Nonultrarelativistic resistive wall wake fields

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Many thanks to...
G. Rumolo, E. Metral, B. Salvant, B. Zotter..

Outline

The ideas...1/2

Ring	**Momentum**	β	γ
PSB @ Injection	0.31 GeV/c	$\sim$ 0.31	$\sim$ 1.05
PSB @ Extraction	1.4 GeV/c	$\sim$ 0.84	$\sim$ 1.79
PS @ Extraction	14 GeV/c	$\sim$ 0.9977	$\sim$ 14.954

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Nonultrarelativistic regime

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Nonultrarelativistic regime

$\Rightarrow$ different approach should be considered

The ideas...2/2

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$\beta \simeq 1$ $\qquad$ $\beta < 1$

Outline

Longitudinal wake field 1/7

Definition of the wake field

$$W_0'(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z_{\parallel}(\omega) e^{i\omega z/\beta c} d\omega \qquad (1)$$

non ultrarelativistic beam F. Zimmermann and K. Oide, PRLSTAB **7**, 044201 (2004)

$$Z_{\parallel}(\omega) = i\frac{Z_0 c k_r^2}{2\pi\omega}\left[K_0(k_r r) + I_0(k_r r)\,\frac{\omega^2\lambda K_1(bk_r)K_0(b\lambda) + k_r c^2\left(\lambda^2 - k^2\right)K_0(bk_r)K_1(b\lambda)}{\omega^2\lambda I_1(bk_r)K_0(b\lambda) - k_r c^2\left(\lambda^2 - k^2\right)I_0(bk_r)K_1(b\lambda)}\right] \qquad (2)$$

$k_r = |\omega|/\gamma\beta c$, $k = \omega/\beta c$, $\lambda^2 = -i\mu_0\sigma\omega + k_r^2$ and $b = \text{beam pipe radius.}$

Longitudinal wake field 2/7

Subtracting the space charge term from Eq. (2) and letting

$$\begin{cases} |\lambda b| \gg 1 & \rightarrow \quad \text{PSB case} \quad \sqrt{\omega\mu_0\sigma} \simeq 2\cdot 10^{-2}[s^{1/2}]\sqrt{\omega} \\ |k_r b| \gg 1 & \rightarrow \quad \text{PSB case} \quad \omega b/\beta\gamma c \simeq 5.3\times 10^{-10}[s]\omega \end{cases} \tag{3}$$

we have

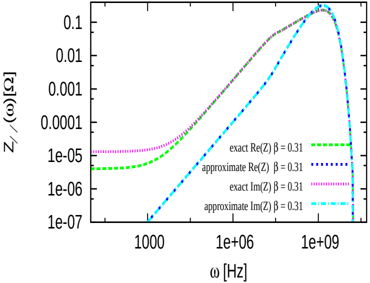
$$Z_{\parallel}(\omega) = i\frac{Z_0\omega}{c\beta^2\gamma^2} I_0(r|\omega|/\gamma\beta c)e^{-2b|\omega|/\gamma\beta c}\ \cdot$$

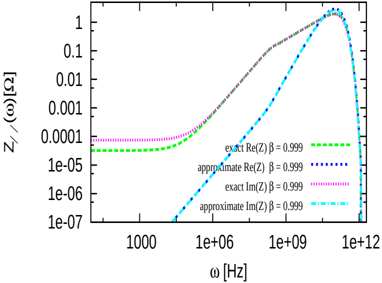
$$\cdot\left[\frac{\sqrt{|\omega|\sigma\mu_0}\,(1-i\cdot\mathrm{sgn}\,(\omega))}{\sqrt{|\omega|\sigma\mu_0}\,(1-i\cdot\mathrm{sgn}\,(\omega))+\sqrt{2}\cdot\mathrm{sgn}\,(\omega)\,c/\beta\gamma\,(i\mu_0\sigma+\omega/c^2)}\right]. \tag{4}$$

range of validity ω such that $|\omega b/\beta\gamma c| \gg 1$: short distances.

