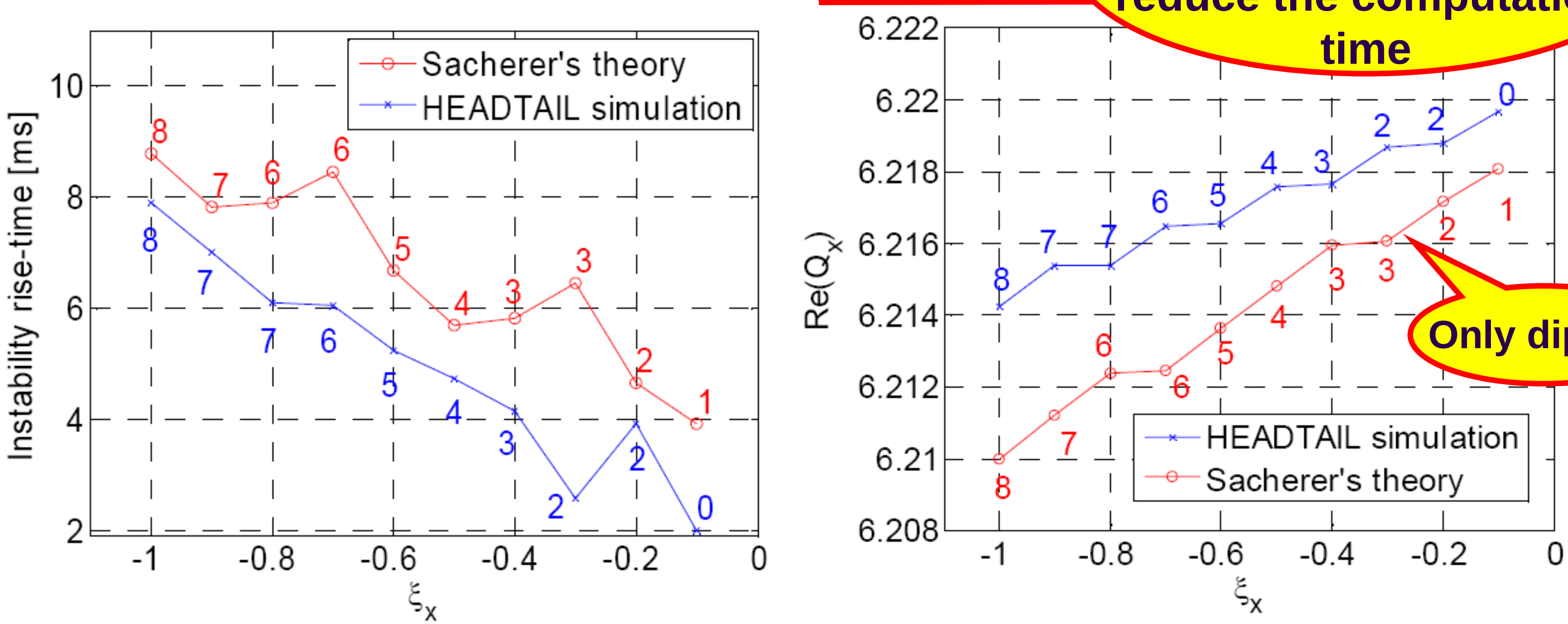
Parameter	Value	Unit
Circumference	628	m
# of bunches	1	
Relativistic γ	2.49	
# of protons / bunch	1.6×10^{12}	
Horiz. tune	6.22	
Vert. tune	6.25	
Horiz. / vert. relative chromaticities	{-1,-0.1} / {-1,-0.1}	
Rms bunch length	12.8	m
Rms long. mom. spread	0.001	
Synchrotron tune	0.00124	
Cavity harmonic number	7	
Mom. compaction factor	0.027	
Beam pipe ½ axes (H,V)	(70,35)	mm
Beam pipe resistivity	10^{-6}	Ωm

} $\Longrightarrow$ **RW impedance**

HEADTAIL SIMULATIONS (2/7)

Effect of chromaticity

Impedance ~ 16 times larger than RW to reduce the computation time



Only dipolar

Figure 5: Comparison of the instability rise-times (left) and real part of the tunes (right) with the associated mode number $|m|$ between Sacherer's theory (using a parabolic bunch) and HEADTAIL simulations vs. chromaticity.

