TMCI FAR ABOVE THE INTENSITY THRESHOLD

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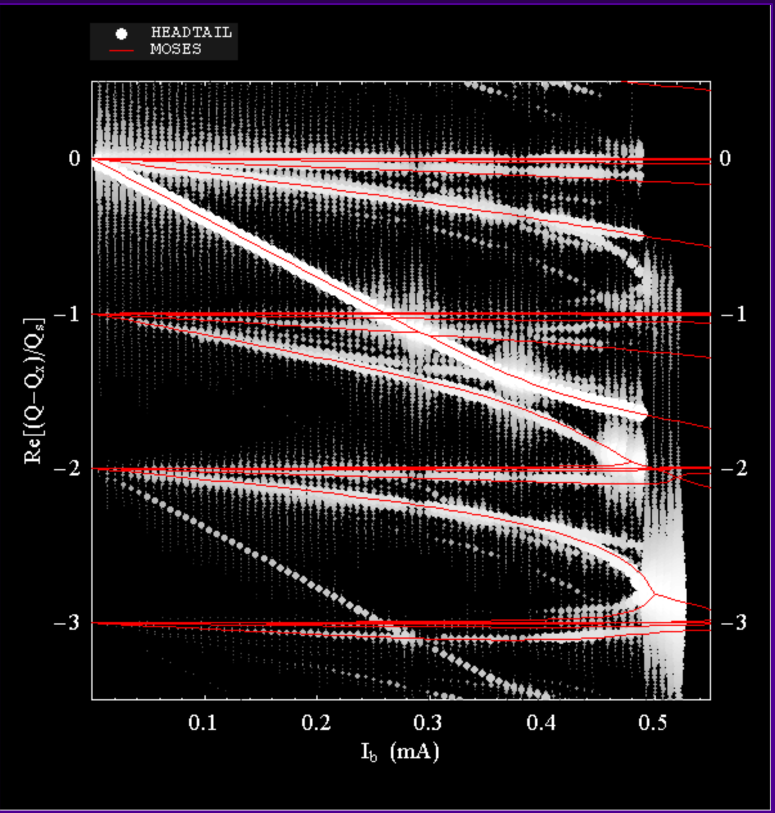
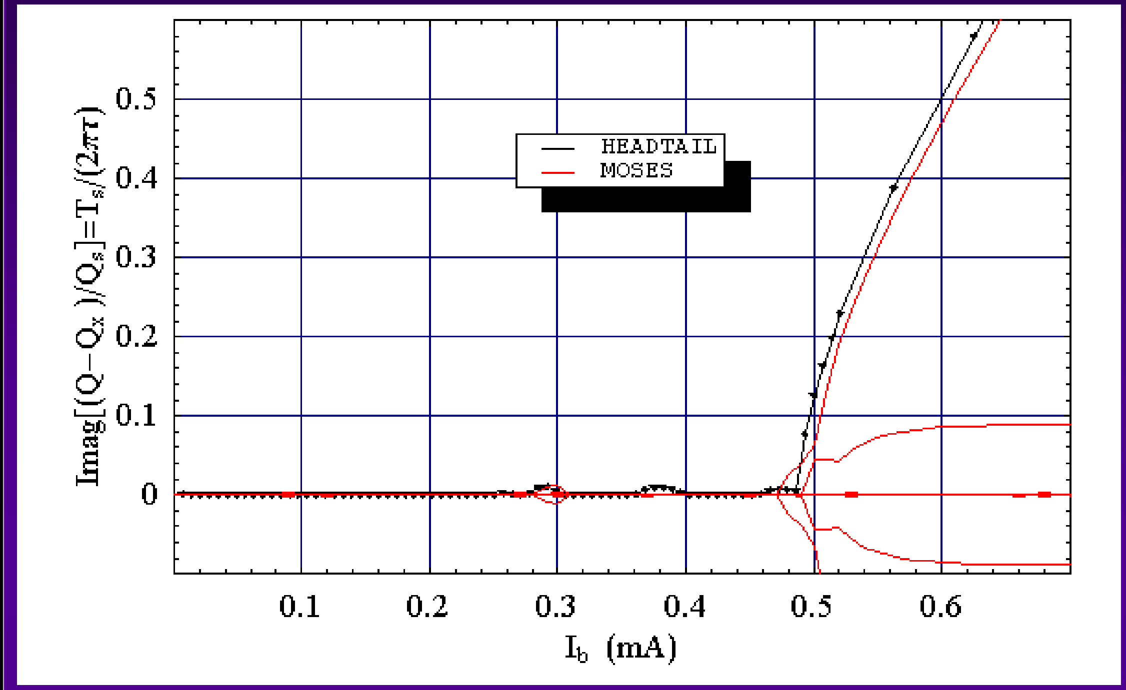
⟹ Discussion with W. Fisher and G. Rumolo at the CARE-HHH workshop (24-25/11/08, Chavannes-de-Bogis)