Longitudinal wake field 3/7

log – log plot of $Z_{||}(\omega)$ function

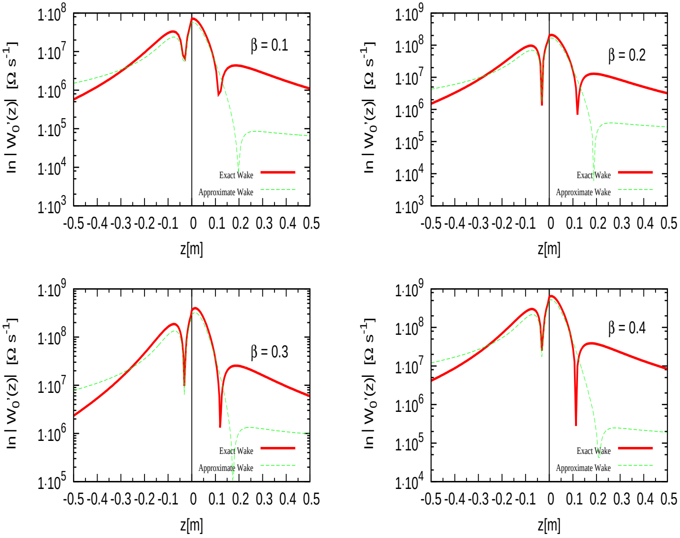




$\omega_{sup} \nearrow$ as $\beta \to 1$.

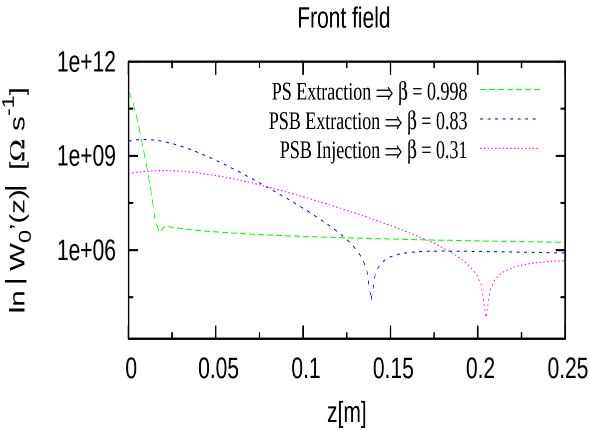
Longitudinal wake field 4/7

FFT of impedance Eq. (2) (Exact) and Eq. (4)(Approximate)



Longitudinal wake field 5/7

Field in front of the bunch, FFT of exact formula...



Longitudinal wake field 5/7

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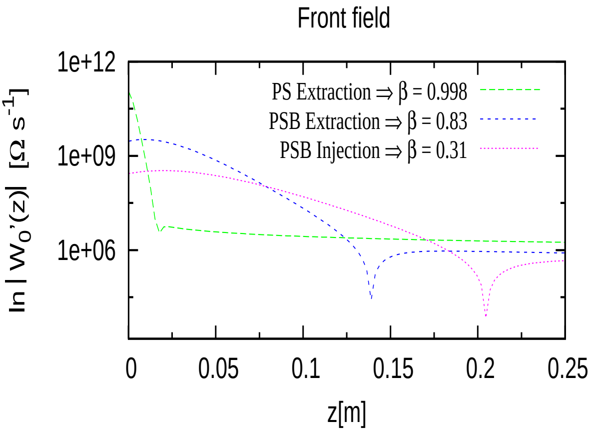
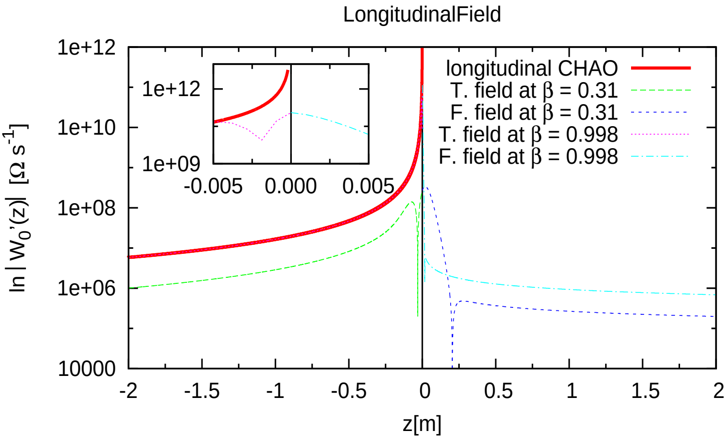


..for the Head and the Tail of the bunch...

