LONGITUDINAL AND TRANSVERSE MICROWAVE INSTABILITIES

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Coasting-beam formalism with peak values

- **Longitudinal**
  - **Rise-time** (neglecting Landau damping)
  - **Intensity threshold** (from Landau damping)
- **Transverse**
  - **Rise-time** (neglecting Landau damping)
  - **Intensity threshold** (from Landau damping)

Longitudinal microwave instability

- **Rise-time**

$$\tau_{\mu W}^{L} = -\frac{1}{p_r^{BB}\,\omega_0} \times \mathrm{Im}\left[\sqrt{-\frac{2\pi\,\beta^2\,E/e}{\eta\,I_p\;j\,Z_l\left(p_r^{BB}\right)/\;p_r^{BB}}}\right]$$

$$p_r^{BB} \approx -\,\omega_r^{BB}/\,\omega_0$$

$$Z_l = Z_l^{SC} + Z_l^{BB}$$

- **Intensity threshold**

$$\Lambda \times \mathrm{Re}\left[\frac{Z_l(p)}{p}\right] = \mathrm{Im}\left[J^{-1}\right] \qquad \Lambda \times \mathrm{Im}\left[\frac{Z_l(p)}{p}\right] = -\mathrm{Re}\left[J^{-1}\right]$$

$$\Lambda = \frac{3\,I_p}{2\,\pi\,\eta\left(E/e\right)\beta^2\left(\frac{\Delta p}{p_0}\right)^2_{\mathrm{FWHH}}}$$

$$J = \int_{x=-1}^{x=1} \frac{x\,dx}{x + \frac{\sqrt{2}\,\omega_{c,p}^{l}}{p\,\omega_0\left|\eta\right|\left(\frac{\Delta p}{p_0}\right)_{\mathrm{FWHH}}}}$$

- **Keil-Schnell approximation**

$$\left|Z_l/p\right| \leq \frac{(E/e)\,\beta^2\left|\eta\right|}{I_p} \times \left(\frac{\Delta p}{p_0}\right)^2_{\mathrm{FWHH}}$$

Transverse microwave instability

- **Rise-time**

$$\tau_{\mu W}^{V} = - \frac{4 \pi Q_y \left(E / e \right)}{I_p \, c \, \mathrm{Re} \left\{ Z_y \left[\left(p_r^{BB} + Q_y \right) \omega_0 \right] \right\}}$$

- **Intensity threshold**

$$\Lambda \times \mathrm{Re} \left[Z_y \left(\omega \right) \right] = \mathrm{Im} \left[J^{-1} \right] \qquad \Lambda \times \mathrm{Im} \left[Z_y \left(\omega \right) \right] = - \mathrm{Re} \left[J^{-1} \right]$$

$$\Lambda = \frac{3 \sqrt{2} \, I_p \, c}{16 \pi Q_y \left(E / e \right) \omega_r^{BB} \, \eta \left(\frac{\Delta p}{p_0} \right)_{\mathrm{FWHH}}} \qquad J = \int_{x=-1}^{x=1} \frac{\left(1 - x^2 \right) dx}{x + \frac{\sqrt{2} \, \omega_{c,p}^{l}}{p \, \omega_0 \left| \eta \right| \left(\frac{\Delta p}{p_0} \right)_{\mathrm{FWHH}}}}$$

- **Keil-Zotter etc. criterion**

$$N_b^{th} \approx \frac{32 \sqrt{2}}{3} \times \frac{Q_y \left| \eta \right| \varepsilon_l}{e \, \beta^2 \, c} \times \frac{f_r}{\left| Z_y^{BB} \right|} \times \left(1 + \frac{f_{\xi_y}}{f_r} \right)$$