HEADTAIL SIMULATIONS (3/7)

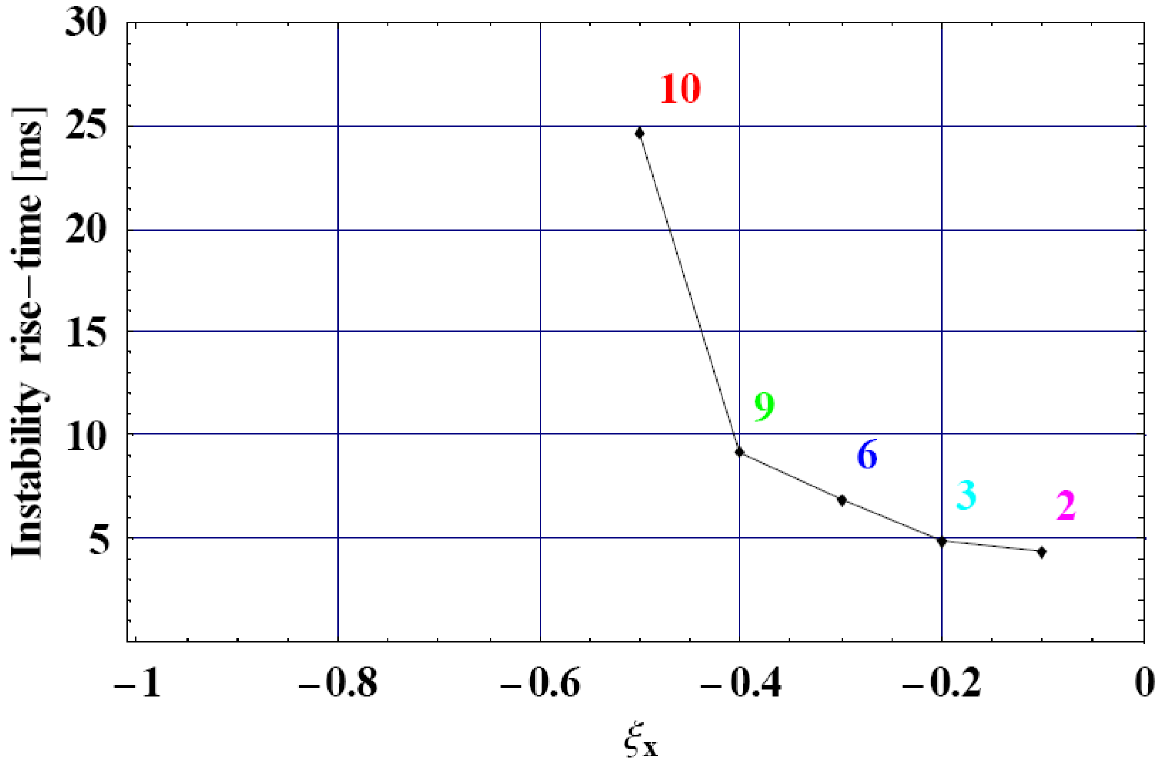


Figure 6: Instability rise-times (with the associated mode number $|m|$) vs. chromaticity predicted from Sacherer's formula using a Gaussian bunch.

HEADTAIL SIMULATIONS (4/7)

