PS BOOSTER

Too many correctors are removed. SVD conditioning is too strong.

A0=

0.94	-0.38	-4.85	1.03	4.71	-1.67	-4.49	2.27	4.18	-2.84	-3.8	3.35
4.74	4.9	-0.38	-4.85	1.03	4.71	-1.67	-4.49	2.27	4.18	-2.84	-3.8
3.35	-0.28	4.9	-0.38	-4.85	1.03	4.71	-1.67	-4.49	2.27	4.18	-2.84
-3.8	-4.86	-0.28	4.9	-0.38	-4.85	1.03	4.71	-1.67	-4.49	2.27	4.18
-2.84	0.94	-4.86	-0.28	4.9	-0.38	-4.85	1.03	4.71	-1.67	-4.49	2.27
4.18	4.74	0.94	-4.86	-0.28	4.9	-0.38	-4.85	1.03	4.71	-1.67	-4.49
2.27	3.35	4.74	0.94	-4.86	-0.28	4.9	-0.38	-4.85	1.03	4.71	-1.67
-4.49	-3.8	3.35	4.74	0.94	-4.86	-0.28	4.9	-0.38	-4.85	1.03	4.71
4.71	4.18	-2.84	-3.8	3.35	4.74	0.94	-4.86	-0.28	4.9	-0.38	-4.85
1.03	2.27	4.18	-2.84	-3.8	3.35	4.74	0.94	-4.86	-0.28	4.9	-0.38
-4.85	-4.49	2.27	4.18	-2.84	-3.8	3.35	4.74	0.94	-4.86	-0.28	4.9
-0.38	-1.67	-4.49	2.27	4.18	-2.84	-3.8	3.35	4.74	0.94	-4.86	-0.28
4.9	4.71	-1.67	-4.49	2.27	4.18	-2.84	-3.8	3.35	4.74	0.94	-4.86
-0.28	1.03	4.71	-1.67	-4.49	2.27	4.18	-2.84	-3.8	3.35	4.74	0.94
-4.86	-4.85	1.03	4.71	-1.67	-4.49	2.27	4.18	-2.84	-3.8	3.35	4.74

12 Correctors and 15 monitors

PS BOOSTER

Too many correctors are removed. SVD conditioning is too strong.

```
SVD = SingularValueDecomposition[A0];
  U = SVD[[1]];
  V = SVD[[3]];
 VT = Transpose[V];
  S = SVD[[2]]; MatrixForm[S]
```

35.445	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	28.4225	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	6.81932	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	6.00346	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	5.17475	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	4.31844	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	3.29707	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	3.27137	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	3.01503	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	2.69539	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	2.21137	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.00211682
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

The SVD conditioning removes all correctors except four, corresponding to the four largest singular values. This is build into MADX

Proof of Singular Value Decomposition

$A = U.S.V^T$, where V is an orthonormal matrix of row vectors (**Det**[V]=1, if quadratic),
U is an orthonormal matrix of column vectors,
S is the matrix with singular values

Proof:

$A=U.S.V^T$ and $A^T=V.S.U^T$

$A^T.A = V.S.U^T.U.S.V^T$

$A.A^T= U.S.V^T.V.S.U^T$

Because U is an orthonormal matrix, then $U^T=U^{-1}$ and $U^T.U = I$

$A^T.A = V.S^2.V^T$

Because V is an orthonormal matrix, then $V^T=V^{-1}$ and $V^T.V = I$

$A.A^T= U.S^2.U^T$

$A^T.A = V.S^2.V^{-1}$

$A.A^T= U.S^2.U^{-1}$

The equations: $A^T.A = V.S^2.V^{-1}$ and $A.A^T= U.S^2.U^{-1}$ is the definition is eigenvalues.

A matrix $M = A^T.A$ or $M = A.A^T$ is by definition symmetric and all symmetric matric always have eigen values.