REMINDER (Benoit's result): HEADTAIL VS. MOSES





- **This instability is therefore clearly a TMCI!**
- **Question from Wolfram Fisher: What happens when intensity much larger than intensity threshold (i.e. instability rise-time much faster than the synchrotron period), i.e. can we still use the concept of modes etc.?**

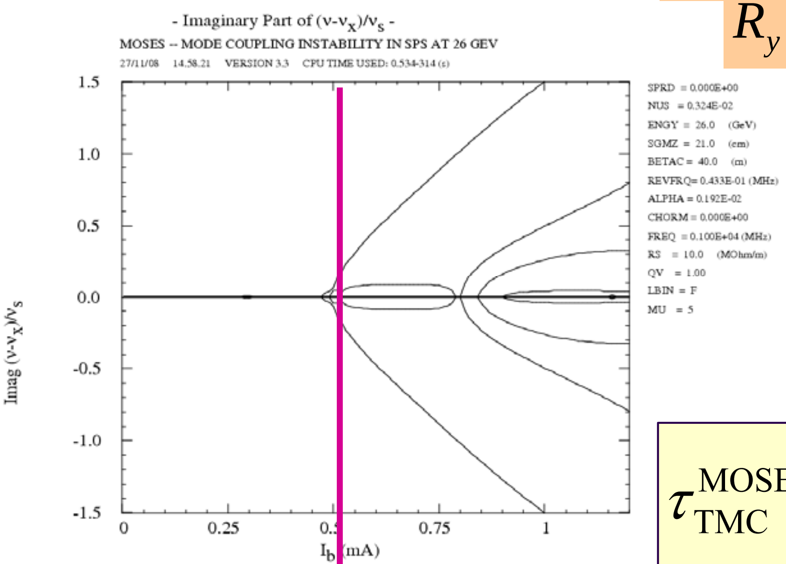
BB resonator impedance

$R_y = 10\ \mathrm{M\Omega/m}$

$f_r = 1\ \mathrm{GHz}$

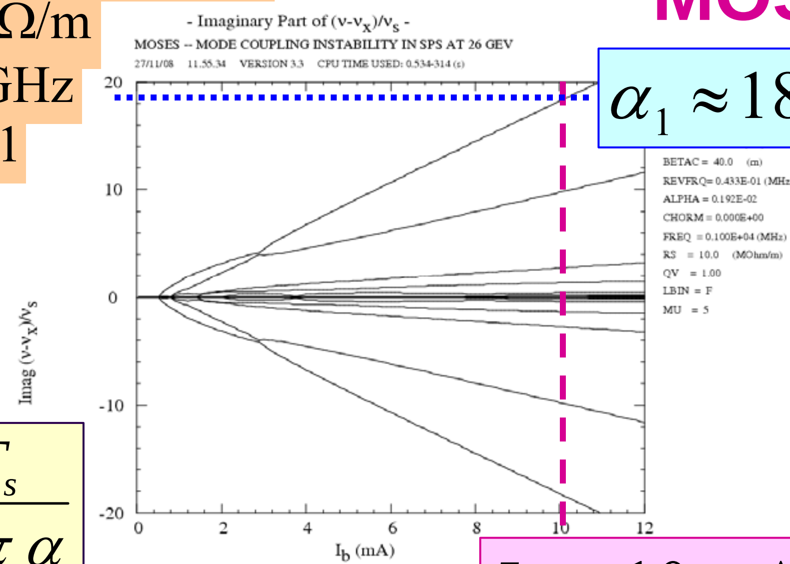
$Q = 1$

MOSES



$I_b^{th} \approx 0.5\ \mathrm{mA}$

$$\tau_{\mathrm{TMC}}^{\mathrm{MOSES}} = \frac{T_s}{2\,\pi\,\alpha}$$

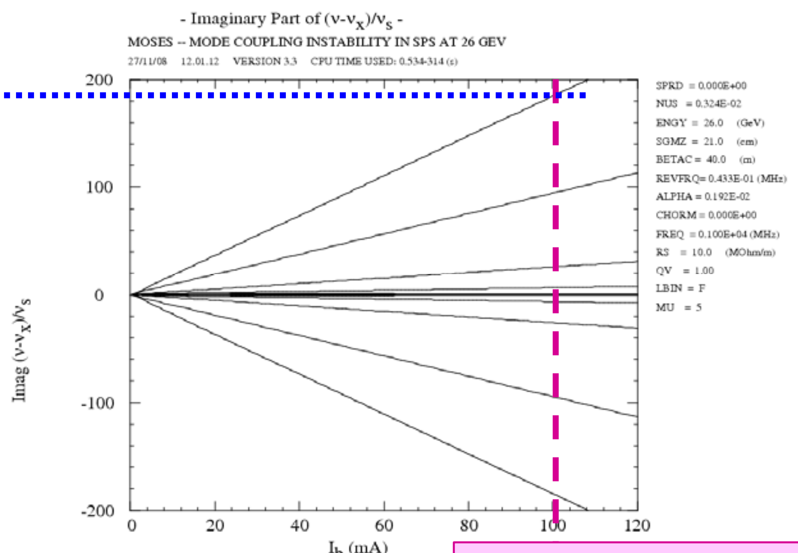