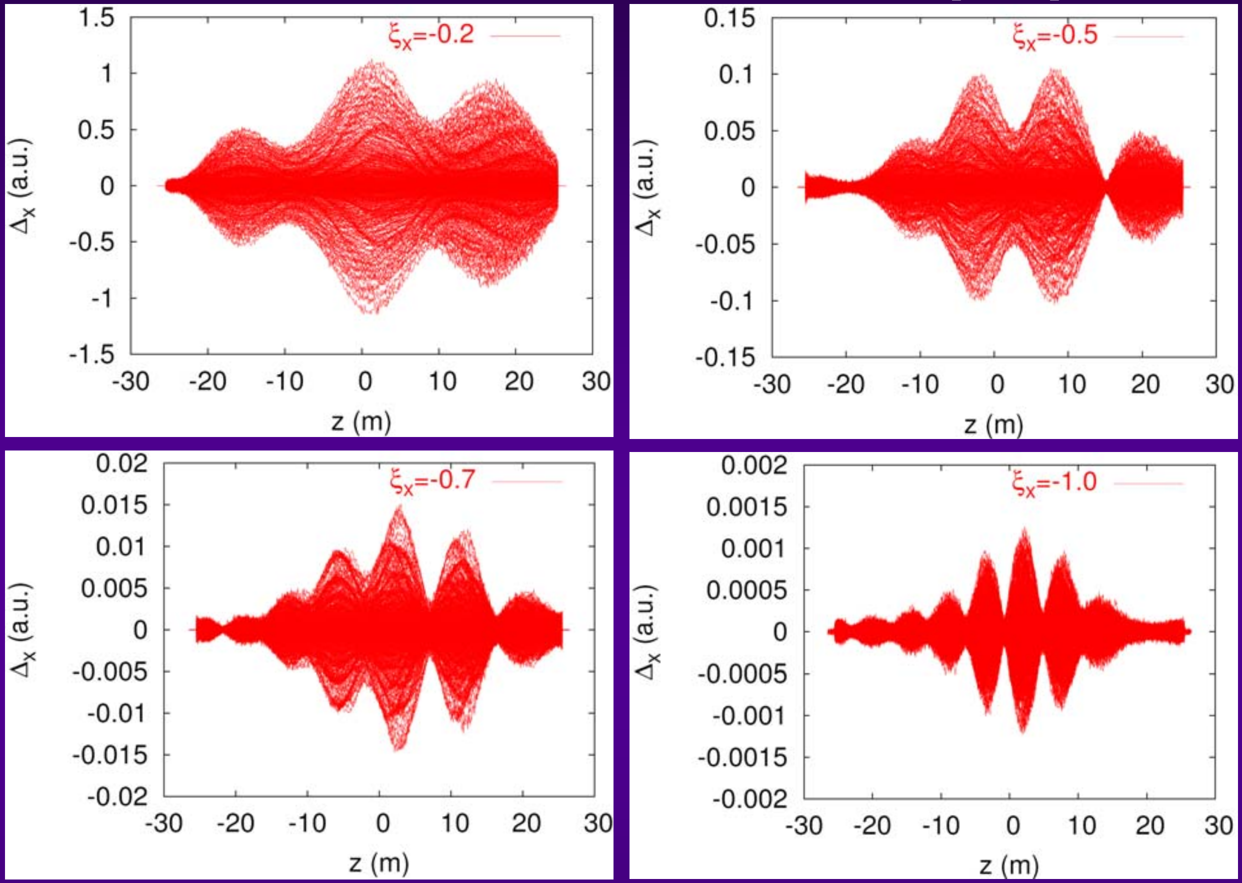


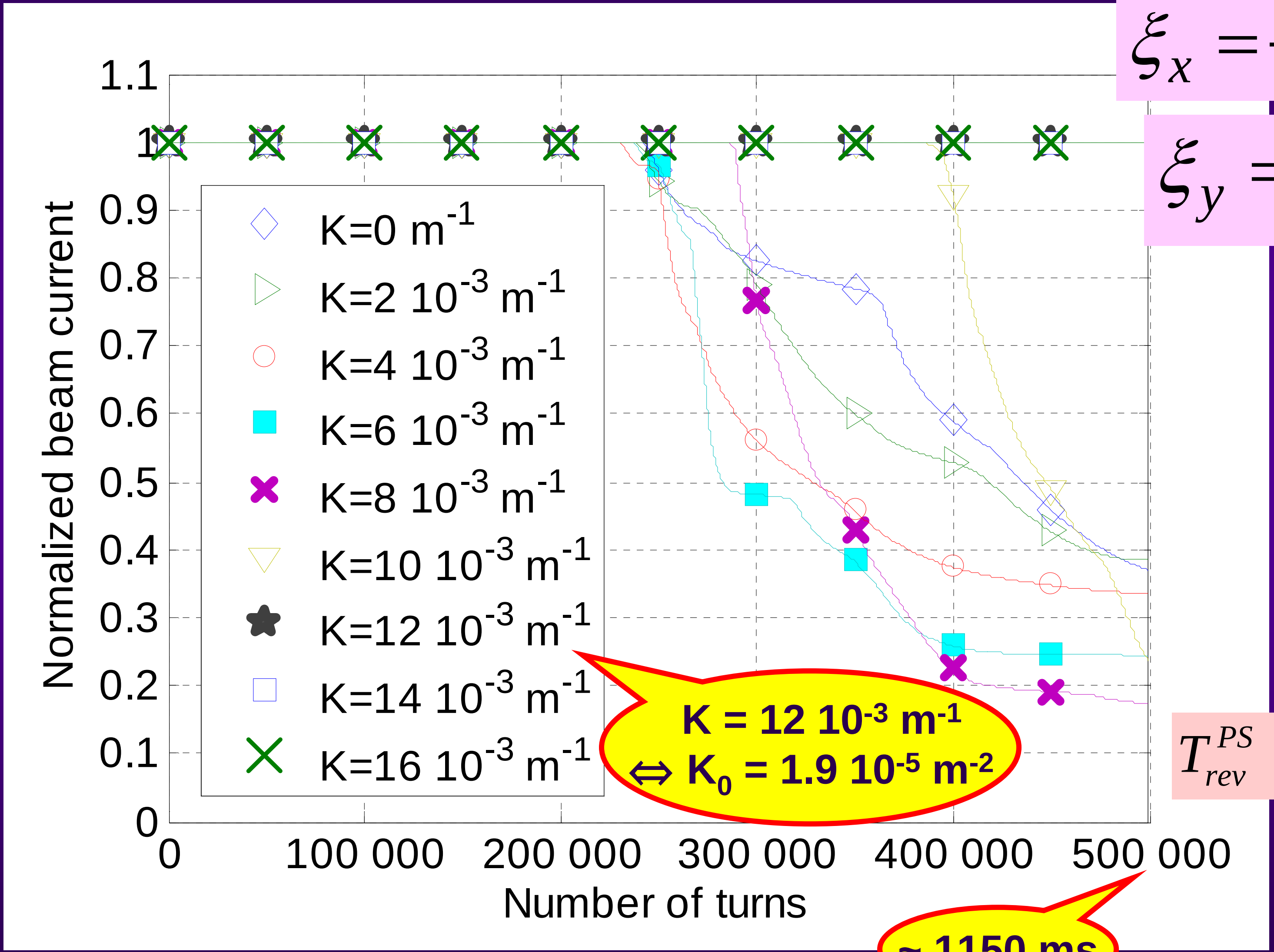
Figure 7: Examples of head-tail modes observed in the HEADTAIL simulations, when superimposing every 10 turns of the 20000 simulated turns at a beam position monitor, for various horizontal chromaticities.

HEADTAIL SIMULATIONS (5/7)