$A=U.S.V^T$; (* The original matrix is regenerated *)

A - A0 (= 0) (* The original matrix and the regenerated matrices are equal *)

Why afraid of singular values close to zero?

M={{1,2,3},{3,4,5},{6,7,8.1}};MatrixForm[M]

1	2	3
3	4	5
6	7	8.1

SVD=SingularValueDecomposition[M];
U=SVD[[1]];
V=SVD[[3]];MatrixForm[V];
VT=Transpose[V];
S=SVD[[2]];MatrixForm[S] (* Singular matrix *)

4.6146	0.	0.
.	1.01088	0.
.	0.	0.0135376

Solve[{1,2,0}==M.{x1,x2,x3},{x1,x2,x3}]
{{ x1= -35, x2 = 70.5, x3 = -35 }}

We are very afraid of singular values close to zero, because they make the correctors work against each other!!!

MR={{1,2},{3,4.}};
Chop[Solve[{1,2}==MR.{x1,x2},{x1,x2}]]
{{ x1 = 0, x2 = 0.5 }}

By removing a corrector (and a monitor), we get small values for the currents of the correctors

SVD is a decomposition, like Fourier transforms

```
S1=S/.{S[[2,2]]=0,S[[3,3]]=0};
S2=S/.{S[[1,1]]=0,S[[3,3]]=0};
S3=S/.{S[[1,1]]=0,S[[2,2]]=0};
M1=U.S1.VT;  M2=U.S2.VT;   M3=U.S3.VT;
M = M1 + M2 + M3;  MatrixForm[M]
```

1	2	3
3	4	5
6	7	8.1

```
SVD1=Outer[Times,U[[{1,2,3},1]],VT[[1]]];  (* Outer product of vectors *)
SVD2=Outer[Times,U[[{1,2,3},2]],VT[[2]]];
SVD3=Outer[Times,U[[{1,2,3},3]],VT[[3]]];
M=S[[1,1]] * SVD1  +  S[[2,2]] * SVD2  +  S[[3,3]] * SVD3;   MatrixForm[M]
```

1	2	3
3	4	5
6	7	8.1

NB! M1 == SVD1, M2 == SVD2, M3 == SVD3

SVD does not order the singular values intrinsically, but is done to make the SVD unique !

```
UO=U;UO[[1,1]]=U[[1,3]];UO[[2,1]]=U[[2,3]];UO[[3,1]]=U[[3,3]];
UO[[1,3]]=U[[1,1]];UO[[2,3]]=U[[2,1]];UO[[3,3]]=U[[3,1]];
```

$$UO = \begin{pmatrix} 0.47004 & -0.846727 & -0.249232 \\ -0.818008 & -0.311823 & -0.483353 \\ 0.331551 & 0.431068 & -0.839198 \end{pmatrix} \quad U = \begin{pmatrix} -0.249232 & -0.846727 & 0.47004 \\ -0.483353 & -0.311823 & -0.818008 \\ -0.839198 & 0.431068 & 0.331551 \end{pmatrix}$$

```
VTO={VT[[3]],VT[[2]],VT[[1]]};
```

$$VTO = \begin{pmatrix} 0.393379 & -0.819277 & 0.417179 \\ 0.795558 & 0.0759062 & -0.601103 \\ -0.460803 & -0.568352 & -0.681642 \end{pmatrix} \quad VT = \begin{pmatrix} -0.460803 & -0.568352 & -0.681642 \\ 0.795558 & 0.0759062 & -0.601103 \\ 0.393379 & -0.819277 & 0.417179 \end{pmatrix}$$

```
SO=S;SO[[1,1]]=S[[3,3]];SO[[3,3]]=S[[1,1]];MatrixForm[SO]
```

$$\begin{pmatrix} 0.0135376 & 0. & 0. \\ 0. & 1.01088 & 0. \\ 0. & 0. & 14.6146 \end{pmatrix}$$

```
M=UO.SO.VTO;MatrixForm[M]
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8.1 \end{pmatrix}$$