$\alpha_1 \approx 18.5$

$I_{b1} = 10\ \mathrm{mA}$

$$\frac{I_{b2}}{I_{b1}} = 10$$

$\alpha_2 \approx 185$



$$\frac{\alpha_2}{\alpha_1} \approx \frac{185}{18.5} = 10$$

$I_{b2} = 100\ \mathrm{mA}$

$$\Longrightarrow \quad \tau_{\mathrm{TMC}}^{\mathrm{MOSES}} \propto \frac{T_s}{2\,\pi\,I_b}$$

SIMPLE TMC MODEL WITH 2 MOST CRITICAL MODES (1/2)

$$\tau_{\text{TMC}}^{sm} = \frac{T_s}{\pi \sqrt{\left(\frac{I_b}{I_b^{th}} - 1 \right) \left(\frac{I_b}{I_b^{th}} \, q + 1 \right)}}$$

with $q \in [\,0\,,1\,]$

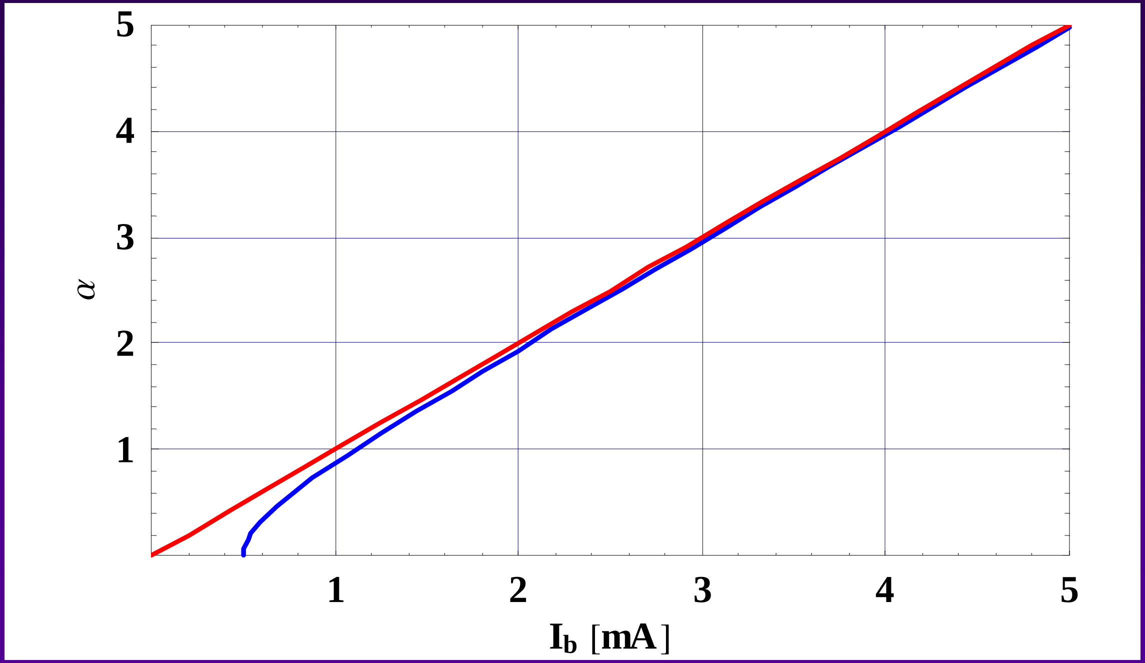
$$q = 0 \text{ for short bunch, i.e. } 2\, f_r\, \tau_b \approx 1$$