Effect of linear coupling between the transverse planes

$$\xi_x = -0.5$$

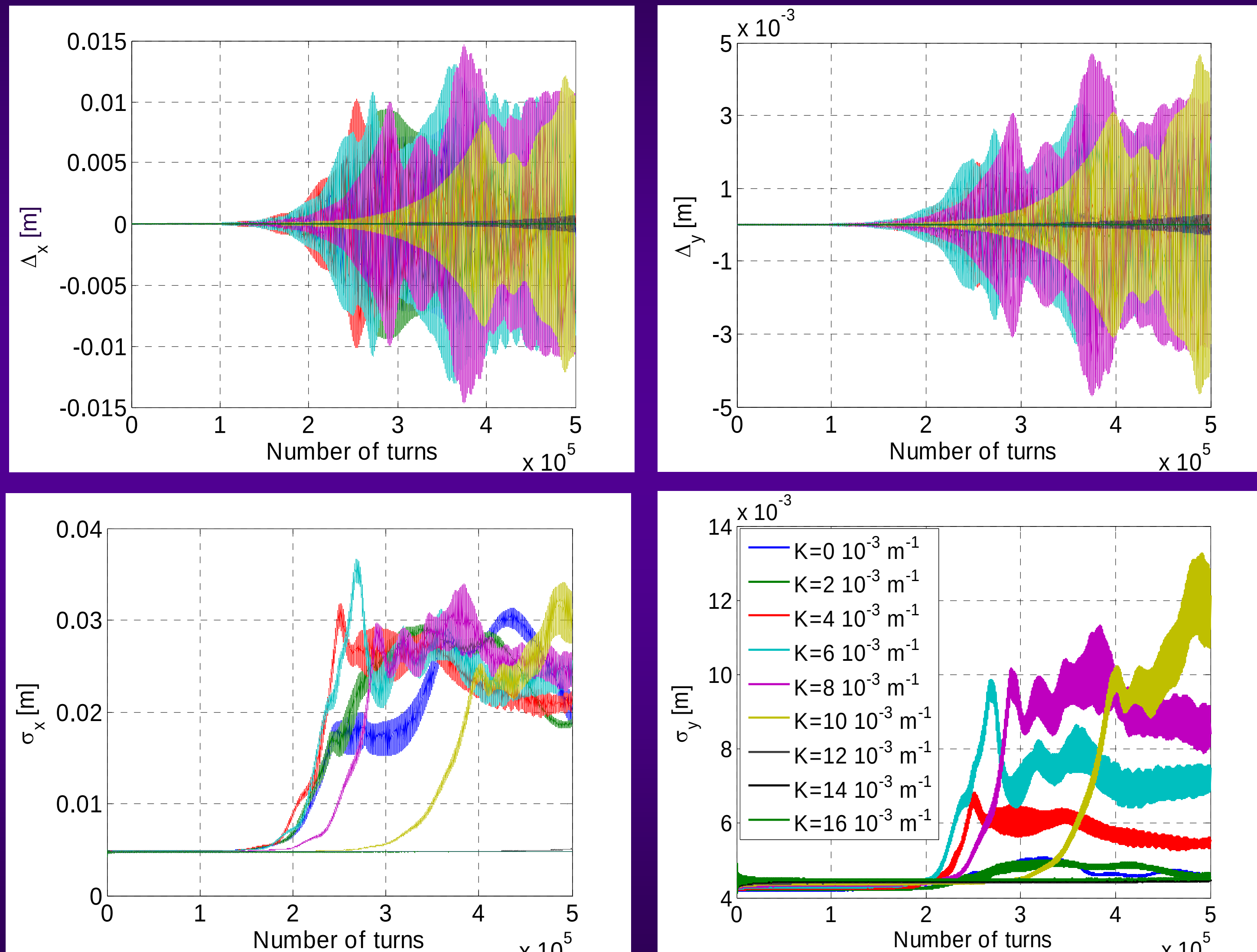
$$\xi_y = -1$$



$$T_{rev}^{PS} \approx 2.3\ \mu s$$

~ 1150 ms

HEADTAIL SIMULATIONS (6/7)



HEADTAIL SIMULATIONS (7/7)

Comparison with (some) theory

Sacherer's formula $\Delta\omega_{m,m}^{x,y} = U_{x,y}^{m} - j\,V_{x,y}^{m}$

Necessary condition for stability $V_x^m + V_y^m \le 0$

Stability criterion

$$\left|\underline{K}_0(l)\right| \ge \frac{2\left[-Q_{x0}\,Q_{y0}\,V_x^m\,V_y^m\right]^{1/2}}{R^2\,\Omega_0} \times \frac{\left[\left(V_x^m + V_y^m\right)^2 + \Omega_0^2\left(Q_h - Q_v - l\right)^2\right]^{1/2}}{-\left(V_x^m + V_y^m\right)}$$

Due to ξ_x = - 0.5

$V_x^4 = 11.2\ \mathrm{s}^{-1}$

$V_y^4 = -41.4\ \mathrm{s}^{-1}$

$\Longrightarrow$ $\left|\underline{K}_0\right|_{\text{limit}}^{\text{theory}} = 2.7\ \ 10^{-5}\ \ \mathrm{m}^{-2}$