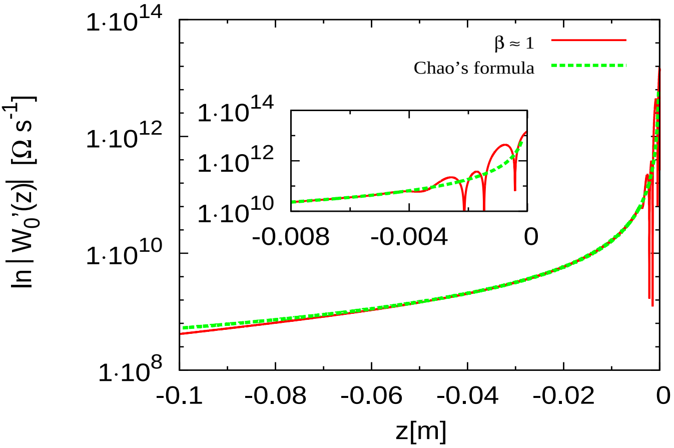
Longitudinal wake field 6/7



Agreement with well-known Chao's ultrarelativistic formula

Longitudinal wake field 7/7

For the ultrarelativistic case...



we have a more complete description of the field close to the bunch.

Outline

Strong head-tail instability 1/3

TRANSVERSE plane simple model

Strong head-tail instability 1/3

TRANSVERSE plane simple model $\longrightarrow$ two particles beam

A. Chao, *Physics of Collective Instabilities in High Energy Accelerators*

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$\vec{x} = (x_1, x_1', x_2, x_2') \Rightarrow \dot{\vec{x}} = \vec{\Phi}(\vec{x})$

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$$\vec{\Phi} = \begin{pmatrix} x_1' \\ -(\omega_\beta/c)^2 x_1 + \alpha_1 x_2 \\ x_2' \\ -(\omega_\beta/c)^2 x_2 + \alpha_2 x_1 \end{pmatrix}, \quad \alpha_j = \frac{N r_0 W_j}{2\gamma C} \quad j = 1, 2.$$

Strong head-tail instability 2/3

Basin of stability obtained from 4-th order Runge-Kutta integration of Eq. (5).

$$\Gamma = \frac{\pi N r_0 W_1 c^2}{4\gamma C \omega_s \omega_\beta}$$

Strong head-tail instability 2/3

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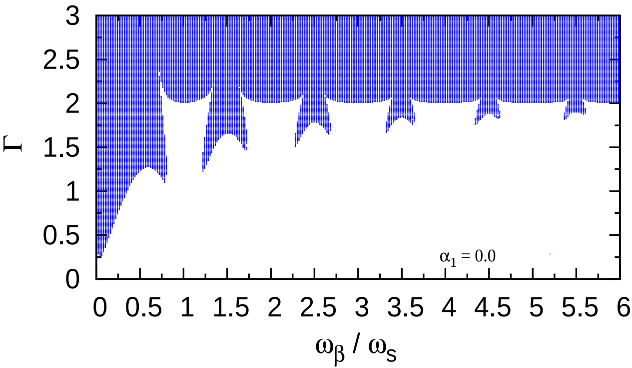
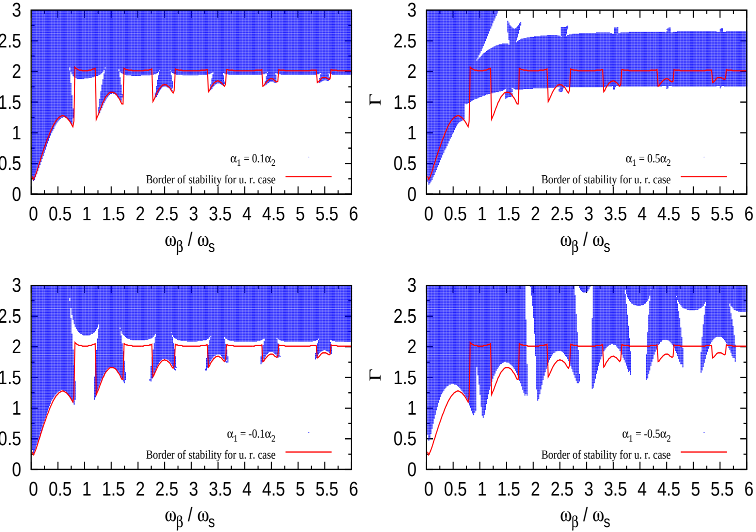


Dotted region $\Rightarrow$ unstable system.

Strong head-tail instability 3/3



Outline

Motivations and concepts

The fields and the their calculation

Possible effects on bunch dynamics

Conclusions

- Extent of the wake field concept for nonultrarelativistic case

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- Numerical evidence of front field for nonultrarelativistic bunches
- For the longitudinal plane the resistive wall front field seems to be relevant
- Work in the transverse plane still ongoing
  However preliminary simulations based on two particle model seem to show a different behaviour concerning the beam stability