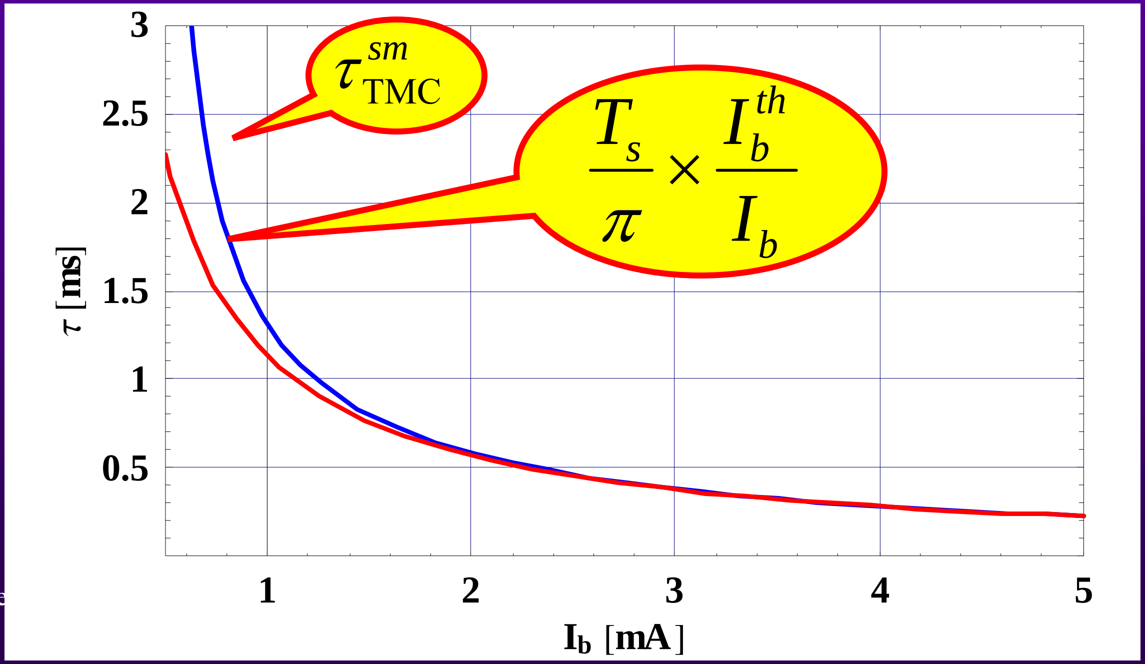
$$q = 1 \text{ for long bunch, i.e. } 2\, f_r\, \tau_b >> 1$$

$I_b >> I_b^{th}$ and long bunch $\Longrightarrow$ $\tau_{\text{TMC}}^{sm} = \frac{T_s}{\pi} \times \frac{I_b^{th}}{I_b}$

Furthermore $I_b^{th} \propto \frac{1}{T_s}$ $\Longrightarrow$ τ_{TMC}^{sm} **is independent of synchrotron motion as could be anticipated (as the instability rise-time is much faster than synchrotron period)**

SIMPLE TMC MODEL WITH 2 MOST CRITICAL MODES (2/2)





SOME NUMERICAL VALUES

$$I_b = 100\ \text{mA}$$

$$T_s = 7.1\ \text{ms}$$

- **From MOSES**

$$\alpha = 185 \Rightarrow \tau_{\text{TMC}}^{\text{MOSES}} = \frac{T_s}{2\,\pi\,\alpha} = \frac{0.0071}{2\,\pi\,185} = 6.1\ \mu\text{s}$$