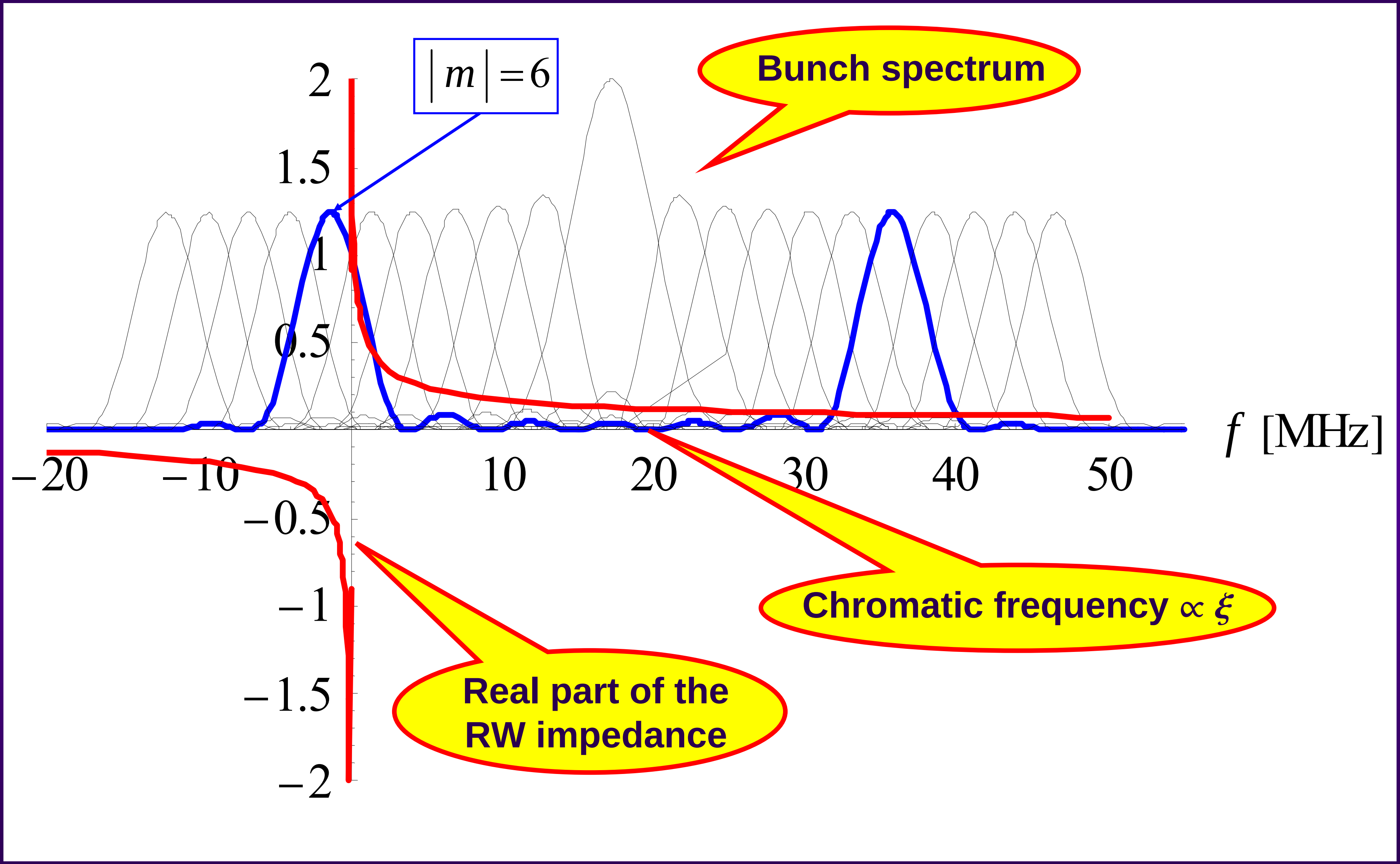
To be compared to 1.9 10^{-5} m^{-2} found from HEADTAIL (over ~ 1.1 s)