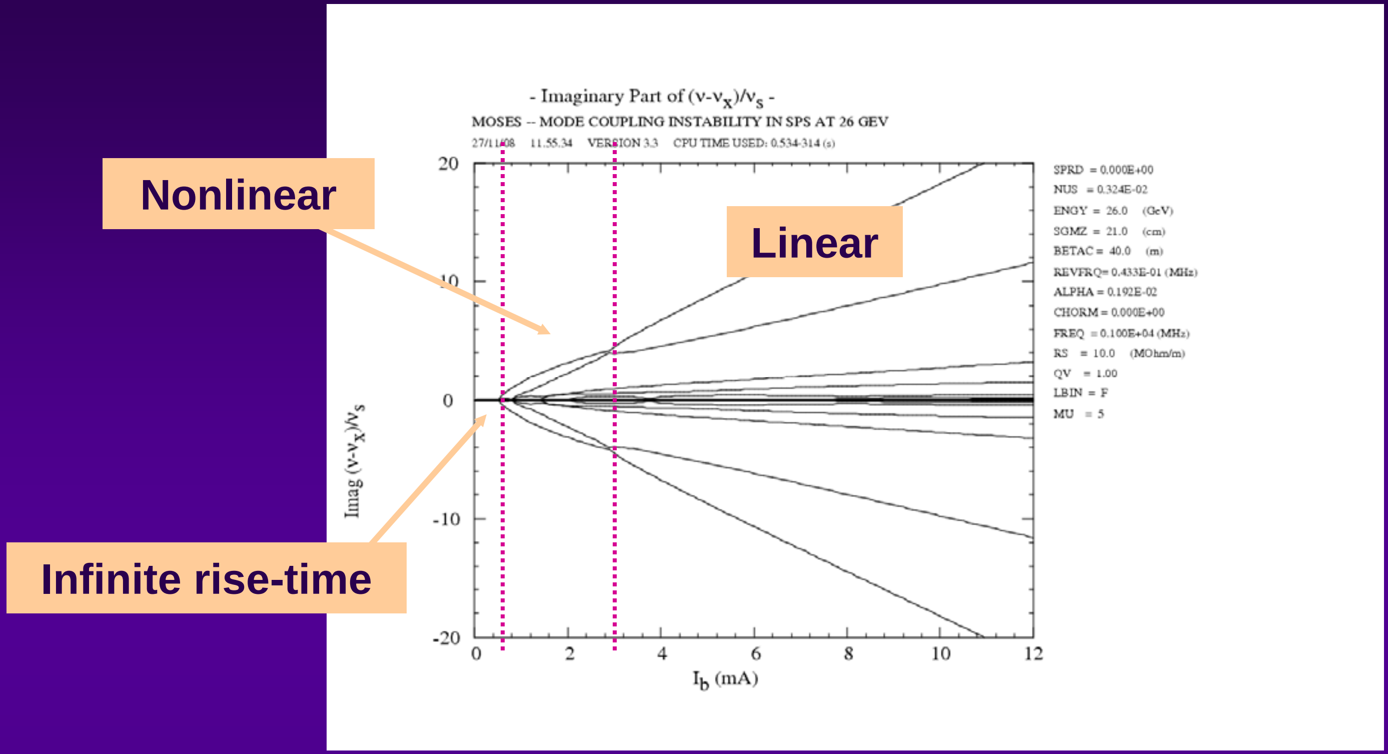
- **From simple TMC model**

$$I_b^{th} = 0.5\ \text{mA} \Rightarrow \tau_{\text{TMC}}^{sm} = \frac{T_s}{\pi} \times \frac{I_b^{th}}{I_b} = \frac{0.0071}{\pi} \times \frac{0.5}{100} = 11.3\ \mu\text{s}$$

- **From coasting-beam formalism with peak values**

$$\tau_{\text{CB}}^{\text{peak}} = \frac{4}{3} \times \frac{\left(E_t / e\right)\,\tau_b}{I_b \left(\dfrac{R}{Q_y}\right) R_y} = 23\ \mu\text{s}$$

ANOTHER POSSIBLE PROOF OF TMCI



- **Measure the instability rise-time vs. intensity above the intensity threshold**
  - **$\Longrightarrow$ 2 regimes:**
    - **Nonlinear ("just" above)**
    - **Then, linear (at much higher intensities)**
  - **Problem: Very difficult to measure!**

CONCLUSION

- **When the intensity is much larger than the TMCI intensity threshold, a simple formula can be used to have an estimate of the instability rise-time**

$$\tau_{\mathrm{TMC}}^{sm} = \frac{T_s}{\pi} \times \frac{I_b^{th}}{I_b}$$

$\Longrightarrow$ **To be checked in more detail**