CONCLUSIONS AND OUTLOOK

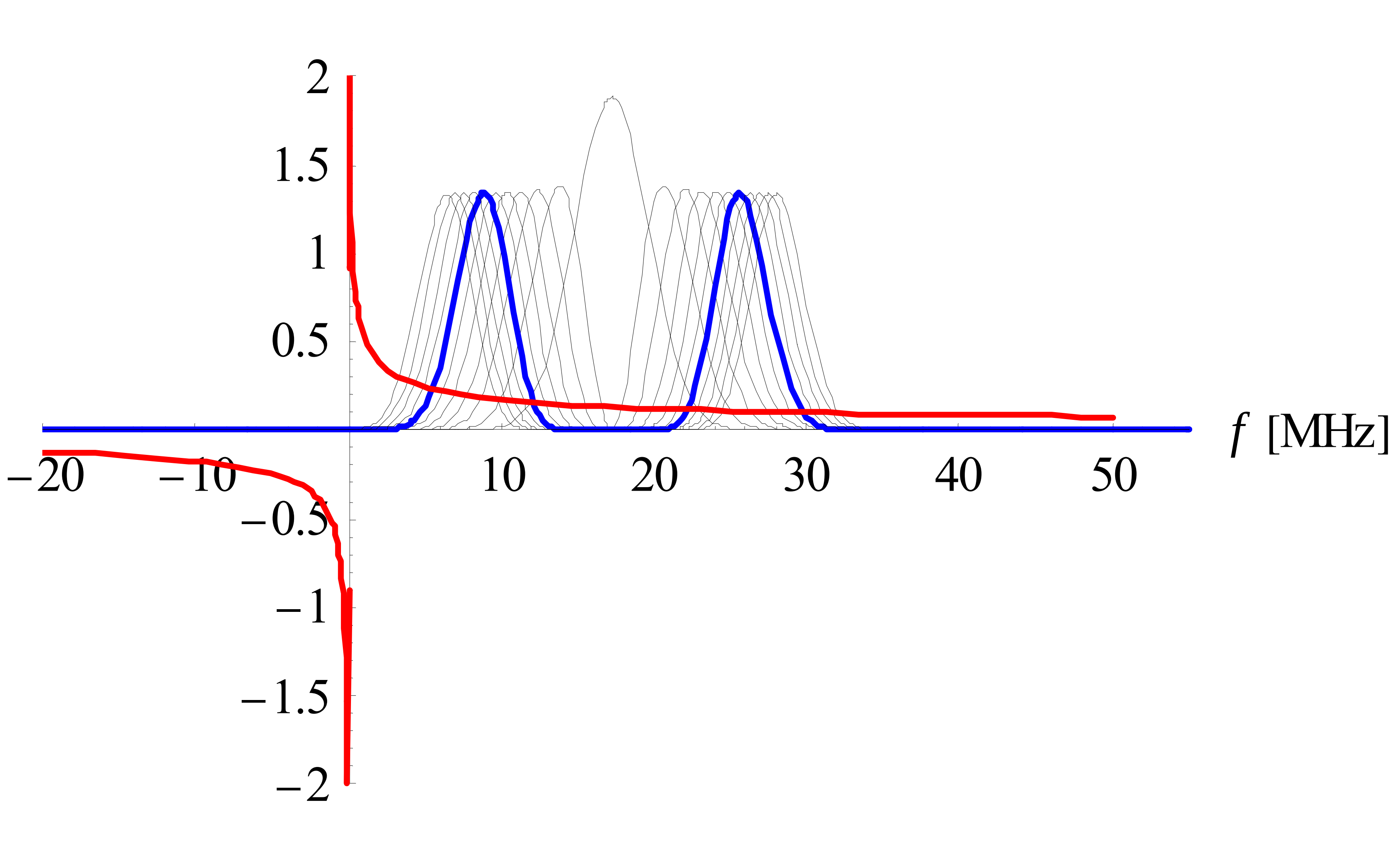
- **The HEADTAIL simulation code has been benchmarked against Sacherer's formula in the case of the CERN PS low-energy horizontal resistive-wall instability (artificially increasing the impedance and therefore decreasing the simulation time by ~16) for various chromaticities**
- **A good agreement was revealed when a parabolic bunch was used for the analytical computations, whereas a poor agreement was obtained for the higher-order head-tail modes using a Gaussian bunch (as already known $\Longrightarrow$ Use parabolic bunches for protons!)**
- **Full-scale HEADTAIL simulations during ~ 1.1 s also revealed the possibility to stabilise the beam by linear coupling (only) when an asymmetry between the two transverse planes is introduced through chromaticities, as predicted theoretically**
- **The simulated case used ξ_x = - 0.5 and ξ_y = - 1.0, whereas the measured chromaticities were (in the past) ξ_x = - 0.9 and ξ_y = -1.3**

$\Longrightarrow$ The next steps will consist to re-measure precisely the chromaticities and simulate cases even closer to reality, introducing also space charge, better model of the impedance…

APPENDIX A: Parabolic bunch



APPENDIX B: Gaussian bunch



APPENDIX C: Studies in 1999